Fama-French three-factor model

Koprčina, Maja

UNIVERSITY OF ZAGREB
FACULTY OF SCIENCE
DEPARTMENT OF MATHEMATICS

Maja Koprčina

FAMA-FRENCH THREE-FACTOR MODEL

Diploma Thesis

Adviser:
prof.dr.sc. Boris Podobnik

Zagreb, 2018

Ovaj diplomski rad obranjen je dana ____________________ pred ispitnim povjerenstvom u sastavu:

1. ____________________________, predsjednik
2. ____________________________, član
3. ____________________________, član

Povjerenstvo je rad ocijenilo ocjenom ________________.

Potpisi članova povjerenstva:

1. ____________________________
2. ____________________________
3. ____________________________

Acknowledgements

I would like to thank my supervisor Boris Podobnik for his support.

To my family.

Contents

List of Tables

1. *Introduction*

The three-factor model of Fama and French has been proven to be a good description of returns on portfolios formed on size and book-to-market equity in the U.S. stock market in addition to capturing much of the cross-sectional variation in average stock return. Fama and French managed to show that even though the model is statistically rejected it still has practical importance and as such, still is a good model. Most of the research regarding the Fama and French three-factor model (hereafter FF3FM) is done using the U.S. data seeing that the model requires specific data to be gathered which is not always easy or plausible.

The Croatian stock market, classified as an emerging capital market, has its own challenges such as thin trading, low liquidity, short history, limited access to data as well as a scarce published research on various topics including the FF3FM. When it comes to asset-pricing, one of the most commonly used asset-pricing models is the Capital Asset Pricing Model. The CAPM is often used to determine the cost of equity which is a crucial input in decision making regarding capital budgeting, in different valuation purposes and in making investment decisions.

The aim of this thesis is to investigate the performance of the FF3FM with regards to the Croatian stock market in addition to comparing the FF3FM to the widely used CAPM. The thesis will attempt to answer the following questions:

1. Does the FF3FM outperform the CAPM in explaining the excess returns in the Croatian stock market from July 2009 to April 2017?

2. Do the CAPM and the FF3FM suffice to explain the average excess stock returns in the Croatian stock market from July 2009 to April 2017?

3. To which extent does the use of different proxies for the market and the selection of the return interval influence the results?

4. Does the exclusion of the financial firms change the conclusions made?

By answering these questions, the thesis aims to provide new insights as well as offer guidance in the use of the asset pricing models for the Croatian stock market.

The structure of the thesis is the following: Chapter 2 provides the theoretical background. Chapter 3, previous research, briefly presents the research results relevant to the thesis. Chapter 4 demonstrates the methods used in conducting the research while chapter 5 provides an insight into the data. Chapter 6 presents the results and finally, chapter 7, offers the conclusions and potential improvements. The appendix briefly presents the data and the results of the research conducted in the same manner but after having excluded the financial firms from the sample.

2. *Theoretical background*

The chapter presents, in short, the theoretical background of the three-factor model. It states the Capital Asset Pricing Model (CAPM) and the Fama and French Three-Factor Model (FF3FM) with all their underlying assumptions.

2.1. *Modern Portfolio Theory*

Suppose there are n securities with the return and the expected return on the security i denoted by R_i and $E[R_i]$ (μ_i), respectively. The variance of the return on the i^{th} security denoted by $Var[R_i]$ ($\sigma_{ii} = \sigma_i^2$) and the covariance of returns between the i^{th} and j^{th} denoted by $Cov(R_i, R_j)$ (σ_{ij}).

Lets form a portfolio consisting of the n available securities. The return on the portoflio R_P is equal to $w_1R_1 + w_2R_2 + ... + w_nR_n$ where w_i is the weight on the security i such that $w_i \geq 0$ for each $i \in 1, 2, .., n$ and $\sum_{i=1}^{n} w_i = 1$. The expected return on the portfolio is then

$$\begin{aligned} E[R_p] &= E[w_1R_1 + w_2R_2 + ... + w_nR_n] \\ &= w_1E[R_1] + w_2E[R_2] + ... + w_nE[R_n] \\ &= \sum_{i=1}^{n} w_iE[R_i] \end{aligned}$$

that is $\mu_P = \sum_{i=1}^{n} w_i\mu_i$. So, the expected return on a portfolio is simply the weighted average of the expected returns of each of the securities in the portfolio. The variance of the portfolio is equal to

$$\begin{aligned} Var(R_p) &= E[(R_P - \mu_P)^2] \\ &= E[(w_1(R_1 - \mu_1) + w_2(R_2 - \mu_2) + ... + w_n(R_n - \mu_n))^2] \end{aligned}$$

Each of the n^2 terms is equal to

$$E[w_iw_j(R_i - \mu_i)(R_j - \mu_j)] = w_iw_jCov(R_i, R_j) = w_iw_j\sigma_{ij} \quad i, j = 1, 2, ..., n$$

when $i = j$

$$E[w_i w_j (R_i - \mu_i)(R_j - \mu_j)] = E[w_i w_i (R_i - \mu_i)(R_i - \mu_i)] = w_i^2 Var(R_i) = w_i^2 \sigma_i^2$$

So, the variance of the portfolio is given by

$$Var(R_p) = \sum_{i=1}^{n} w_i^2 Var(R_i) + \sum_{1 \leq i < j \geq n} w_i w_j Cov(R_i, R_j)$$

that is

$$\sigma_P^2 = \sum_{i=1}^{n} w_i^2 \sigma_i^2 + \sum_{1 \leq i < j \geq n} w_i w_j \sigma_{ij}$$

Covariances could be rewritten in terms of correlations for easier interpretation.

$$Cov(R_i, R_j) = \sigma_i \sigma_j \rho_{ij}$$

$$\sigma_P^2 = \sum_{i=1}^{n} w_i^2 \sigma_i^2 + \sum_{1 \leq i < j \geq n} w_i w_j \sigma_i \sigma_j \rho_{ij}$$

Portfolio variance is the weighted sum of all the variances and covariances. There are exactly n variances and $n^2 - n$ covariances in this sum. Thus, the covariances dominate the total variance of a portfolio. Positive covariances increase the variance of the portfolio whereas negative covariances decrease the variance of the portfolio (since for each i, $w_i \geq 0$, that is no short selling). Additionally, the weights assigned to each security influence the total portfolio variance as well as the expected return of the portfolio. Maximizing expected return for a given level of risk is at the core of the Markowitz modern portfolio theory.

Assumptions of the modern portfolio theory: (1) returns are normally distributed random variables (2) investors are rational and risk-averse (3) all available information is public (4) investors share beliefs on expected returns (5) there are no fees (6) individual investors are not influential enough to influence the price (7) borrowing and lending rates are equal (8) investors have unlimited access to capital.

Now, assume all n securities are risky, $\sigma_i^2 > 0$, i=1,2,..,n and that no security can be represented as a linear combination of the other securities. Then, the variance-covariance matrix of returns $\Sigma = [\sigma_{ij}]$ s nonsingular. In the mean-variance space, the feasible region is the set of all possible pairs of the form (σ_P, μ_P) for all possible combinations of individual security weights such that $\sum_{i=1}^{n} w_i = 1$. The set of all feasible portfolios which can be constructed from these n securities such that they have the smallest variance for a given level of expected return is the frontier. A frontier portfolio is a portfolio that for a given level of expected return has the minimum variance. For a given μ_P,

$$minimize \frac{1}{2} \sum_{i,j=1}^{n} w_i w_j \sigma_{ij}$$

subject to $\sum_{i=1}^{n} w_i \mu_i = \mu_P$ and $\sum_{i=1}^{n} w_i = 1$.

Definition. *Asset A mean-variance dominates asset B if $E[R_A] \geq E[R_B]$ and $Var(R_A) < Var(R_B)$ or $E[R_A] > E[R_B]$ and $Var(R_A) \leq Var(R_B)$.*

Definition. *Set of all non-dominated portfolios in the mean-variance space is called the efficient frontier.*

Thus, the efficient frontier consists of portfolios that satisfy the condition that no other portfolio exists with a higher expected return for a given level of risk, or a lower risk for a given level of expected return. Portfolios that do not lie on the efficient frontier can be strictly improved. Consequently, no rational mean-variance investor would choose to hold a portfolio not located on the efficient frontier.

Theorem (A Mutual Fund Theorem)**.** *Given n assets satisfying the above conditions, there are two portfolios ("mutual funds") constructed from these n assets, such that all risk-averse individuals, who choose their portfolios so as to maximize utility functions dependent only on the mean and variance of their portfolios, will be indifferent in choosing between portfolios from among the original n asset or from these two funds.*

Proof. For a proof, see [17]. □

So, as described in the theorem, in the presence of risky assets only, an optimal portfolio for any individual will be an efficient portfolio. Given a level of expected return every minimum-variance portfolio, that is, every efficient portfolio can be formed as a combination of any two efficient portfolios.

Suppose we add a riskless asset to are set of n securities. The return on the $(n+1)^{th}$ asset with a guaranteed return R_f. Also, since the $(n+1)^{th}$ asset is riskless $\sigma_{n+1} = 0$ and $\sigma_{i,n+1} = 0$ for i=1,2,...,n. The following theorem is a stronger version of the Mutual Fund Theorem.

Theorem. *Given n assets satisfying the above conditions and a riskless asset with return R_f, there exists a unique pair of efficient mutual funds, one containing only risky assets and the other only the riskless asset, such that all risk-averse individuals, who choose their portfolios so as to maximize utility functions dependent only on the mean and variance of their portfolios, will be indifferent in choosing between portfolios from among the original n+1 assets or from these two funds, if and only if $R_f < E$.*

Proof. For a proof, see [17]. □

A consequence of the theorem is that in the presence of a risk-free asset, the efficient frontier in the mean-variance model is a straight line.

Capital market line is the tangent line on the frontier curve drawn from the point of the risk-free asset, that is

$$E[R_p] = R_f + \sigma_p \frac{E[R_M] - R_f}{\sigma_M}$$

Theorem (The Capital Asset Pricing Model (CAPM)). *The expected return on any asset i, $E[R_i]$, satisfies*

$$E(R_i) = R_f + \beta_i [E(R_M) - R_f]$$

where

$$\beta_i = \frac{Cov(R_i, R_M)}{Var(R_M)}$$

Proof. Let R_i be the return on the asset under consideration. Furthermore, asset i is assumed to be inefficient, in the mean-standard deviation space it does not lie on the efficient frontier but it does lie in the feasible region. Lets consider the portfolio consisting of the asset i and the market portfolio

$$R_\alpha = \alpha R_i + (1 - \alpha) R_M \quad \text{where} \quad \alpha \in [0, 1]$$

The expected return on this portfolio is

$$E[R_\alpha] = \alpha E[R_i] + (1 - \alpha) E[R_M]$$

and the variance

$$\sigma_\alpha^2 = Var(R_\alpha) = \alpha^2 Var(R_i) + 2\alpha(1 - \alpha) Cov(R_i, R_M) + (1 - \alpha)^2 Var(R_M)$$

That is, we have

$$\mu(\alpha) = \alpha \mu_i + (1 - \alpha) \mu_M$$

$$\sigma^2(\alpha) = \alpha^2 \sigma_i^2 + 2\alpha(1 - \alpha)\sigma_{iM} + (1 - \alpha)^2 \sigma_M^2$$

The curve $\alpha \mapsto (\sigma(\alpha), \mu(\alpha)))$ is in the feasible region. For $\alpha = 0$, $(\sigma(0), \mu(0))) = (\sigma_M, \mu_M)$ and for $\alpha = 1$, $(\sigma(1), \mu(1))) = (\sigma_i, \mu_i)$. So the curve touches the capital market line at the market point (σ_M, μ_M) and is a tangent to the capital market line at for $\alpha = 0$. touches the capital market line at $(\sigma_M, E[R_M])$. Therefore, the derivative of the curve at $\alpha = 0$ is equal to the slope of the capital market line at the market point M.

$$\frac{d\mu(\alpha)}{d\sigma(\alpha)}_{\alpha=0} = \frac{\mu_M - R_f}{\sigma_M}$$

$$\frac{d\mu(\alpha)}{d\alpha} = \mu_i - \mu_M$$

$$\frac{d\sigma(\alpha)}{d\alpha} = \frac{1}{\sigma(\alpha)}[\alpha\sigma_i^2 + (1-2\alpha)\sigma_{iM}) + (\alpha - 1)\sigma_M^2]$$

$$\frac{d\mu(\alpha)}{d\sigma(\alpha)}_{\alpha=0} = \frac{\sigma(0)(\mu_i - \mu_M)}{\sigma_{iM} - \sigma_M^2}$$

We have,

$$\frac{\sigma(0)(\mu_i - \mu_M)}{\sigma_{iM} - \sigma_M^2} = \frac{\mu_M - R_f}{\sigma_M}$$

Solving for μ_i yields the CAPM formula for asset i,

$$\begin{aligned}\mu_i &= R_f + \frac{\sigma_{iM}}{\sigma_M^2}(\mu_M - R_f) \\ &= R_f + \beta_i(\mu_M - R_f)\end{aligned}$$

□

Beta contains information about the risk associated with the security's fluctuations with respect to the market portfolio. When $\beta_p = 1$ then $\mu_p = \mu_M$, that is the expected return on the portfolio p is the same as the expected return on the market portolio. Additionally, $\beta_p = 1 \iff \frac{\sigma_{iM}}{\sigma_M^2} = 1 \iff \sigma_{iM} = \sigma_M^2$. Furthermore, an asset negatively correlated with the market portolio, $\sigma_{iM} < 0$ has $\beta_i < 0$ making the expected return on asset i, $\mu_i < R_f$. From the CAPM formula, it follows

$$R_i = R_f + \beta_i(R_M - R_f) + \epsilon_i$$

where $E[\epsilon_i] = 0$, $\forall i$

$$\begin{aligned}Cov(\epsilon_i, R_M) &= Cov(R_i - R_f - \beta_i(R_M - R_f), R_M) \\ &= Cov(R_i - \beta_i(R_M - R_f), R_M) \\ &= Cov(R_i, R_M) - \beta_i Cov(R_M, R_M) \\ &= \sigma_{iM} - \frac{\sigma_{iM}}{\sigma_M^2}\sigma_M^2 \\ &= \sigma_{iM} - \sigma_{iM} \\ &= 0\end{aligned}$$

It follows that

$$\begin{aligned}
Var(R_i) &= Var(R_f - \beta_i(R_M - R_f) + \epsilon_i) \\
&= Var(R_f) + \beta_i^2 Var(R_M - R_f) + Var(\epsilon_i) + 2Cov(R_f, (R_M - R_f)) \\
&+ 2Cov(R_f, \epsilon_i) + 2Cov((R_M - R_f), \epsilon_i) \\
&= \beta_i^2 Var(R_M) + Var(\epsilon_i)
\end{aligned}$$

The variance of the asset i can be broken into two orthogonal components, $\beta_i^2 Var(R_M)$ also referred to as the systemic risk and $Var(\epsilon_i)$ also refferd to as nonsystemic risk. The systemic risk is the part of the risk related to the market portfolio and is implicit in the economy. The systemic risk cannot be diversified away.

2.2. *Fama-French Three-Factor Model*

In "The Cross-Section of Expected Stock Returns" (1992) Fama and French state the following : "The central prediction of the model (the CAPM) is that the market portfolio of invested wealth is mean-variance efficient in the sense of Markowitz (1959). The efficiency of the market portfolio implies that (a) expected returns on securities are a positive linear function of their market βs (the slope in the regression of a security's return on the market return), and (b) market βs suffice to describe the cross-section of expected returns." Based on the several empirical contradictions of the CAPM, such as the size effect, as well as other relationships between average returns and different variables, such as leverage, book-to-market equity, they explore the roles of size, book-to-market equity, earnings-to-price ration and leverage in average returns.

In the paper, they provide evidence that two easily measured variables, size and book-to-market equity provide a characterization of the cross-section of average stock returns in the US stock market for the 1963 -1990 period. They find that there is a relationship between size and average return, and when controlling for size, the relationship between β and the average return is flat. Additionally, they discover a stronger cross-sectional relation between average returns and book-to-market equity. As for leverage and earnings-to-price ratio, they conclude that the combination of size and book-to-market equity absorbs their roles in average returns. Finally, they conclude that their tests do not support the central prediction of the CAPM.

Motivated by their findings, Fama and French construct the three-factor model. The model states that the expected return on a portfolio in excess of the risk-free rate ($E[R_i] - R_f$) is explained by three factors: (1) the expected return on a market portfolio in excess of the risk-free rate ($E[R_M] - R_f$); (2) the difference between the return on a portfolio consisting of small stocks and the return on a portfolio consisting of large stocks ("small minus big", SMB); and (3) the difference between the return on a portfolio consisting of high book-to-market stocks and the return on a portfolio consisting of low book-to-market stocks ("high minus low", HML). Specifically,

$$E[R_i] - R_f = b_i(E[R_M] - R_f) + s_iE[SMB] + h_iE[HML]$$

$E[R_M] - R_f$,$E[SMB]$ and $E[HML]$ are expected premiums and the b_i,s_i and h_i are the slopes in the time-series regression,

$$R_i - R_f = \alpha_i + b_i(R_M - R_f) + s_iSMB + h_iHML + \epsilon_i$$

. In "Multifactor Explanations of Asset Pricing Anomalies" (1996) Fama and French state: "At a minimum, the available evidence suggests that the three-factor model, with intercepts equal to 0.0, is a parsimonious description of returns and average returns." Fama and French are making both points, that the model explains variation over time in returns (measured by high R^2 values) in addition to explaining the variation across the portfolios in the average returns (measured by the intercepts being indistinguishable from 0.0). Importantly, the true test of the model is whether the intercepts are all jointly zero. The model is about the average returns versus betas.

3. *Previous Research*

Lunden (2007) uses FF3FM and an extended six-factor international model to capture the variation in Brazilian stock returns from 1995 to 2006. Nine equally-weighted portfolios are formed through a dependent sorting procedure, where stocks are first sorted according to size and then according to book-to-market values, thus resulting in equally large portfolios. The BOVESPA index, a value-weighted index that reflects the most traded stocks, is used as a proxy for the market. Lunden emphasises that the Brasilian data he is working with is given in real numbers, adjusted for inflation, whereas the U.S. excess market return, the SMB and the HML factor are all calculated in nominal terms, so noise was added to the regressions in the international model. Lunden finds a negative relationship between size and average return and a positive relationship between the B/M ratio and average return, a pattern not found by another study on the Brasilian market, which he contributes to a different approach to forming the portfolios. He concludes that the domestic three-factor model better explains the variation of returns than the traditional CAPM while the inclusion of foreign factors somewhat increases the explanatory power of the model.

Akgul (2013) explores the power the FF3FM has in explaining the average returns on European stock markets over the period of June 1990 to May 2011, an important period since it includes the European Monetary Union (EMU) period and the integration period before. He includes 10 EMU countries (Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Netherlands, and Spain) and for 3 non-EMU but EU countries (Denmark, Sweden, Switzerland) in his research. He uses a eurozone model on the 10 EMU countries and a country-specific model for each country in his sample. In the eurozone model, he combines 10 EMU countries to form a Eurozone market and uses Euribor for the risk-free rate starting from 1999 and Germany Euro-Mark 1 Month for the earlier period. Akgul concludes that the FF3FM can be accepted as a fairly good model for European countries looking at country-specific model results (only in Greece and Ireland does the model fail). He also finds that the results get better for 7 out of 10 eurozone countries with the eurozone model and does not reject the hypothesis of conversion within the Eurozone countries after monetary integration.

Kilsgård and Wittorf (2010) test the FF3FM and the CAPM in the Swedish stock market. They also test the models in different economic conditions, limiting the time period used. They form 16 stock portfolios using the time period from

2005 to 2010, 58 observations. Using the whole time period from 2005 to 2010, they find that on average companies with high BE/ME ratios tend to have lower returns than companies with low BE/ME ratios. This result contradicts Fama and French's findings that values stocks outperform growth stocks. However, after excluding the entire year of 2007 from the sample, their results align with those of Fama and French. Finally, they conclude that the FF3FM provides higher explanatory power than the CAPM in both stable and less stable market conditions. They also find that during a period of financial turmoil, the FF3FM does not perform well on the Swedish stock market.

Manolakis (2012) examines the FF3FM for the Greek stock exchange as well as a non-linear method, GARCH model. The sample consists of 227 stocks and 120 monthly observations for the time period from 2001 to 2011. Greece Government 10-year bond index is used as a proxy for the risk-free rate and the total market return is calculated as a weighted average of all included stock returns, plus the negative book-to-market stocks. Manolakis forms six portfolios and finds evidence that market, size and book-to-market have explanatory power in the Greek stock market and goes on to conclude that the three-factor model better explains the common variation in the stock returns than the CAPM. Manolakis questions the validity of the results as he finds residual distributions being far from normal as well as signs of residual autocorrelation. After constructing GARCH models, resulting in better coefficient estimates, solving heteroskedasticity and autocorrelation problems, he finds that coefficient estimates from GARCH models agree with those of linear regression models and are consistent with the theory.

Eraslan (2013) tests the validity of the FF3FM on the Istanbul Stock Exchange (ISE) over the time period from 2003 to 2010. His sample consists of 274 stocks and 96 monthly observations. Eraslan constructs 9 portfolios with an independent sorting procedure according to size and book-to-market equity and uses ISE-all index to proxy for the market portfolio and three and six-month Turkish Treasury bill rates to proxy for the risk-free rate. He finds that portfolios containing large firms have higher average returns than portfolios containing small size firms as well as that portfolios containing low book-to-market equity firms perform better than those containing high book-to-market equity firms. He also finds that medium size portfolios outperform small size portfolios on the ISE in the observed time period. Eraslan concludes that the size factor has no effect on portfolios containing large capitalization firms but can explain the excess return variations on portfolios containing small and medium-sized firms whereas the book-to-market factor has an effect on portfolios with high book-to-market equity firms. He points out that the FF3FM has a limited potential to explain variations in the returns on portfolios consisting of stocks on the ISE from 2003 to 2010. Eraslan states that the market risk factor has a wider and stronger effect on portfolio returns than the other two risk factors.

Dolinar (2013) finds that the FF3FM has greater explanatory power compared to the CAPM in the Croatian stock market. Dolinar uses a dataset consisting of monthly observations in the time period from April 2007 to March 2013. He sets the following conditions for stocks to be included: a stock has to be common, the stock issuer has to be a nonfinancial company and a stock has to have at least one trading record per month (in the period from March of year t to March of year t+1). Market equity values at the end of March of each year and that is when the portfolios are formed as well. Due to the small number of stocks left in the sample, he chooses to split the stocks into two groups according to size (Small and Big) and two groups according to book-to-market equity (Low and High) using the median as a breakpoint for both. Out of these four portfolios, the SMB and the HML factors are constructed. Dolinar does not test the model on portfolios, instead, he selects 37 stocks to test the model on. He also tests a one-factor model (equivalent to CAPM) and a two-factor model (factors being size and book-to-market). Based on the R^2 values of the one-factor model he concludes that the market factor leaves much variation in returns that can be explained by other factors. In the two-factor model, he finds that book-to-market shows much greater explanatory power and concludes that size is of little use in explaining common variation in stock returns. FF3FM proves to be the best in capturing the strong common variation in returns, although the overall adjusted R^2 values would have been lower than the R^2 values he is making his conclusions on. He also states that even though the size and the BE/ME factors have not proved to be statistically significant for all selected stocks, the SMB and HML factors individually capture a small amount of shared variation missed by the market factor. Dolinar emphasises that still a large portion of common variation in stock returns could be explained by other factors.

Effects of the return interval on estimated betas have also been discussed. Cohen et al. (1980) documented that security beta estimates calculated employing the OLS method are sensitive to changes in the return measurement interval. They argued that delays in the security price adjustment to new information produce cross-correlations among security returns, resulting in autocorrelation in market returns and biased beta estimates. Additionally, adjustment delays being greater for the infrequently traded securities renders those beta estimations less reliable. Scholes and Williams (1977) that betas are biased downward for infrequently traded and biased upward for frequently traded securities. Roll (1981) also found that betas of infrequently traded securities are underestimated when returns are measured for a short time interval.

Diacogniannis and Makri (2008) examine the intervalling effect bias in OLS beta estimates in the thinner market, specifically the Athens Stock Exchange. Their sample consists of 187 securities over a four year time period, 2001-2004. They estimate beta using OLS technique and compare the results for different return intervals (daily, biweekly and monthly) where they observe that the magnitude of the intervalling effect is inversely related to the market value of the firms. They also test the Hawawini (1983) model with biweekly and monthly return data and the models proposed by Scholes and Williams (1977) and Cohen et al. (1983) using daily returns. They conclude that, in the Athens Stock Exchange, there are no statistically significant differences between the mean beta estimated using the OLS method and the mean beta obtained via the model of Scholes and Williams (1977). They also find that the results using the Hawawini's models indicate a good performance for estimating betas for longer return intervals for high-cap portfolios.

Lambert and Hübner (2014) investigate whether there is really a value premium in the US market after the effects of the market and size are removed. They demonstrate that a theoretical bias is created when the independent sorting procedure is used to form the portfolios which results in the size factor being underestimated and the book-to-market being overestimated. By making a modification to the Fama and French methodology and using conditional instead of independent rankings, they observe a much stronger size effect than previously documented. Using a sequential sorting procedure they succeed in removing most of the correlation in the data. Additionally, they use all listed stocks while determining the quantile limits for portfolios and not only NYSE stocks as Fama and French do and rebalance the portfolios on a monthly instead on a yearly basis. They discover a strong size effect but an insignificant value effect over the period. Furthermore, they state "We also documented the superior accuracy of our alternative premiums for pricing individual stocks."(Lambert and Hübner, 2014).

In their 2012 study, Pettengill, Chang and Hueng test the ability of the FF3FM to predict stock return and return variation following the approach of Fama and MacBeth (1973). They find that expected returns, particularly for extreme portfolios, are poor predictors of actual returns. Further, they conclude that a model consisting of the market beta and the factor loading related to size seems to predict more efficiently than either the three-factor model or the CAPM.

4. *Methodology*

The chapter explains the methods used in conducting the study. Different approaches to the portfolio constructions are chosen based on previous research and previously known characteristics of the Croatian stock market. First, the construction process, or better yet processes, of the explanatory variables are presented. Secondly, the same is shown for the dependent variable, the excess returns on Croatian stocks. Finally, it is explained how the models are tested in addition to what specific regressions are run.

4.1. *Explanatory variables*

4.1.1. *SMB and HML factor construction*

Two different ways of computing the mimicking portfolios are presented in addition to alternative risk premiums for the size and book-to-market. First, the approach of Fama and French is further examined. Secondly, a modified sequential (conditional) approach with a different number of formed portfolios is presented.

4.1.1.1. *2x3 independent sorts*

The Fama and French approach (Fama and French, 1993) is used to construct the factor mimicking portfolios. Six portfolios (Small/Low, Small/Medium, Small/High, Big/Low, Big/Medium, Big/High) are constructed from the intersections of the two ME groups and the three BE/ME groups. For instance, the Small/Low portfolio contains stocks which are found in the Small size group as well as the Low book-to-market group. This also implies that each company will be present in only one of the six portfolios. The reasoning for dividing the stocks into three BE/ME portfolios instead of two, as for size (ME), is that book-to-market equity proved to exhibit stronger power in explaining average stock returns than size (Fama and French, 1992).

For the entire time period (2009-2017), the procedure is as follows: in July of year t, all suitable stocks are sorted according to size, measured as the market value of the company (number of shares outstanding times price per share as of July of year t). Then, the sample is split into two groups based on the median of the size-sorted sample. One group of stocks with low ME values (Small), and the other with high ME values (Big). A firm's book equity, as well as its market equity, at the end of December of year t-1, is used to calculate the firm's book-to-market ratio. In a similar manner, using the book-to-market equity ratio of December year t-1, stocks are sorted according to BE/ME ratios. The market value of equity is calculated in December of year t-1 and July of year t whereas the book value of equity is calculated only for December of year t-1. The six months gap between the ME values (calculated in July of year t), used to measure size, and BE/ME values (calculated in December of year t-1) is needed to ensure that the accounting data is publically available. The BE/ME-sorted stocks are then divided into three groups, Low, Medium and High. The BE/ME breakpoints are the 30th and 70th percentiles of BE/ME values. Portfolios are formed at the intersections of the sorts. Finally, the monthly value-weighted returns on the six portfolios are calculated from July of year t to June of year t+1. New portfolios are again formed in June of year t+1. Companies with a negative book value of equity are excluded.

Small minus big (SMB) portfolio, intended to mimic the risk factor in returns related to size, is the difference between the arithmetic mean of the returns on the three Small portfolios (Small/Low, Small/Medium, Small/High) and the arithmetic mean of the returns on the three Big portfolios (Big/Low, Big/Medium, Big/High). For each month t,

$$R_t^{SMB} = \frac{1}{3}(R_t^{Small/Low} + R_t^{Small/2} + R_t^{Small/High}) - \frac{1}{3}(R_t^{Big/Low} + R_t^{Big/2} + R_t^{Big/High}) \quad (4.1)$$

Hence, SMB is the difference between the mean returns on Small- and Big-stock portfolios, after controlling for the BE/ME. This difference should be largely free of the influence of BE/ME, focusing instead on the different return behaviours of small and big stocks (Fama and French, 1993).

Likewise, the high minus low (HML) portfolio, intended to mimic the risk factor in returns related to book-to-market equity, is calculated as the difference between the arithmetic mean of the returns on the two High portfolios (Small/High, Big/High) and the arithmetic mean of the returns on the two Low portfolios (Small/Low, Big/Low). For each month t,

$$R_t^{HML} = \frac{1}{2}(R_t^{Small/High} + R_t^{Big/High}) - \frac{1}{2}(R_t^{Small/Low} + R_t^{Big/Low}) \quad (4.2)$$

Hence, HML is the difference between the mean returns on High- and Low-BE/ME portfolios, after controlling for the size. This difference should be free of size influence and focus largely on the behaviour of high and low BE/ME stocks (Fama and French, 1993).

4.1.1.2. *3x3 conditional sorts*

Under independent sorting, the six portfolios will have approximately the same number of stocks only if size and book-to-market are unrelated characteristics; that is if there is no significant correlation between the risk fundamentals (Lambert and Hübner, 2014). Even Fama and French point out that using independent size and book-to-market sorts of NYSE stocks to form portfolio means that the highest book-to-market/market equity quintile is tilted toward the smallest stocks (Fama and French, 1993). A pattern observed in the Croatian data as well (see Table 5.2). Thus, a different sorting procedure is also going to be employed. The alternative methodology for constructing mimicking portfolios suggested by Lambert and Hübner (2014), the sequential sorting technique. First, all stocks are sorted according to size and then divided into three groups (Small, Medium and Big), the ME breakpoints being $1/3^{rd}$ and $2/3^{rd}$ quantile of ME values. Then, stocks in each size quantile are sorted according to BE/ME ratios and divided into three groups (Low, Medium and High) with the BE/ME breakpoints being $1/3^{rd}$ and $2/3^{rd}$ quantile of BE/ME values. The rest of the procedure is the same as in the independent sorting case with the total number of portfolios now being nine instead of six.

The SMB portfolio is the difference between the arithmetic mean of the returns on the three Small portfolios (Small/Low, Small/Medium, Small/High) and the arithmetic mean of the returns on the three Big portfolios (Big/Low, Big/Medium, Big/High). For each month t,

$$R_t^{SMB} = \frac{1}{3}(R_t^{Small/Low} + R_t^{Small/2} + R_t^{Small/High}) - \frac{1}{3}(R_t^{Big/Low} + R_t^{Big/2} + R_t^{Big/High}) \quad (4.3)$$

In the same manner, the HML portfolio is calculated as the difference between the arithmetic mean of the returns on the three High portfolios (Small/High, Medium/High, Big/High) and the arithmetic mean of the returns on the three Low portfolios (Small/Low, Medium/Low, Big/Low). For each month t,

$$R_t^{HML} = \frac{1}{3}(R_t^{Small/High} + R_t^{2/High} + R_t^{Big/High}) - \frac{1}{3}(R_t^{Small/Low} + R_t^{2/Low} + R_t^{Big/Low}) \quad (4.4)$$

4.1.2. *Market factor*

4.1.2.1. *Risk-free rate of return*

A completely risk-free rate of return is next to impossible to find on the market. Usually, government bonds are utilized as a proxy for the risk-free rate, but only when considering developed countries. Events such as inflation may affect the rate of return on bonds making them even less risk-free than before. A Croatian Government bond is used as a proxy for the risk-free rate of return.

4.1.2.2. *Market portfolio*

By definition, a market portfolio is a portfolio comprised of all investments possible. It should incorporate every type of asset available with each asset weighted in proportion to its total presence in the market so that the expected return on a market portfolio is identical to the expected return of the market as a whole. Due to the lack of data, constructing a true market portfolio is not plausible. Different stock indices might be used to proxy for the market portfolio in the stock market. A value-weighted portfolio of all stocks might also be considered. A value-weighted portfolio consisting of all stocks in the sample, from now on referred to as Market Portfolio, and CROBEX, the Zagreb Stock Exchange equity index, are both going to be used to proxy for the market portfolio. CROBEX is a value-weighted equity index containing 25 most traded shares with individual weights reflecting the free float market capitalization of a stock with a maximal value of 10%.

Lastly, the monthly excess return on the market portfolio over the risk-free rate of return is calculated to represent the market risk factor.

4.2. *Dependent variable*

Different approaches to the portfolio constructions for dependent variables are also chosen. Three different sorting procedures with different numbers of resulting portfolios are explained, the independent sorting procedure resulting in 9 portfolios, the sequential (conditional) sorting procedure resulting in 9 portfolios and the double independent sorting procedure resulting in 16 portfolios.

4.2.1. *Independent sorts*

The nine portfolios for the dependent variables are constructed in a similar manner as the portfolios for the independent variables following the approach of Fama and French (1993). For the entire time period (2009-2017), the procedure is as follows: in July of year t, all suitable stocks are sorted according to size, measured as the market value of the company (number of shares outstanding times price per share as of July of year t). Then, the sample is split into three groups (Small, Medium, Big) with the ME breakpoints being $1/3^{rd}$ and $2/3^{rd}$ quantile of ME values. Similarly, using the book-to-market equity ratio of December year t-1, stocks are sorted according to BE/ME ratios. The BE/ME-sorted stocks are then divided into three groups (Low, Medium and High) with the BE/ME breakpoints being $1/3^{rd}$ and $2/3^{rd}$ quantile of BE/ME values. Portfolios are constructed at the intersection of the sorts. Finally, the monthly value-weighted returns on the nine portfolios are calculated from July of year t to June of year t+1. New portfolios are again formed in June of year t+1. Companies with a negative book value of equity are excluded. Given the nine portfolios, the monthly excess returns are calculated by subtracting the risk-free rate of return from the value-weighted portfolio returns. This result in a time-series of monthly excess returns on the nine stock portfolios from July 2009 to April 2017.

4.2.2. *Conditional sorts*

The methodology of Fama and French, where the stocks are sorted simultaneously on size and book-to-market and then portfolios are constructed at the intersection of the sorts, results in a not completely satisfactory number of stocks per portfolio, especially in the Small/2 and the Big/High portfolios. In fact, with independent sorts, there are years when the Big/High portfolio contains no stocks at all. Therefore, the second approach is utilized where the stocks are not sorted according to size and book-to-market equity independently, the conditional sorting procedure described earlier. By grouping the stocks sequentially, first according to size and then according to the book-to-market value, the resulting portfolios will consist of an equal number of securities (give or take one). The sequential sorting procedure is as follows: in July of year t, all stocks are sorted according to size and then divided into three groups (Small, Medium and Big), the ME breakpoints being $1/3^{rd}$ and $2/3^{rd}$ quantile of ME values. Then, stocks in each size quantile are sorted according to BE/ME ratios and divided into three groups (Low, Medium and High) with the BE/ME breakpoints being $1/3^{rd}$ and $2/3^{rd}$ quantile of BE/ME values. This results in a total number of nine portfolios. Finally, the monthly value-weighted returns on the nine portfolios are calculated from July of year t to June of year t+1. New portfolios are again formed in June of year t+1. Again, companies with a negative book value of equity are excluded. Given the nine portfolios, the monthly excess returns are calculated by subtracting the risk-free rate of return from the value-weighted portfolio returns.

4.2.3. *Double independent sorts*

In an attempt to increase the number of stocks per portfolio and the total number of portfolios, another sorting procedure is employed for the portfolio constructing process. The double independent sorting procedure entails the following: in July of year t, all stocks are sorted independently according to size and book-to-market equity and divided into four groups using quartiles as breakpoints. Then, 16 portfolios are formed by joining the intersecting groups together. For instance, the Small/Low portfolio contains all stocks found in the Small size group as well as all stocks found in the Low book-to-market group. This also implies that each Small size company will be present in all three Small size portfolios (Small/Low, Small/Medium, Small/High). Lastly, the monthly value-weighted returns on the nine portfolios are calculated from July of year t to June of year t+1. New portfolios are again formed in June of year t+1. Again, companies with a negative book value of equity are excluded. The monthly excess returns on the 16 portfolios are calculated by subtracting the risk-free rate of return from the value-weighted portfolio returns.

4.3. *Regressions and model tests*

In order to test and compare the explanatory power of the CAPM and the FF3FM on the Croatian stock market, OLS time-series regressions are run. As stated by Fama and French (1993):

"The time-series regressions are also convenient for studying two important asset-pricing issues. (a) One of our central themes is that if assets are priced rationally, variables that are related to average returns, such as size and book-to-market equity, must proxy for sensitivity to common (shared and thus undiversifiable) risk factors in returns. The time-series regressions give direct evidence on this issue. In particular, the slopes and R^2 values show whether mimicking portfolios for risk factors related to size and BE/ME capture shared variation in stock and bond returns not explained by other factors. (b) The time-series regressions use excess returns (monthly stock or bond returns minus the one-month Treasury bill rate) as dependent variables and either excess returns or returns on zero-investment portfolios as explanatory variables. In such regressions, a well-specified asset-pricing model produces intercepts that are indistinguishable from 0 (Merton (1973)). The estimated intercepts provide a simple return metric and a formal test of how well different combinations of the common factors capture the cross-section of average returns. Moreover, judging asset-pricing models on the basis of the intercepts in excess return regressions imposes a stringent standard."

In line with Fama and French (1993), after running the regressions, it is tested whether the intercepts and the coefficients of the risk factors are significantly different from 0.

Three different sorting procedures are put to the test. First, the regressions are run for the scenario where both the dependent variable and the explanatory variables are constructed through the independent sorting procedure. Then, the regressions are run for the scenario where both the dependent variable and the explanatory variables are constructed through the sequential sorting procedure. Lastly, the regressions are run for the scenario where the dependent variable is constructed through a double independent sorting procedure and the explanatory variables are constructed through the independent sorting procedure.

In all three scenarios, the dependent variable in both the CAPM and the FF3FM regressions is the excess returns on 9 (16 in the double independent sorting case) stock portfolios formed according to size and book-to-market equity. The explanatory variable in the CAPM is the excess return on a broad market portfolio of stocks whereas the explanatory variables in the FF3FM include two additional variables, returns on mimicking portfolios for size and book-to-market equity. The time-series regression slopes are factor loadings that, unlike size or BE/ME, have a clear interpretation as risk-factor sensitivities for stocks (Fama and French, 1993). Furthermore, in doing so, two different data sets are used, monthly and biweekly returns, and two different proxies for the broader market as well, the Market Portfolio and CROBEX.

5. *Data*

The chapter presents the data that is used for conducting the study. First, it is explained what time period is used. Secondly, the sample and the sample characteristics are presented. Lastly, descriptive statistics for constructed portfolios, in addition to some supplementary information regarding the data, are presented.

5.1. *Time period*

The time period used is the result of the time period available for the stock price data on the Zagreb stock exchange website. Stock prices prior to January 2nd, 2008 are not publicly available. The data consists of 112 monthly observations and 225 biweekly observations from January 2008 to April 2017.

5.2. *Sample construction*

The initial sample consisted of 145 stocks listed on the Zagreb stock exchange. The sample period is January 2008 to April 2017. For the time period of 10 years, monthly and biweekly returns of these stocks, as well as yearly observations of the respective market capitalization and the book-to-market equity ratios, were gathered and calculated. Book values of equity and the number of shares outstanding were extracted from the firms' annual financial statements (audited and consolidated if available) disclosed on the ZSE. The changes in the number of stocks of a specific security where also accounted for. Monthly and biweekly stock returns are not adjusted for dividends. The individual stock return at time t is calculated as the difference between the stock price at time t and t-1 further divided by the stock price at time t-1, t-1 and t being the endings of two consecutive monthly or biweekly periods. Contrary to the Fama and French approach, financial firms were not excluded from the sample (corresponding tables for the data without the financial firms as well as results of the OLS regressions are given in the appendix).

To be included in the portfolios in year t, a security must: (1) have been listed on the ZSE for at least a year, (2) have a non-negative value of book equity at the end of December of year t-1 and (3) have been traded more than x times per quartile in year t-1, where x is defined as

$$x = 0.1 \frac{\sum_i number \quad different \quad days \quad stock \quad i \quad is \quad traded}{total \quad number \quad of \quad trading \quad days} \tag{5.1}$$

Companies that have gone bankrupt, or have been delisted from the Zagreb stock exchange for some other reason, are not included in the sample. Due to the way ZSE keeps records of the price data for those types of securities, the data of the company's historical stock prices could not be obtained, and therefore, neither the firm's past market values of equity could be calculated. In those cases, in any given year, the company could not be included in any of the portfolios. For this reason, it might be the case that survivorship bias is present in the final sample.

Due to thin trading and the requirement for positive book-to-market ratios of securities, the sample is further reduced to an average number of 77 stock per year. The number of included stocks per year ranges from 71 to 80. It is also worth noting that most of the infrequently traded stocks were also characterized by small values of market capitalization, so by excluding those stocks the sample might only be containing medium and large size stocks.

5.3. *Descriptive statistics*

5.3.1. *Explanatory variables*

Table 5.1 shows summary statistics for the explanatory variables.

With the Market Portfolio proxying for the market, equity premium for 2009-2017 (the average difference between monthly/biweekly value-weight market return and the risk-free rate) is -0.40% (t=-0.51) and -0.21% (t=-0.57) for monthly and biweekly data, respectively. The excess return that investing in Croatian stock market provides over a risk-free rate is negative over a time period from 2009 to 2017.

With the CROBEX proxying for the market, equity premium for 2009-2017 is -0.17% (t=-0.36) and -0.10% (t=-0.45) for monthly and biweekly data, respectively. Again, the excess return that investing in Croatian stock market provides over a risk-free rate is negative but halved in value in comparison to the Market Portfolio proxying for the market, over a time period from 2009 to 2017. Additionally, equity premium is more imprecise when Market Portfolio proxies for the market. The correlation between the excess return on a market portfolio when CROBEX proxies for the market and the excess return on a market portfolio when the Market Portfolio proxies for the market is 0.67 and 0.61 for the monthly and biweekly data, respectively. It is worth noting that the correlation between the excess return on a market portfolio and SMB and HML is above 0.38 (except for the HML_C) when the Market Portfolio proxies for the market whereas in the CROBEX case it is less than 0.1. Also, the

SMB and the HML, constructed both through independent and conditional sorting procedures, are negatively correlated with the excess return on a market portfolio.

Table 5.1: Summary statistics for explanatory returns: July 2009 to April 2017

Panel A: Monthly data, 94 Observations

	mean	sd	t_{mean}	Correlation					
				MP-r_{rf}	CRO-r_{rf}	SMB_I	HML_I	SMB_C	HML_C
MP-r_{rf}	-0.40	7.51	-0.51						
CRO-r_{rf}	-0.17	4.47	-0.36	0.67	1.00				
SMB_I	0.26	3.51	0.71	-0.44	-0.10	1.00			
HML_I	0.27	5.59	0.48	-0.39	-0.07	0.13	1.00		
SMB_C	0.13	4.74	0.28	-0.46	-0.08	0.65	0.53	1.00	
HML_C	0.55	4.10	1.29	-0.07	-0.01	-0.04	0.70	0.29	1.00

Panel B: Biweekly data, 190 Observations

	mean	sd	t_{mean}	Correlation					
				MP-r_{rf}	CRO-r_{rf}	SMB_I	HML_I	SMB_C	HML_C
MP-r_{rf}	-0.21	5.09	-0.57						
CRO-r_{rf}	-0.10	2.98	-0.45	0.61	1.00				
SMB_I	0.13	2.81	0.65	-0.38	-0.03	1.00			
HML_I	0.06	3.87	0.22	-0.44	-0.05	0.12	1.00		
SMB_C	0.10	3.74	0.35	-0.51	-0.09	0.70	0.47	1.00	
HML_C	0.16	2.99	0.75	-0.06	-0.05	-0.01	0.59	0.17	1.00

There seems to be neither size nor value premium in the Croatian stock market during the observed time period. Using the independent sorting procedure, the average premium for the size-related factor is 0.26% (t=0.71) per month and 0.13% (t=0.65) biweekly while the average premium for the book-to-market factor is 0.27% (t=0.48) per month and 0.06% (t=0.22) biweekly.

Using the dependent sorting procedure, the average premium for the size-related factor is 0.13% (t=0.28) and 0.10% (t=0.35) biweekly while the average premium for the book-to-market factor is 0.55% (t=1.29) per month and 0.16% (t=0.75) biweekly.

Average premiums are halved when biweekly returns are used as opposed to monthly returns while their standard deviations decline only by 20 to 30 %.

Average premiums for the size- and the book-to-market-related factor being positive is an indicator that, on average, small stocks slightly outperformed big stocks and value stocks slightly outperformed growth stocks during the observed time period.

The correlation between the SMB and the HML factor is around 0.13 when independent sorts are used to construct the portfolios and somewhat higher with conditional sorts 0.29 and 0.17 for monthly and biweekly data, respectively. Moreover, the correlation between the SMB_I and the SMB_C is 0.65 using monthly and 0.70 using biweekly data. Similarly, the correlation between the HML_I and the HML_C is 0.70 and 0.59 for monthly and biweekly data, respectively.

5.3.2. *Dependent variables*

In table 5.2, the descriptive statistics for the dependent variable are presented. The table shows descriptive statistics for portfolios formed through three different procedures explained in the previous chapter.

With independent sorts, the average number of firms in portfolio increases with size in the Low and Medium BE/ME quantile whereas for the High BE/ME quantile the opposite holds. The largest number of firms in the Small size quantile falls in the High BE/ME quantile, whereas almost all firms in the Big quantile fall in the Low and Medium BE/ME quantile leaving the Big/High portfolio with an average of 1.75 firms per year. Additionally, the average firm size for portfolios decreases with book-to-market equity. The highest BE/ME quintile is tilted towards the smallest stocks. Only 1.55% of the total market value lands in the Small size quantile whereas the Big size quantile contains 91.81% of the total market value. The Low BE/ME quantile adds up to 54.84% and High BE/ME quantile to only 8.6% of total market value.

In contrast to independent sorting, sequential sorting ensures approximately the same number of stocks in all nine portfolios each year. Average of firm size and the average of annual market value in portfolio both decreases with book-to-market. Similarly, the average of BE/ME ratio decreases with size. The Low BE/ME quantile adds up to 38.25% and High BE/ME quantile to 24.83% of total market value.

Double independent sorting results in almost the same number of stocks in all 16 portfolios. Not surprisingly, average firm size and annual percent of market value in portfolio decrease with book-to-market. Portfolios with the lowest percentage of market value belong to the High BE/ME quartile. Average BE/ME ratios decline with size making the Big/Low portfolio a portfolio with the lowest BE/ME ration, and the Small/Low portfolio the portfolio with the largest one.

Table 5.3 presents the means and the standard deviations for the excess returns on portfolios formed according to size and book-to-market formed through three different procedures explained in the previous chapter. The table presents both biweekly and monthly excess return data for the independent and dependent sorting procedure for the 9 resulting portfolios and only biweekly excess return data for the double independent sorting procedure that results in 16 portfolios.

Fama and French (1993) observe a negative relation between size and average return and a stronger positive relation between average return and book-to-market equity in the US market. No such statement can be made for the Croatian stock market, at least for the observed time period. With independent sorts, the average portfolio monthly excess returns range from -1.03 to 0.88 % whereas the biweekly excess returns range from -0.44 to 0.38 % in the time period from July 2009 to April 2017. The means of the excess returns in the Medium size quantile increase with BE/ME. Portfolios in the High BE/ME quantile experience higher average excess returns than those in the Low BE/ME quantile with the exception of the Big size quantile where the opposite holds. Small/High portfolio seems to experience the

Table 5.2: Descriptive statistics for portfolios formed on size and book-to-market equity: July 2009 to April 2017

Size quantile	Low	2	3	High	Low	2	3	High	
	Book-to-market equity (BE/ME) quantiles								
	Average of annual averages of firm size				Average of annual number of firms in portfolio				Independent sorts
Small	5.69	6.64		5.12	5.50	4.50		15.12	
2	30.06	27.71		23.07	6.62	9.75		8.25	
Big	453.71	272.62		263.57	12.88	10.38		1.75	
	Average of annual percent of market value in portfolio				Average of annual B/E ratios (for portfolio)				
Small	0.35	0.34		0.86	0.58	1.47		4.57	
2	2.27	2.96		2.12	0.75	1.45		3.32	
Big	52.22	33.97		5.62	0.72	1.38		4.04	
	Average of annual averages of firm size				Average of annual number of firms in portfolio				Conditional sorts
Small	6.04	5.70		4.84	8.50	7.88		8.75	
2	29.82	26.02		24.70	8.00	7.88		8.75	
Big	434.93	431.09		239.31	8.38	7.88		8.75	
	Average of annual percent of market value in portfolio				Average of annual B/E ratios (for portfolio)				
Small	0.56	0.51		0.49	0.86	2.46		5.98	
2	2.67	2.25		2.42	0.82	1.51		3.20	
Big	35.02	34.16		21.92	0.60	1.05		1.98	
	Low	2	3	High	Low	2	3	High	
	Average of annual averages of firm size				Average of annual number of firms in portfolio				Independent double sorts
Small	142.23	73.69	46.88	9.15	37.75	37.50	37.75	38.25	
2	149.46	80.04	52.83	14.51	37.25	37.00	37.25	37.75	
3	165.56	95.78	68.28	29.50	37.00	36.75	37.00	37.50	
Big	376.41	308.52	279.78	238.46	37.25	37.00	37.25	37.75	
	Average of annual percent of market value in portfolio				Average of annual B/E ratios (for portfolio)				
Small	6.83	4.34	2.85	0.54	1.96	2.25	2.55	4.03	
2	7.14	4.65	3.16	0.85	1.45	1.73	2.04	3.56	
3	8.00	5.51	4.02	1.71	1.05	1.34	1.65	3.18	
Big	18.92	16.43	14.94	12.63	0.88	1.16	1.47	2.99	

highest excess returns overall while the Big/High portfolio the lowest. The Big/High portfolio contains on average 1.75 firms (see table 5.2) which should be taken into consideration. The average standard deviations range from 4.22 to 9.19% for monthly and from 3.04 to 6.44% for biweekly excess returns. Standard deviations in the Small quantile decline when moving from Low to High BE/ME with the highest standard deviation overall belonging to the Small/Low portfolio.

With dependent sorts, the average portfolio monthly excess returns range from -0.30 to 0.83% whereas the biweekly excess returns range from -0.16 to 0.41 % for the observed time period. The average excess returns for portfolios in the Medium size quantile seems to be increasing with BE/ME. The Big/Low portfolio experiences the highest average excess returns. With monthly data High BE/ME quantile experiences higher returns than the Low BE/ME quantile for all size quantiles and Big size quantile experiences higher returns than the Small size quantile, with the exception of Medium BE/ME quantile. The pattern seems to change when switching to biweekly data. In the Low BE/ME quantile the difference in average excess returns between Big and Small size portfolios is reduced from 0.48 to 0.03% and in the Small size quantile, the difference in average excess returns between High and Low BE/ME portfolios is reduced from 0.44 to 0.02%. The average standard deviations range from 4.37 to 8.45% for monthly and from 2.95 to 5.20% for biweekly excess returns.

With double independent sorts, the average portfolio biweekly excess returns range from -0.19 to 0.39% for the observed time period. The average excess returns seem to decrease with size with the exception of the Low BE/ME quantile. In the Low BE/ME quantile, the average excess returns seem not to differ amongst different size quantiles as much. An inverted U shape pattern can be observed in the average excess returns in different size quantiles with the exception of the Big size quantile where the returns seem not to differ as much. The standard deviations for the biweekly excess returns range from 2.83 to 5.48% with the lowest average standard deviations being observed at the intersections of the two medium size quantiles and the two medium BE/ME quantiles. The standard deviations for the Low BE/ME quantile tend to be higher than those of the High BE/ME quantile.

Stock returns having high standard deviations, relative to their average returns, results in the fact that even large average returns are often not reliably different from zero.

Table 5.3: Summary statistics for the size-B/M excess returns :
July 2009 to April 2017

Panel A: Monthly excess returns for 9 portfolios formed on size and B/M

	Mean			Standard deviation		
	Low	2	High	Low	2	High
Independent sorts						
Small	-0.22	-0.72	0.88	9.19	7.09	6.55
2	-0.35	0.44	0.74	6.47	4.22	6.61
Big	-0.24	0.86	-1.03	7.38	4.65	5.74
Conditional sorts						
Small	0.11	0.81	0.55	5.73	6.94	7.89
2	-0.19	0.37	0.75	6.08	4.37	6.14
Big	0.59	-0.30	0.83	8.00	8.45	4.64

Panel B.a: Biweekly excess returns for 9 portfolios formed on size and B/M

	Mean			Standard deviation		
	Low	2	High	Low	2	High
Independent sorts						
Small	0.09	-0.33	0.38	6.44	5.43	4.15
2	-0.16	0.23	0.27	4.41	3.04	4.51
Big	-0.15	0.38	-0.44	5.05	3.00	4.40
Conditional sorts						
Small	0.23	0.36	0.21	4.69	4.64	5.20
2	-0.08	0.17	0.28	4.12	3.19	4.29
Big	0.26	-0.16	0.41	5.09	5.88	2.95

Panel B.b: Biweekly excess returns for 16 portfolios formed on size and B/M

	Mean				Standard deviation			
	Low	2	3	High	Low	2	3	High
Independent double sorts								
Small	-0.19	0.34	0.39	0.12	5.48	3.01	3.06	3.46
2	-0.19	0.31	0.35	0.12	5.37	2.97	3.03	3.45
3	-0.18	0.27	0.30	0.11	5.14	2.83	2.96	2.96
Big	-0.09	-0.01	0.03	-0.03	4.97	4.22	4.47	4.56

6. *Results*

6.1. *Independent sorts*

Table 6.1 presents results of the regressions for all nine stock portfolios for the entire sample period from July 2009 to April 2017, both using different proxies for the market portfolio (Market Portfolio, as defined in previous chapters, and CROBEX, the Zagreb Stock Exchange equity index) as well as different data sets (monthly and biweekly returns/data). The table allows for comparison of the two models, CAPM and its expansion FF3FM. Additionally, it gives further insights into the behaviour of the models when different market proxies or data sets are used.

For the nine portfolios formed independently, the adjusted R^2 values for the CAPM seem to be quite low for most portfolios for all cases of market proxies and data sets used. The exceptions being Big/Low portfolio when using Market Portfolio (Adj.R^2 $\geq$0.893) and Big/2 portfolio when using CROBEX (adj.R^2 is 0.618 using monthly returns and 0.592 using biweekly returns) as a proxy for the market. Due to the fact that all portfolios are value-weighted and that the Market Portfolio is a value-weighted portfolio of all stocks, it is to be expected that the highest percentage of explained variation is found in the portfolio with the highest market capitalization values that in the Big/Low portfolio case amount to 55.22% of the total market value (see Table 5.2). As for the CROBEX case, since it contains 25 stocks, each with a maximal weight of 0.1 it might be the case that the portfolios containing those stocks for a multiple year period would have higher adj. R^2 values. Seeing that neither the index composition nor the stocks weights through time are publicly available, the statement cannot be further tested.

It is also worth noting that the explanatory power of the model tends to increase with size. The adjusted R^2 values are higher for the Big size quantile than for the Small size quantile. As an example, in the case of biweekly data and the Market portfolio, the adjusted R^2 values in the Small size quantile range from 0.010 to 0.089, whereas in the Big size quantile they range from 0.114 to 0.896. Similar patterns are found for other combinations of data sets and market proxies as well. This can be attributed to the construction of the portfolios and the market proxies, as well as the characteristics of the market, discussed earlier. Additionally, the adjusted R^2 values seem to be higher when monthly data are used as opposed to biweekly data.

The intercepts are statistically significant at a 10% level for the Big/High portfolio when using monthly data and at a 5% level for the Big/2 portfolio when CROBEX proxies for the market. In all other cases, intercept values are not statistically significant. The significance of the intercept for the Big/High portfolio disappears when switching from monthly to biweekly data and consequently doubling the number of observations. It is also worth noting that, with independent sorts, the Big/High portfolio consists of on average 1.75 stocks which might be influencing the results.

All beta coefficients are highly statistically significant at a 0.1% level with the exception of the S/2 portfolio when the Market Portfolio is used to proxy for the market.

When Market Portfolio is used, the beta coefficients for all portfolios except the Big/Low portfolio range from 0.246 to 0.547 and from 0.133 to 0.391 using monthly and biweekly returns, respectively. For the Big/Low portfolio, the beta coefficient is 0.918 with monthly data and 0.935 with biweekly data. Due to the way the betas are calculated, this can be attributed to the portfolio construction method as well as the construction of the market proxy. Furthermore, beta coefficients in the Low BE/ME quantile tend to be higher than those of the High BE/ME quantile. With independent sorting, most of the stocks belonging to the Low BE/ME quantile are big size stocks whereas the High BE/ME quantile is tilted towards small size stocks. In fact, in the Low BE/ME quantile there are on average 5.50 stocks in the Small size quantile, 6.62 stocks in the Medium size quantile and 12.88 stocks in the Big size quantile while in the High BE/ME quantile there are on average 15.12 stocks in the Small size quantile, 8.25 stocks in the Medium size quantile and 1.75 stocks in the Big size quantile (see Table 5.2).

When CROBEX proxies for the market portfolio, betas range from 0.709 to 1.318 and from 0.700 to 1.084 for monthly and biweekly data, respectively. With the exception of the 2/High portfolio, the highest beta values are produced in the Low BE/ME quantile with values greater than 1.

After adding the SMB and HML factors to the overall regressions, the adjusted R^2 values increase for all portfolios with the exception of the Big/2 portfolio when using CROBEX as a market proxy where the adjusted R^2 values decrease by 0.003 and 0.002 using monthly and biweekly data, respectively. In the Market Portfolio case, the Big/Low portfolio experiences the smallest increase in the adjusted R^2 values overall of exactly 0.008 in both data scenarios. Adding two more factors to the regression seems not to increase the percentage of explained variation for the Big/Low portfolio. In fact, the entire Big size quantile experiences the smallest increase in the adjusted R^2 values with the increase ranging from 0.008 to 0.055 and from 0.008 to 0.103 using monthly and biweekly data, respectively. Having in mind that the Market Portfolio, as well as all other portfolios in the study, is value-weighted and the stocks in the Big size quantile get the highest weights assigned to them in the Market Portfolio, it is reasonable to expect that the inclusion of the two additional factors is not going to add to the percentage of explained variation in the excess stock returns especially when accounting for the correlation between the Market Portfolio and the SMB and HML, respectively (see Table 5.1). For the remaining portfolios, the increase in the

Table 6.1: Results of the CAPM and FF3FM results : Independent sorts, July 2009 to April 2017

		monthly returns							
		CAPM			FF3FM				
		Intercept	R_m-R_f	Adj.R^2	Intercept	R_m-R_f	SMB	HML	Adj.R^2
Market Portfolio	S-L	-0.874	0.547 ***	0.187	-1.038	0.647 ***	1.093 ***	-0.348 .	0.339
	S-2	-0.962	0.246 *	0.058	-1.035	0.524 ***	0.867 ***	0.276 .	0.223
	S-H	0.392	0.312 ***	0.136	0.359	0.711 ***	0.840 ***	0.627 ***	0.526
	2-L	-0.714	0.391 ***	0.187	-0.814	0.672 ***	1.027 ***	0.193	0.431
	2-2	-0.011	0.262 ***	0.208	-0.035	0.410 ***	0.380 **	0.194 *	0.316
	2-H	0.315	0.368 ***	0.187	0.295	0.793 ***	0.814 ***	0.715 ***	0.617
	B-L	-0.077	0.918 ***	0.893	-0.076	0.842 ***	-0.128	-0.139 *	0.901
	B-2	0.666	0.346 ***	0.329	0.662	0.468 ***	0.223 .	0.211 *	0.384
	B-H	-1.171 .	0.318 ***	0.141	-1.148 .	0.468 ***	0.118	0.352 **	0.192
CROBEX	S-L	-0.735	1.318 ***	0.357	-0.865	1.220 ***	0.656 ***	-0.566 ***	0.471
	S-2	-0.828	0.790 ***	0.218	-0.945	0.838 ***	0.508 *	0.077	0.275
	S-H	0.517	0.878 ***	0.361	0.428	0.979 ***	0.349 *	0.333 ***	0.492
	2-L	-0.612	0.951 ***	0.363	-0.736	0.965 ***	0.564 ***	-0.079	0.435
	2-2	0.084	0.709 ***	0.502	0.060	0.726 ***	0.102	0.049	0.503
	2-H	0.454	1.011 ***	0.466	0.381	1.121 ***	0.266 *	0.391 ***	0.605
	B-L	-0.248	1.109 ***	0.408	-0.069	0.941 ***	-0.718 ***	-0.520 ***	0.713
	B-2	0.754 *	0.835 ***	0.618	0.774 *	0.841 ***	-0.094	0.047	0.615
	B-H	-1.072 .	0.815 ***	0.308	-1.034 .	0.849 ***	-0.199	0.189	0.322
		biweekly returns							
		CAPM			FF3FM				
		Intercept	R_m-R_f	Adj.R^2	Intercept	R_m-R_f	SMB	HML	Adj.R^2
Market Portfolio	S-L	-0.258	0.391 ***	0.089	-0.350	0.534 ***	1.037 ***	-0.289 *	0.231
	S-2	-0.445	0.133	0.010	-0.465	0.437 ***	0.728 ***	0.390 **	0.164
	S-H	0.166	0.238 ***	0.081	0.154	0.628 ***	0.828 ***	0.576 ***	0.520
	2-L	-0.291	0.323 ***	0.129	-0.327	0.538 ***	0.701 ***	0.150	0.273
	2-2	0.049	0.257 ***	0.177	0.034	0.447 ***	0.486 ***	0.225 ***	0.375
	2-H	0.152	0.345 ***	0.148	0.176	0.772 ***	0.594 ***	0.843 ***	0.608
	B-L	-0.049	0.935 ***	0.896	-0.046	0.868 ***	-0.154 ***	-0.091 *	0.904
	B-2	0.279	0.308 ***	0.279	0.283	0.421 ***	0.174	0.213 ***	0.346
	B-H	-0.468	0.307 ***	0.114	-0.429	0.469 ***	-0.024	0.492 ***	0.217
CROBEX	S-L	-0.190	1.084 ***	0.230	-0.280	1.077 ***	0.747 ***	-0.533 ***	0.347
	S-2	-0.361	0.700 ***	0.136	-0.431	0.748 ***	0.484 **	0.183	0.213
	S-H	0.231	0.793 ***	0.303	0.163	0.847 ***	0.465 ***	0.266 ***	0.474
	2-L	-0.220	0.980 ***	0.393	-0.273	0.994 ***	0.404 ***	-0.101	0.437
	2-2	0.103	0.760 ***	0.510	0.071	0.776 ***	0.237 ***	0.014	0.547
	2-H	0.228	1.048 ***	0.450	0.198	1.103 ***	0.151 *	0.466 ***	0.605
	B-L	-0.174	1.037 ***	0.351	-0.072	0.943 ***	-0.668 ***	-0.532 ***	0.685
	B-2	0.320 *	0.786 ***	0.592	0.327 *	0.784 ***	-0.058	0.017	0.590
	B-H	-0.411	0.873 ***	0.307	-0.378	0.883 ***	-0.282 *	0.274 **	0.351

. p<0.1, * p<0.05, ** p<0.01, *** p<0.001

adjusted R^2 values ranges from 0.108 to 0.430 and from 0.142 to 0.460 using monthly and biweekly data, respectively. Additionally, the increase in the adjusted R^2 values tends to grow with BE/ME.

In the CROBEX case, Medium BE/ME quantile experiences the smallest increase in the adjusted R^2 values with the increase values ranging from -0.003 to 0.057 and from -0.002 to 0.077 using monthly and biweekly data, respectively. The Big/Low portfolio experiences the largest increase in the adjusted R^2 values overall with the increase being 0.305 for monthly and 0.334 for biweekly data. The remaining portfolios' increase ranges from 0.044 to 0.171 and from 0.072 to 0.139 using monthly and biweekly data, respectively.

The significance of the intercepts remains unchanged. Moreover, the significance of the beta coefficients increases with the inclusion of the SMB and HML factors and all are now highly significant. Beta coefficients for the Market Portfolio case now range from 0.410 to 0.842 and from 0.421 to 0.868 when using monthly and biweekly data, respectively. It is also worth noting that for some portfolios the beta coefficient increases quite significantly as is the case with Small/High and 2/High portfolio, beta coefficients increase by more than 0.400. For the Big/Low portfolio, the beta coefficient declines. In the CROBEX case, the beta coefficients range from 0.726 to 1.220 and from 0.748 to 1.103 when using monthly and biweekly data, respectively.

Fama and French find that the values of the beta coefficients tend to be approaching 1 after the SMB and the HML factor have been added to the regression. No such definite statement can be made with this sample.

Using monthly data, the SMB factor is very highly significant at a 0.1% level for all Small size quantile portfolios and 2 out of 3 Medium size quantile portfolios when Market Portfolio is used to proxy the market. The coefficient values for the SMB variable range from 0.840 to 1.093 in the Small size quantile and from -0.128 to 0.223 in the Big size quantile. The coefficient values for the SMB variable decline with size.

When CROBEX is used, the SMB factor is statistically significant at a 1% level for the portfolios in the Low BE/ME quantile and statistically significant for the portfolios in the Small size quantile. The coefficient values for the SMB variable decline with size and are higher, in absolute value, in the Low than the High BE/ME quantile. Additionally, the coefficients for the Big size quantiles are all negative.

Using the biweekly data and Market Portfolio as a market proxy, the SMB factor is very highly significant at a 0.1% level for 7 out of 9 portfolio regressions, the two exceptions being Big/2 and Big/High portfolios for which the SMB factor is not significant at all. The coefficient values for the SMB variable range from 0.728 to 1.037 in the Small size quantile and from -0.154 to 0.174 in the Big size quantile. When CROBEX proxies for the market, the only portfolio for which the SMB factor is not statistically significant is the Big/2 portfolio. In all other portfolio regressions, the SMB factor is significant at a 10% level or lower. For all portfolios in the Low BE/ME quantile, the SMB factor is very highly significant. Again, same as with monthly data, Small size quantiles exhibit higher coefficient values than Big size quantile where coefficients are mostly negative.

In total, the coefficients of the SMB factor decrease from Small to Big size quantiles for each book-to-market quantile and thereby rewarding the small firms. Thus, the SMB factor captures common variation left out by the market factor. Also, when switching from monthly to biweekly returns, the significance of the SMB factor increases. One of the reasons for this increase in the significance of the SMB factor might be an increase in the number of observations. The biweekly data consists of a double number of observations than that of monthly data.

Using the monthly data, the HML factor is highly statistically significant at a 1% level (or lower) for all High BE/ME quantile portfolios when Market Portfolio is used to proxy the market. In total, the HML factor is significant for 8 out of 9 portfolio regressions. When CROBEX is used, the HML factor is not significant in 5 out of 9 portfolio regressions with the Small/Low, Small/High, 2/High and Big/High portfolio holding HML factor as very highly statistically significant.

Using the biweekly data and Market Portfolio as a market proxy, the HML factor is not statistically significant only for one portfolio, the 2/Low. When CROBEX proxies for the market, the HML factor is not significant for 4 out of 9 portfolio regressions. For all portfolios in the Low and High BE/ME quantile, with the exception of the 2/Low portfolio, the HML factor is significant.

Again, an increase in significance is present when switching from monthly to biweekly data. The coefficient values for the HML variable increase in each size quantile with BE/ME in addition to experiencing negative values in the Low BE/ME quantile. With the average premium for the book-to-market factor being positive (see Table 5.1), this is an indicator of the market rewarding High BE/ME quantile firms as opposed to Low BE/ME firms. Consequently, the HML factor captures common variation left out by the market and the SMB factor.

6.2. *Conditional sorts*

Table 6.2 presents results of the regressions for all the 9 stock portfolios, formed with a sequential sorting procedure, for the entire sample period from July 2009 to April 2017, both using different proxies for the market portfolio (Market Portfolio, as defined earlier, and CROBEX, the Zagreb Stock Exchange equity index) as well as different data sets (monthly and biweekly returns/data). SMB and HML mimicking portfolios were also constructed through a sequential sorting procedure as suggested by Lambert (2014). Similar as table 6.1, table 6.3 allows for comparison of the two models, CAPM and FF3FM as well.

For the 9 portfolios formed sequentially, the adjusted R^2 values for the CAPM are less than 0.330 for almost all portfolios (except the Big/2 portfolio) in the Market Portfolio case whereas in the CROBEX case the adjusted R^2 values range from 0.279 to 0.592 and from 0.227 to 0.558 using monthly and biweekly data, respectively. The explanatory power of the model tends to increase with size as well as when using monthly, rather than biweekly data. Additionally, the values of adjusted R^2

are greater when CROBEX is used to proxy the market with the exception of the Big/2 portfolio where adjusted R^2 in the Market Portfolio case is 0.858 and 0.867 for monthly and biweekly data, respectively. Again, since all portfolios are value-weighted, including the Market Portfolio, it is intuitive that the highest percentage of explained variation is found in the portfolio containing stocks with the highest market capitalization values (see Table 5.2). Interestingly, when CROBEX proxies for the market the highest values of the adjusted R^2 are achieved in the Big size quantile with the exception of the Big/2 portfolio where the value is 0.279 for monthly and 0.227 for biweekly data. This can be attributed to the construction of the portfolios and the market proxies, as well as the characteristics of the market, discussed earlier.

Big/High portfolio seems to be experiencing statistically significant intercepts at a 5% level for CROBEX and 10% level for the Market Portfolio. In all other cases, intercept values are not statistically significant.

All beta coefficients are highly statistically significant at a 0.1% level. Beta coefficients tend to be higher when using monthly data, as opposed to biweekly data. When Market Portfolio is used, beta coefficients for all portfolios, except the Big/Low and Big/2 portfolio, range from 0.316 to 0.455 and from 0.246 to 0.351 using monthly and biweekly returns, respectively. Beta coefficients are around 0.600 for the Big/Low and around 1.050 for the Big/2 portfolio. Again, a consequence of the portfolio construction method as well as the construction of the market proxy. Furthermore, the beta coefficients increase with size except in the High BE/ME quantile where the opposite holds. When CROBEX proxies for the market portfolio, betas range from 0.744 to 1.288 and from 0.752 to 1.176 for monthly and biweekly data, respectively. Additionally, the highest beta values for the Low and Medium BE/ME quantile are produced in the Big size quantile while the highest beta values for the Small and Medium size quantile are produced in the High BE/ME quantile.

With the inclusion of the SMB and HML factors the adjusted R^2 values increase for all portfolios. The largest increase in the adjusted R^2 values occurs in the Small size quantile where for the monthly data the increase in values ranges from 0.459 to 0.508 and from 0.214 to 0.311 when using Market Portfolio and CROBEX, respectively. It is also worth noting that with the CROBEX proxying for the market, the Medium size quantile experiences the smallest increase in adjusted R^2 values whereas with the Market Portfolio proxying for the market, that is the Medium BE/ME quantile (Small/Medium portfolio excluded). Furthermore, the significance of the intercepts remains unchanged exclusive of the Big/Low portfolio in the CROBEX case where the significance changes from not significant to significant at a 5% level. Beta coefficients are all still highly statistically significant at a 0.1% level and mostly increase in value.

When CROBEX is used, the SMB factor is significant at a 0.1% level for the portfolios in the Small size quantile and the Big/Low and Big/2 portfolio. The SMB factor is also statistically significant for the portfolios in the Low BE/ME quantile. The coefficient values for the SMB variable are the highest in the Small size quantile, where they range from 0.469 to 0.720, and the two Big size quantile portfolios, Big/Low and Big/2, with values turning negative. Moreover, all coefficients for the Big size quantiles are negative.

Table 6.2: Results of the CAPM and FF3FM results : Conditional sorts, July 2009 to April 2017

monthly returns

		CAPM			FF3FM				
		Intercept	R_m-R_f	Adj.R^2	Intercept	R_m-R_f	SMB	HML	Adj.R^2
Market Portfolio	S-L	-0.049	0.329 ***	0.167	0.218	0.601 ***	1.012 ***	-0.541 ***	0.675
	S-2	0.729	0.368 ***	0.148	0.822	0.697 ***	1.167 ***	-0.218 .	0.607
	S-H	0.450	0.455 ***	0.175	0.043	0.757 ***	0.948 ***	0.731 ***	0.674
	2-L	-0.323	0.383 ***	0.214	-0.136	0.521 ***	0.525 ***	-0.372 **	0.343
	2-2	0.209	0.316 ***	0.250	0.166	0.374 ***	0.191 .	0.073	0.275
	2-H	0.620	0.380 ***	0.198	0.412	0.495 ***	0.347 **	0.379 **	0.334
	B-L	0.546	0.610 ***	0.312	0.853	0.566 ***	-0.078	-0.576 ***	0.393
	B-2	-0.173	1.036 ***	0.858	-0.251	1.053 ***	0.041	0.146 .	0.862
	B-H	0.697 .	0.373 ***	0.327	0.481 .	0.436 ***	0.165 .	0.401 ***	0.480
CROBEX	S-L	-0.046	0.809 ***	0.371	0.141	0.858 ***	0.627 ***	-0.483 ***	0.635
	S-2	0.737	0.938 ***	0.356	0.732	0.998 ***	0.720 ***	-0.151	0.570
	S-H	0.455	1.128 ***	0.395	-0.039	1.173 ***	0.469 ***	0.802 ***	0.706
	2-L	-0.334	0.857 ***	0.388	-0.182	0.872 ***	0.200 .	-0.324 **	0.428
	2-2	0.206	0.744 ***	0.504	0.153	0.742 ***	-0.032	0.105	0.504
	2-H	0.629	0.970 ***	0.473	0.393	0.977 ***	0.050	0.422 ***	0.547
	B-L	0.515	1.288 ***	0.500	0.853 .	1.250 ***	-0.406 ***	-0.529 ***	0.659
	B-2	-0.419	1.004 ***	0.279	-0.478	0.948 ***	-0.680 ***	0.258	0.402
	B-H	0.687 *	0.836 ***	0.592	0.460 .	0.830 ***	-0.099	0.439 ***	0.714

biweekly returns

		CAPM			FF3FM				
		Intercept	R_m-R_f	Adj.R^2	Intercept	R_m-R_f	SMB	HML	Adj.R^2
Market Portfolio	S-L	0.167	0.246 ***	0.059	0.251	0.633 ***	1.108 ***	-0.675 ***	0.650
	S-2	0.306	0.256 ***	0.065	0.320	0.639 ***	1.048 ***	-0.212 *	0.521
	S-H	0.167	0.324 ***	0.092	0.040	0.697 ***	0.930 ***	0.711 ***	0.645
	2-L	-0.118	0.333 ***	0.156	-0.077	0.483 ***	0.434 ***	-0.316 ***	0.286
	2-2	0.123	0.278 ***	0.173	0.102	0.378 ***	0.258 ***	0.106	0.243
	2-H	0.243	0.351 ***	0.155	0.175	0.500 ***	0.362 ***	0.401 ***	0.311
	B-L	0.264	0.536 ***	0.280	0.365	0.485 ***	-0.073	-0.641 ***	0.423
	B-2	-0.045	1.081 ***	0.867	-0.077	1.081 ***	-0.019	0.210 ***	0.877
	B-H	0.366 .	0.327 ***	0.270	0.324 .	0.403 ***	0.177 **	0.255 ***	0.367
CROBEX	S-L	0.195	0.815 ***	0.237	0.230	0.866 ***	0.726 ***	-0.616 ***	0.603
	S-2	0.332	0.824 ***	0.245	0.300	0.893 ***	0.664 ***	-0.152 .	0.486
	S-H	0.191	0.941 ***	0.277	0.025	1.044 ***	0.516 ***	0.779 ***	0.653
	2-L	-0.097	0.930 ***	0.427	-0.069	0.934 ***	0.162 **	-0.261 ***	0.467
	2-2	0.138	0.752 ***	0.441	0.111	0.766 ***	0.047	0.151 *	0.459
	2-H	0.269	1.015 ***	0.456	0.189	1.050 ***	0.085	0.462 ***	0.559
	B-L	0.267	1.176 ***	0.465	0.386 .	1.105 ***	-0.335 ***	-0.579 ***	0.663
	B-2	-0.178	0.954 ***	0.227	-0.163	0.887 ***	-0.711 ***	0.286 *	0.423
	B-H	0.376 *	0.800 ***	0.558	0.332 *	0.811 ***	-0.048	0.303 ***	0.633

. $p<0.1$, * $p<0.05$, ** $p<0.01$, *** $p<0.001$

When Market Portfolio proxies for the market, the SMB factor is very highly significant at a 0.1% level for the portfolios in the Small size quantile and the Low BE/ME quantile with the exception of Big/Low portfolio. The coefficient values for the SMB factor are higher in the Small size quantile, where they range from 0.948 to 1.167, than the Big size quantile, where they range from -0.078 to 0.165. The SMB factor is statistically significant for all portfolios in the High BE/ME quantile as well.

In total, the coefficients of the SMB factor decrease from smaller to bigger size quantiles for each book-to-market quantile. Thus, the SMB factor captures common variation left out by the market factor. Again, when switching from monthly returns to biweekly returns, the significance of the SMB factor increases with the increase in the number of observations.

Using CROBEX as a proxy for the market, the HML factor is very highly significant at a 0.1% level for all Low and High BE/ME quantile portfolios. When monthly data are used, the HML factor is not statistically significant for 3 out of 9 portfolio regressions all of which belong to the Medium BE/ME quantile. When biweekly data are used the HML factor is statistically significant for all portfolios. The coefficients for the Low and High BE/ME quantile portfolios experience opposite signs and are greater, in absolute value, than for the Medium BE/ME quantile. Additionally, the values of the coefficients in each size quantile tend to increase with BE/ME.

When Market Portfolio proxies for the market, the HML factor is very highly significant at a 0.1% level for all Low and High BE/ME quantile portfolios. Again, the coefficients for the Low and High BE/ME quantile portfolios experience opposite signs and are greater, in absolute value, than for the Medium BE/ME quantile. As in the CROBEX case, the values of the coefficients in each size quantile tend to increase with BE/ME.

Similarly to the SMB factor, an increase in the significance of the coefficients is present when switching from monthly to biweekly data.

In all four cases observed, the coefficient values for the HML variable increase in each size quantile with BE/ME in addition to experiencing negative values in the Low BE/ME quantile. Thus, the HML factor captures common variation left out by the market and the SMB factor.

6.3. *Double independent sorts*

Table 6.3 presents results of the regressions for all 16 stock portfolios formed through a double independent sorting procedure for the entire sample period from July 2009 to April 2017 using biweekly returns and different proxies for the market portfolio (Market Portfolio, as defined earlier, and CROBEX,The Zagreb Stock Exchange equity index). The SMB and HML factors are formed through an independent sorting procedure.

Adjusted R^2 values are quite higher than when portfolios are constructed through independent sorting. Moreover, a strong pattern can be observed in the values themselves. Looking at the adjusted R^2 values, two distinct groups of portfolios may be observed, those with high and those with low adjusted R^2 values. All portfolios in the Low BE/ME quantile and the Big size quantile form one group, and the remaining portfolios form the other. When Market Portfolio is used to proxy for the market, the latter group has higher adjusted R^2 values ranging from 0.887 to 0.959 while the former group's adjusted R^2 values range 0.180 to 0.329. In the CROBEX case, the opposite holds, with the first group's adjusted R^2 values ranging from 0.358 to 0.471 and the second from 0.539 to 0.705. A possible explanation for the above pattern can be found in the portfolio constructing process as well as in the construction of the market proxy. As observed in an earlier chapter, with the independent sorting procedure Low BE/ME quantile is tilted towards the big size stocks whereas the High BE/ME quantile is tilted towards the small size stocks. Because the portfolios are value-weighted, with the double independent sorts, each portfolio in the Low BE/ME quantile contains the big size stocks that will have the highest weights and therefore, have a greater influence on the portfolio returns. Additionally, the group of Big size stocks with Low BE/ME holds the total market cap value of 43.94% which translates into a high influence of those stocks on the Market Portfolio returns as well. Hence, the high percentage of explained variation for the Market Portfolio case for each Low BE/ME quantile and each Big size quantile. CROBEX, on the other hand, does not contain every stock present on the market at a given year, but a total of 25 most frequently traded stocks with a maximal weight of a 10% for a single stock and not a weight proportional to the total market cap of the stock in the market. Considering that in a simple linear regression environment the R^2 value is simply the square of the sample correlation coefficient between the dependent and the explanatory variable, the pattern in the adjusted R^2 values might be explained by the composition of CROBEX and the dependent variable portfolios.

It is also worth noting that when Market Portfolio is used, the explanatory power of the model tends to increase with size.

In the Market Portfolio case, the only statistically significant intercept appears in the regression for the Small/3 portfolio. On the other hand, 6 out of 16 intercepts are statistically significant at a 5% level in the CROBEX case with all six portfolios belonging to the 2 and 3 BE/ME quantile.

All beta coefficients are highly statistically significant. When Market Portfolio is used, the beta coefficients for portfolios in the Low BE/ME and Big size quantile range from 0.818 to 1.012 while for the remaining portfolios they range from 0.303 to 0.346. When CROBEX proxies for the market portfolio, betas range from 0.852 to 1.105. Additionally, the highest beta values are produced in the Low BE/ME and Big size quantile. Beta coefficients tend to rise with size for all BE/ME quantiles except the Low BE/ME quantile, where the opposite holds.

After adding the SMB and HML factors to the overall regressions, the adjusted R^2 values increase for all portfolios. The increase in adjusted R^2 values ranges from 0.001 to 0.590 and from 0.002 to 0.318 using Market Portfolio and CROBEX as a market proxy, respectively. High BE/ME quantile experiences greater increase in the Market Portfolio case, with the exception of the Big/High portfolio, whereas in

Table 6.3: Results of the CAPM and FF3FM results : Double independent sorts, July 2009 to April 2017

		biweekly returns							
		CAPM			FF3FM				
		Intercept	R_m-R_f	Adj.R^2	Intercept	R_m-R_f	SMB	HML	Adj.R^2
Market Portfolio	S-L	-0.090	1.012 ***	0.887	-0.082	0.943 ***	-0.105 *	-0.142 ***	0.896
	S-2	0.300	0.346 ***	0.298	0.275	0.479 ***	0.308 ***	0.206 ***	0.399
	S-3	0.345 .	0.330 ***	0.253	0.321 .	0.509 ***	0.299 ***	0.352 ***	0.430
	S-H	0.076	0.303 ***	0.168	0.025	0.657 ***	0.629 ***	0.666 ***	0.719
	2-L	-0.096	0.995 ***	0.894	-0.093	0.943 ***	-0.050	-0.125 ***	0.900
	2-2	0.275	0.344 ***	0.303	0.244	0.497 ***	0.368 ***	0.228 ***	0.444
	2-3	0.307	0.334 ***	0.270	0.274	0.535 ***	0.393 ***	0.360 ***	0.500
	2-H	0.069	0.305 ***	0.180	0.002	0.659 ***	0.802 ***	0.561 ***	0.769
	3-L	-0.087	0.961 ***	0.907	-0.082	0.919 ***	-0.068	-0.085 **	0.910
	3-2	0.229	0.343 ***	0.329	0.204	0.483 ***	0.301 ***	0.232 ***	0.454
	3-3	0.264	0.341 ***	0.296	0.240	0.514 ***	0.301 ***	0.330 ***	0.473
	3-H	0.070	0.318 ***	0.260	0.037	0.540 ***	0.408 ***	0.410 ***	0.558
	B-L	-0.001	0.957 ***	0.951	0.008	0.915 ***	-0.102 ***	-0.061 **	0.955
	B-2	0.054	0.818 ***	0.941	0.055	0.826 ***	-0.008	0.029	0.941
	B-3	0.104	0.870 ***	0.954	0.109	0.869 ***	-0.052 .	0.030	0.955
	B-H	0.041	0.888 ***	0.959	0.046	0.880 ***	-0.055 *	0.010	0.960
CROBEX	S-L	-0.194	1.105 ***	0.358	-0.076	1.047 ***	-0.652 ***	-0.592 ***	0.677
	S-2	0.311 *	0.852 ***	0.624	0.307 *	0.852 ***	0.038	-0.010	0.621
	S-3	0.361 *	0.869 ***	0.608	0.353 *	0.877 ***	0.011	0.121 **	0.625
	S-H	0.102	0.920 ***	0.540	0.049	0.952 ***	0.254 ***	0.361 ***	0.733
	2-L	-0.198	1.086 ***	0.361	-0.089	1.031 ***	-0.597 ***	-0.576 ***	0.661
	2-2	0.287 *	0.853 ***	0.644	0.275 *	0.856 ***	0.087 .	0.003	0.646
	2-3	0.322 *	0.876 ***	0.644	0.304 *	0.886 ***	0.089 .	0.115 **	0.668
	2-H	0.092	0.893 ***	0.539	0.023	0.922 ***	0.424 ***	0.254 ***	0.740
	3-L	-0.182	1.079 ***	0.388	-0.076	1.027 ***	-0.601 ***	-0.523 ***	0.680
	3-2	0.241 *	0.855 ***	0.705	0.236 .	0.857 ***	0.028	0.014	0.703
	3-3	0.280 *	0.890 ***	0.695	0.273 *	0.898 ***	0.010	0.098 **	0.707
	3-H	0.089	0.880 ***	0.688	0.067	0.894 ***	0.101 *	0.164 ***	0.739
	B-L	-0.0960	1.072 ***	0.405	0.014	1.021 ***	-0.632 ***	-0.498 ***	0.711
	B-2	-0.020	0.991 ***	0.471	0.063	0.953 ***	-0.485 ***	-0.364 ***	0.702
	B-3	0.022	1.023 ***	0.449	0.116	0.982 ***	-0.555 ***	-0.384 ***	0.699
	B-H	-0.044	1.031 ***	0.440	0.052	0.988 ***	-0.565 ***	-0.410 ***	0.702

. p<0.1, * p<0.05, ** p<0.01, *** p<0.001

the CROBEX case the highest increase in the explained variation occurs in the Low BE/ME and Big size quantile. Furthermore, the significance of the intercepts remains unchanged exclusive of the 3/2 portfolio in the CROBEX case where the significance changes from significant at a 5% level to significant at a 10% level.

The beta coefficients range from 0.479 to 0.943 and from 0.852 to 1.047 when using Market Portfolio and CROBEX, respectively. It is also worth noting that in the Market Portfolio case, beta coefficients above 0.880 in the CAPM, have now decreased, and those under 0.880 have increased. In the CROBEX case, beta values do not change as much for most portfolios.

When Market Portfolio proxies for the market, the SMB factor is very highly significant at a 0.1% level for 10 out of 16 portfolios, most of which belong to the 2, 3 and High BE/ME quantile. The SMB factor is not statistically significant for almost portfolios in the Low BE/ME and the Big size quantile, with the exception of Big/Low portfolio where it is significant at a 0.1% level. all Small size quantile portfolios and 3 out of 4 medium size quantile portfolios when Market Portfolio is used to proxy the market. The coefficient values for the SMB variable range from 0.691 to 1.442 in the Small size quantile and from -0.172 to 0.481 in the Big size quantile.

When CROBEX is used, the SMB factor is significant at a 0.1% level for 9 out of 16 portfolios, all of which belong to the Low or High BE/ME quantile and the Big size quantile. 7 out of 9 highly significant SMB slopes are negative, with the two positive ones belonging to the Small/High and 2/High portfolio which mostly contain no Big size stocks at all.

In total, in each size quantile, with the exception of the Big size quantile, the coefficients for the SMB variable increase with BE/ME. Based on the significance levels and the absolute values of the coefficients, it is safe to say that the SMB factor captures common variation left out by the market factor both in the Market Portfolio and the CROBEX case.

When Market Portfolio proxies for the market, the HML factor is statistically significant at a 0.1% level for 11 out of 16 portfolios. The HML factor is not statistically significant for 3 portfolios in the Big size quantile, the Big/2, the Big/3 and the Big/High portfolio. Again, HML slopes experience negative values in the Low BE/ME quantile and increase when moving to High BE/ME quantile in all four cases observed.

When CROBEX is used, the HML factor is statistically significant at a 0.1% level for 10 out of 16 portfolios. The HML factor is not statistically significant for 3 portfolios in the 2 BE/ME quantile, the Small/2, the 2/2 and the 3/2 portfolio. 7 out of 10 highly significant HML slopes are negative, with the three positive ones belonging to the Small/High, 2/High and 3/High portfolio. In each size quantile, with the exception of the Big size quantile, the coefficient for the HML variable increases with BE/ME and therefore the HML factor captures common variation left out by the market and the SMB factor.

7. *Conclusion*

The FF3FM is a better description of returns on portfolios formed on size and book-to-market equity than the CAPM. The SMB and the HML factor are not always statistically significant but they do capture common variation in returns that is missed by the market factor.

With that being said, CAPM is not a bad model. For most portfolios, the intercepts are not statistically significant and for some portfolios, the FF3FM provides only slightly higher percentage of explained variation despite the inclusion of the two additional factors. That is especially the case with all the portfolios in the Low BE/ME quantile and all the portfolios in the Big size quantile when the double independent sorting procedure is used and the Market Portfolio proxies for the market. Similar is true for the two Medium BE/ME quantiles, with the exception of the Big size quantile, when CROBEX is used.

Biweekly returns might be preferred to monthly returns given that they provide for twice as many observations and ensure lower volatility in the returns (at least for the observed time period).

The Market Portfolio, which might be regarded as a better approximation of the market than an equity index consisting of 25 stocks with individual weights constrained to 10%, exhibits higher correlation with the SMB and the HML factors and is far inferior to the CROBEX equity index in the CAPM in terms of explained variation, except when the double independent sorting procedure is used.

Independent sorting procedure emphasizes one of the features of the Croatian stock market, most of the small cap firms inherently having high BE/ME ratios and most of the large cap firms having low BE/ME ratios, and therefore results in a not completely adequate number of stocks in each of the 9 portfolios (Big/High portfolio consists on average of 1.75 stocks). CAPM performs well, almost all intercepts are not statistically significant. CAPM performs on average better, in terms of explained variation, when CROBEX proxies for the market as opposed to the Market Portfolio.

As opposed to independent sorting, the conditional sorting procedure ensures an equal number of stocks in each portfolio. The Big/High portfolio experiences statistically significant intercepts for all cases of market proxies and data sets used. The additional factors seem to increase the explanatory power of the model the most in the Small size quantile.

With double independent sorting, the FF3FM seems to capture much of the cross-sectional variation in average stock returns but fares poorly with the middle BE/ME quantile portfolios in the CROBEX case. The additional factors seem to increase the explanatory power of the model the most in the High the Low BE/ME quantile when Market Portfolio and CROBEX are used, respectively. Also, the adjusted R^2 values have the lowest spread in the CROBEX case and are all quite high around 0.7 while in the Market Portfolio case the adjusted R^2 values for the Low BE/ME and Big size portfolios are around 0.9 and higher.

The three-factor model might work better when further adapted in a way that is more meaningful for the developing markets. It would be interesting to see the effects of the monthly rebalancing of the portfolios since that might help to better capture some of the time-varying dimensions of risk such as low liquidity or distress in the market.

Also, seeing that different design choices for factor and portfolio construction result in different portfolio exposures to risk, it would be useful to explore this further by changing the breakpoints during the portfolio construction process. Shorter return intervals might also help.

A. *Appendix*

It is common practice to exclude the financial firms from the sample since they normally use a high level of leverage that for a nonfinancial firm would more likely be interpreted as a sign of distress. However, other capital-intensive industries, such as telecommunications or oil and gas refining typically experience a higher level of leverage. Given the characteristics of the Croatian stock market, the exclusion of the financial firms may not be as justified as is in the case of the U.S. stock market. Therefore, the corresponding tables of the data and the results of the OLS regressions, with financial firms excluded from the sample, are presented in the appendix.

Table A.1: Summary statistics for explanatory returns: July 2009 to April 2017

Panel A: Monthly data, 94 Observations

	mean	sd	t_{mean}	Correlation					
				MP-r_{rf}	CRO-r_{rf}	SMB_I	HML_I	SMB_C	HML_C
MP-r_{rf}	0.28	5.60	0.48						
CRO-r_{rf}	-0.17	4.47	-0.36	0.88	1.00				
SMB_I	0.11	3.92	0.27	-0.24	-0.05	1.00			
HML_I	-0.13	6.06	-0.21	-0.31	-0.12	-0.02	1.00		
SMB_C	0.05	4.85	0.10	-0.15	0.02	0.60	0.40	1.00	
HML_C	0.76	4.33	1.71	-0.25	-0.05	0.01	0.69	0.16	1.00

Panel B: Biweekly data, 190 Observations

	mean	sd	t_{mean}	Correlation					
				MP-r_{rf}	CRO-r_{rf}	SMB_I	HML_I	SMB_C	HML_C
MP-r_{rf}	0.13	3.58	0.52						
CRO-r_{rf}	-0.10	2.98	-0.45	0.85	1.00				
SMB_I	0.07	3.12	0.29	-0.14	0.02	1.00			
HML_I	-0.20	4.26	-0.64	-0.29	-0.09	-0.17	1.00		
SMB_C	0.07	3.53	0.29	-0.23	-0.06	0.62	0.29	1.00	
HML_C	0.27	3.28	1.14	-0.23	-0.04	-0.02	0.66	0.09	1.00

Table A.2: Descriptive statistics for portfolios formed on size and book-to-market equity: July 2009 to April 2017

Size quantile	Book-to-market equity (BE/ME) quantiles: Low	2	High	Low	2	High	
	Average of annual averages of firm size			Average of annual number of firms in portfolio			Independent sorts
Small	5.69	6.80	5.79	4.50	4.50	12.75	
2	31.10	30.21	24.48	5.88	7.50	7.38	
Big	366.76	170.08	113.04	10.88	8.75	1.62	
	Average of annual percent of market value in portfolio			Average of annual B/E ratios (for portfolio)			
Small	0.43	0.56	1.33	0.55	1.37	4.65	
2	3.19	4.00	3.07	0.69	1.40	3.22	
Big	57.87	26.21	3.34	0.70	1.35	4.19	
	Average of annual averages of firm size			Average of annual number of firms in portfolio			Conditional sorts
Small	6.52	6.23	5.23	7.25	6.88	7.62	
2	31.16	29.37	24.97	6.88	6.62	7.25	
Big	491.46	171.00	141.64	7.00	6.88	7.38	
	Average of annual percent of market value in portfolio			Average of annual B/E ratios (for portfolio)			
Small	0.84	0.75	0.73	0.80	2.43	5.96	
2	3.73	3.39	3.14	0.77	1.47	3.22	
Big	49.26	20.43	17.72	0.57	1.02	2.06	

	Low	2	3	High	Low	2	3	High	
	Average of annual averages of firm size				Average of annual number of firms in portfolio				Independent double sorts
Small	120.22	43.07	33.84	9.99	32.38	31.88	32.00	32.50	
2	127.67	49.40	40.00	15.68	31.88	31.38	31.50	32.00	
3	142.68	64.34	54.84	30.19	31.75	31.25	31.38	31.88	
Big	284.19	209.32	199.40	172.90	32.25	31.75	31.88	32.38	
	Average of annual percent of market value in portfolio				Average of annual B/E ratios (for portfolio)				
Small	8.13	3.30	2.55	0.79	1.91	2.20	2.52	4.03	
2	8.55	3.71	2.97	1.20	1.46	1.75	2.07	3.62	
3	9.64	4.80	4.05	2.29	1.02	1.30	1.63	3.18	
Big	19.57	14.73	13.99	12.22	0.87	1.14	1.46	3.00	

Table A.3: Summary statistics for the size-B/M excess returns :
July 2009 to April 2017

Panel A: Monthly excess returns for 9 portfolios formed on size and B/M

	Mean			Standard deviation		
	Low	2	High	Low	2	High
Independent sorts						
Small	-0.39	-0.15	0.92	9.91	7.59	6.74
2	-0.89	0.63	0.24	7.50	4.70	6.79
Big	0.34	0.70	-1.28	7.50	4.34	6.18
Conditional sorts						
Small	-0.23	0.70	0.91	7.92	6.87	8.40
2	-0.74	0.54	0.34	6.52	4.94	6.56
Big	0.36	0.44	0.43	8.40	4.71	4.43

Panel B.a: Biweekly excess returns for 9 portfolios formed on size and B/M

	Mean			Standard deviation		
	Low	2	High	Low	2	High
Independent sorts						
Small	0.10	0.09	0.46	7.50	5.64	4.67
2	-0.37	0.32	0.06	5.08	3.39	4.78
Big	0.16	0.38	-0.59	4.77	2.88	4.56
Conditional sorts						
Small	0.11	0.41	0.40	5.74	5.08	5.61
2	-0.32	0.30	0.09	4.47	3.66	4.62
Big	0.17	0.25	0.28	5.31	3.22	3.06

Panel B.b: Biweekly excess returns for 16 portfolios formed on size and B/M

	Mean				Standard deviation			
	Low	2	3	High	Low	2	3	High
Independent double sorts								
Small	0.11	0.33	0.32	0.03	4.86	2.98	3.21	3.89
2	0.11	0.31	0.29	0.05	4.68	2.97	3.18	3.78
3	0.07	0.24	0.20	0.01	4.36	2.91	3.10	3.29
Big	0.20	0.22	0.26	0.21	4.21	3.39	3.58	3.72

Table A.4: Results of the CAPM and FF3FM : Independent sorts, July 2009 to April 2017

		monthly returns							
		CAPM			FF3FM				
		Intercept	R_m-R_f	Adj.R^2	Intercept	R_m-R_f	SMB	HML	Adj.R^2
Market Portfolio	S-L	-0.655	0.970 ***	0.293	-0.789	1.061 ***	0.830 ***	-0.139	0.394
	S-2	-0.299	0.533 ***	0.145	-0.445	0.712 ***	0.957 ***	0.061	0.362
	S-H	0.736	0.650 ***	0.284	0.656 .	0.971 ***	0.673 ***	0.634 ***	0.679
	2-L	-1.072	0.647 ***	0.225	-1.202 *	0.882 ***	0.901 ***	0.261 *	0.447
	2-2	0.489	0.510 ***	0.362	0.443	0.625 ***	0.340 ***	0.178 **	0.463
	2-H	0.039	0.735 ***	0.361	-0.043	1.059 ***	0.684 ***	0.637 ***	0.759
	B-L	0.008	1.199 ***	0.799	0.044	1.063 ***	-0.297 ***	-0.262 ***	0.854
	B-2	0.554 .	0.541 ***	0.482	0.545 .	0.602 ***	0.091	0.139 *	0.509
	B-H	-1.420 *	0.489 ***	0.187	-1.444 **	0.652 ***	0.248 .	0.370 ***	0.306
CROBEX	S-L	-0.170	1.304 ***	0.339	-0.276	1.272 ***	0.536 **	-0.325 *	0.414
	S-2	-0.029	0.739 ***	0.181	-0.117	0.759 ***	0.754 ***	-0.072	0.324
	S-H	1.086 *	1.020 ***	0.453	1.118 **	1.115 ***	0.402 ***	0.459 ***	0.665
	2-L	-0.720	1.041 ***	0.379	-0.770	1.088 ***	0.659 ***	0.109	0.495
	2-2	0.756 *	0.754 ***	0.509	0.750 *	0.774 ***	0.169 .	0.070	0.526
	2-H	0.427	1.111 ***	0.531	0.458	1.203 ***	0.388 ***	0.445 ***	0.728
	B-L	0.542	1.212 ***	0.517	0.530	1.107 ***	-0.600 ***	-0.464 ***	0.745
	B-2	0.835 **	0.792 ***	0.662	0.849 **	0.795 ***	-0.071	0.040	0.662
	B-H	-1.143 *	0.849 ***	0.371	-1.109 *	0.897 ***	0.075	0.266 **	0.428

		biweekly returns							
		CAPM			FF3FM				
		Intercept	R_m-R_f	Adj.R^2	Intercept	R_m-R_f	SMB	HML	Adj.R^2
Market Portfolio	S-L	-0.010	0.805 ***	0.143	-0.110	0.838 ***	0.843 ***	-0.205 .	0.286
	S-2	0.030	0.431 ***	0.070	-0.008	0.598 ***	0.815 ***	0.192 *	0.254
	S-H	0.371	0.636 ***	0.234	0.397 .	0.919 ***	0.712 ***	0.559 ***	0.589
	2-L	-0.469	0.736 ***	0.265	-0.479	0.883 ***	0.545 ***	0.228 **	0.375
	2-2	0.251	0.512 ***	0.289	0.249	0.633 ***	0.404 ***	0.204 ***	0.440
	2-H	-0.035	0.725 ***	0.291	0.021	1.038 ***	0.583 ***	0.693 ***	0.681
	B-L	0.001	1.191 ***	0.800	-0.011	1.078 ***	-0.273 ***	-0.228 ***	0.852
	B-2	0.302 .	0.549 ***	0.463	0.315 *	0.619 ***	0.126 *	0.154 ***	0.510
	B-H	-0.656 *	0.511 ***	0.156	-0.611 *	0.656 ***	0.134	0.370 ***	0.253
CROBEX	S-L	0.211	1.160 ***	0.208	0.091	1.101 ***	0.654 ***	-0.363 ***	0.336
	S-2	0.152	0.659 ***	0.116	0.122	0.657 ***	0.680 ***	0.071	0.246
	S-H	0.545 .	0.909 ***	0.332	0.589 *	0.950 ***	0.505 ***	0.369 ***	0.516
	2-L	-0.264	1.084 ***	0.400	-0.275	1.086 ***	0.346 ***	0.056	0.438
	2-2	0.399 *	0.806 ***	0.498	0.399 *	0.812 ***	0.262 ***	0.083 *	0.554
	2-H	0.171	1.119 ***	0.483	0.250	1.178 ***	0.348 ***	0.484 ***	0.684
	B-L	0.271	1.127 ***	0.492	0.210	1.075 ***	-0.516 ***	-0.453 ***	0.722
	B-2	0.452 ***	0.778 ***	0.642	0.460 ***	0.782 ***	-0.013	0.035	0.642
	B-H	-0.502 .	0.871 ***	0.320	-0.449 .	0.905 ***	-0.014	0.249 ***	0.368

. $p<0.1$, * $p<0.05$, ** $p<0.01$, *** $p<0.001$

Table A.5: Results of the CAPM and FF3FM : Conditional sorts, July 2009 to April 2017

monthly returns

		CAPM			FF3FM				
		Intercept	R_m-R_f	Adj.R^2	Intercept	R_m-R_f	SMB	HML	Adj.R^2
Market Portfolio	S-L	-0.408	0.645 ***	0.199	0.060	0.665 ***	1.198 ***	-0.698 ***	0.792
	S-2	0.542	0.555 ***	0.196	0.452	0.674 ***	0.884 ***	0.017	0.572
	S-H	0.684	0.815 ***	0.288	-0.203	1.116 ***	0.812 ***	0.999 ***	0.811
	2-L	-0.916	0.624 ***	0.279	-0.832	0.647 ***	0.388 **	-0.144	0.349
	2-2	0.396	0.518 ***	0.337	0.233	0.574 ***	0.157 .	0.182 .	0.377
	2-H	0.140	0.701 ***	0.351	-0.273	0.841 ***	0.373 ***	0.465 ***	0.526
	B-L	-0.011	1.320 ***	0.771	0.314	1.213 ***	-0.258 ***	-0.371 ***	0.831
	B-2	0.274	0.602 ***	0.507	-0.024	0.674 ***	0.009	0.364 ***	0.605
	B-H	0.296	0.486 ***	0.371	0.019	0.568 ***	0.143 *	0.324 ***	0.491
CROBEX	S-L	-0.076	0.923 ***	0.264	0.508	0.855 ***	1.086 ***	-0.851 ***	0.822
	S-2	0.845	0.895 ***	0.332	0.907 *	0.870 ***	0.771 ***	-0.137	0.615
	S-H	1.110	1.204 ***	0.405	0.523	1.225 ***	0.631 ***	0.731 ***	0.721
	2-L	-0.583	0.960 ***	0.427	-0.382	0.940 ***	0.277 **	-0.285 *	0.482
	2-2	0.663 .	0.745 ***	0.449	0.621	0.746 ***	0.061	0.051	0.443
	2-H	0.513	1.077 ***	0.534	0.295	1.085 ***	0.232 *	0.273 **	0.598
	B-L	0.572	1.307 ***	0.479	1.099 *	1.285 ***	-0.453 ***	-0.664 ***	0.688
	B-2	0.588 *	0.883 ***	0.700	0.434 .	0.896 ***	-0.105 *	0.211 ***	0.738
	B-H	0.557 .	0.761 ***	0.587	0.406	0.770 ***	0.047	0.196 **	0.621

biweekly returns

		CAPM			FF3FM				
		Intercept	R_m-R_f	Adj.R^2	Intercept	R_m-R_f	SMB	HML	Adj.R^2
Market Portfolio	S-L	0.041	0.532 ***	0.105	0.146	0.632 ***	1.124 ***	-0.748 ***	0.708
	S-2	0.332	0.549 ***	0.145	0.221	0.766 ***	0.901 ***	0.055	0.515
	S-H	0.311	0.680 ***	0.184	-0.066	1.072 ***	0.827 ***	0.965 ***	0.767
	2-L	-0.418	0.705 ***	0.315	-0.419	0.754 ***	0.310 ***	-0.104	0.370
	2-2	0.231	0.529 ***	0.264	0.147	0.622 ***	0.223 ***	0.203 **	0.336
	2-H	-0.003	0.691 ***	0.282	-0.202	0.876 ***	0.297 ***	0.560 ***	0.483
	B-L	-0.007	1.298 ***	0.764	0.113	1.175 ***	-0.248 ***	-0.312 ***	0.825
	B-2	0.166	0.616 ***	0.465	0.081	0.681 ***	0.038	0.272 ***	0.536
	B-H	0.208	0.533 ***	0.385	0.108	0.613 ***	0.062	0.312 ***	0.493
CROBEX	S-L	0.192	0.821 ***	0.177	0.354 .	0.857 ***	1.030 ***	-0.866 ***	0.765
	S-2	0.484	0.797 ***	0.213	0.456 .	0.849 ***	0.777 ***	-0.094	0.498
	S-H	0.501	1.014 ***	0.286	0.256	1.094 ***	0.649 ***	0.754 ***	0.678
	2-L	-0.223	1.019 ***	0.457	-0.172	1.022 ***	0.197 **	-0.244 ***	0.503
	2-2	0.380 .	0.802 ***	0.423	0.349 .	0.815 ***	0.129 *	0.086	0.440
	2-H	0.196	1.090 ***	0.489	0.080	1.119 ***	0.163 **	0.395 ***	0.585
	B-L	0.285	1.197 ***	0.447	0.460 *	1.140 ***	-0.446 ***	-0.545 ***	0.663
	B-2	0.332 *	0.852 ***	0.617	0.298 *	0.853 ***	-0.068 .	0.143 **	0.638
	B-H	0.358 *	0.800 ***	0.602	0.307 *	0.806 ***	-0.031	0.196 ***	0.642

. p<0.1, * p<0.05, ** p<0.01, *** p<0.001

Table A.6: Results of the CAPM and FF3FM : Double independent sorts, July 2009 to April 2017

		biweekly returns							
		CAPM			FF3FM				
		Intercept	R_m-R_f	Adj.R^2	Intercept	R_m-R_f	SMB	HML	Adj.R^2
Market Portfolio	S-L	-0.048	1.208 ***	0.792	-0.063	1.094 ***	-0.248 ***	-0.238 ***	0.842
	S-2	0.252	0.590 ***	0.500	0.262 .	0.684 ***	0.229 ***	0.189 ***	0.592
	S-3	0.248	0.571 ***	0.404	0.258	0.684 ***	0.284 ***	0.223 ***	0.520
	S-H	-0.056	0.609 ***	0.311	-0.013	0.885 ***	0.570 ***	0.590 ***	0.773
	2-L	-0.048	1.171 ***	0.802	-0.064	1.077 ***	-0.178 ***	-0.207 ***	0.838
	2-2	0.231	0.592 ***	0.507	0.237 .	0.706 ***	0.319 ***	0.212 ***	0.657
	2-3	0.209	0.582 ***	0.427	0.215	0.719 ***	0.394 ***	0.253 ***	0.621
	2-H	-0.029	0.601 ***	0.321	-0.013	0.866 ***	0.731 ***	0.501 ***	0.827
	3-L	-0.076	1.116 ***	0.838	-0.085	1.041 ***	-0.169 ***	-0.154 ***	0.864
	3-2	0.157	0.594 ***	0.532	0.171	0.697 ***	0.230 ***	0.216 ***	0.646
	3-3	0.124	0.601 ***	0.481	0.136	0.716 ***	0.277 ***	0.232 ***	0.609
	3-H	-0.077	0.611 ***	0.439	-0.051	0.784 ***	0.363 ***	0.368 ***	0.692
	B-L	0.058	1.088 ***	0.854	0.052	0.997 ***	-0.243 ***	-0.173 ***	0.900
	B-2	0.098	0.898 ***	0.896	0.100	0.865 ***	-0.116 ***	-0.053 **	0.907
	B-3	0.131	0.936 ***	0.877	0.132	0.890 ***	-0.147 ***	-0.078 ***	0.895
	B-H	0.082	0.977 ***	0.885	0.083	0.928 ***	-0.163 ***	-0.084 ***	0.905
CROBEX	S-L	0.226	1.143 ***	0.488	0.161	1.089 ***	-0.495 ***	-0.467 ***	0.710
	S-2	0.411 **	0.816 ***	0.663	0.417 ***	0.822 ***	0.075 .	0.055 .	0.670
	S-3	0.408 **	0.856 ***	0.629	0.419 **	0.866 ***	0.129 **	0.092 **	0.651
	S-H	0.120	0.958 ***	0.535	0.182	1.007 ***	0.370 ***	0.413 ***	0.780
	2-L	0.219	1.119 ***	0.504	0.156	1.069 ***	-0.421 ***	-0.433 ***	0.698
	2-2	0.392 **	0.833 ***	0.695	0.397 ***	0.840 ***	0.160 ***	0.073 **	0.726
	2-3	0.373 **	0.874 ***	0.669	0.381 **	0.886 ***	0.232 ***	0.114 ***	0.730
	2-H	0.144	0.940 ***	0.546	0.177	0.975 ***	0.536 ***	0.327 ***	0.821
	3-L	0.182	1.105 ***	0.566	0.131	1.062 ***	-0.404 ***	-0.370 ***	0.742
	3-2	0.319 **	0.845 ***	0.745	0.331 **	0.854 ***	0.072 *	0.080 **	0.759
	3-3	0.292 *	0.893 ***	0.737	0.304 **	0.904 ***	0.115 **	0.094 ***	0.760
	3-H	0.096	0.925 ***	0.699	0.129	0.950 ***	0.186 ***	0.215 ***	0.789
	B-L	0.307	1.053 ***	0.552	0.258 .	1.010 ***	-0.469 ***	-0.381 ***	0.773
	B-2	0.310 *	0.936 ***	0.671	0.282 **	0.910 ***	-0.311 ***	-0.231 ***	0.808
	B-3	0.352 *	0.974 ***	0.655	0.321 **	0.945 ***	-0.348 ***	-0.261 ***	0.810
	B-H	0.310 .	0.990 ***	0.626	0.277 *	0.960 ***	-0.372 ***	-0.276 ***	0.789

. $p<0.1$, * $p<0.05$, ** $p<0.01$, *** $p<0.001$

Bibliography

[1] B. Akgul. Fama French Three Factor Regression on European Stock Markets Before and After EMU. Master's thesis, Tilburg School of Economics and Management, 2013. `http://arno.uvt.nl/show.cgi?fid=128520`.

[2] G. Diacogiannis and P. Makri. Estimating Betas in Thinner Markets: The Case of the Athens Stock Exchange. *International Research Journal of Finance and Economics*, 2008.

[3] D. Dolinar. Test of the Fama-French three-factor model in Crotia. *UTMS Journal of Economics, Vol. 4, No. 2*, pages 101–112, 2013.

[4] V. Eraslan. Fama and French Three-Factor Model: Evidence from Istanbul Stock Exchange. *Business and Economics Research Journal, Vol. 4, No. 2*, pages 11–22, 2013.

[5] Cohen et al. Implications of Microstructure Theory for Empirical Research on Stock Price Behaviour. *Journal of Finance, Vol. 35*, pages 249–257, 1980.

[6] Cohen et al. Friction in the Trading Process and the Estimation of the Systematic Risk. *Journal of Financial Economics, Vol. 12*, pages 263–278, 1983.

[7] E. F. Fama and K. R. French. The Cross-section of Expected Stock Returns. *Journal of Finance, Vol. 47*, pages 427–465, 1992.

[8] E. F. Fama and K. R. French. Common Risk Factors in the Returns on Stocks and Bonds. *Journal of Financial Economics, Vol. 33*, pages 3–56, 1993.

[9] E. F. Fama and K. R. French. Multifactor Explanations of Asset Pricing Anomalies. *Journal of Finance, Vol. 51*, pages 55–84, 1996.

[10] E. F. Fama and K. R. French. Size, Value, and Momentum in International Stock Returns. *Journal of Financial Economics, Vol. 105*, pages 457–472, 2012.

[11] G. Chang G. Pettengill and J. Hueng. Risk-return Predictions with the Fama-french Three-factor Model Betas. *International Journal of Economics and Finance, Vol. 5, No. 1*, 2012.

[12] J. M. Griffin. Are the Fama and French Factors Global or Country Specific? *Review of Financial Studies, Vol. 15*, page 783803, 2002.

[13] G. Hawawini. Why Beta Shifts as the Return Interval Changes. *Financial Analysts Journal, Vol. 39*, pages 73–77, May 1983.

[14] D. Kilsgård and F. Wittorf. The Fama and French Three-Factor Model - Evidence from the Swedish Stock Market. Master's thesis, Lund University, 2010. .

[15] M. Lambert and G. Hübner. Size Matters, Book Value Does Not! The Fama-French Empirical CAPM Revisited. 2014.

[16] L. P. Lunden. The Brazilian Stock Market. Master's thesis, University of Oslo, 2007. `https://www.duo.uio.no/bitstream/handle/10852/17455/Thesis2.pdf?sequence=1&isAllowed=y`.

[17] R. C. Merton. An Analytic Derivation of the Efficient Portfolio Frontier. *The Journal of Financial and Quantitative Analysis, Vol. 7, No. 4*, pages 1851–1872, 1972.

[18] R. Roll. A Possible Explanation of the Small Firm Effect. *Journal of Finance, Vol. 36*, pages 879–888, 1981.

[19] M. Scholes and J. Williams. Estimating Betas from Non-synchronous Data. *Journal of Financial Economics, Vol. 5*, pages 309–327, 1977.

Abstract

The thesis tests the Fama and French Three-Factor Model on the Croatian stock market. The performance of the model is compared with that of the Capital Asset Pricing Model. Based on previous attempts of researchers to apply the three-factor model of Fama and French to developing markets, three different portfolio constructing procedures were chosen and compared. Additionally, the implications of using two different proxies for the market and two different return intervals are studied and compared. Overall, the size and the book-to-market factor add to the explanation of the cross-section of average stock returns provided by the market factor. Fama and French Three-Factor Model explains the cross-section of average stock returns in the Croatian market and provides a greater explanatory power in comparison with the Capital Asset Pricing Model. However, much of the common variation in stock returns still remains to be explained by additional factors that reflect the defining characteristics of the emerging markets.

Sažetak

Rad testira Fama-French trofaktorski model na Hrvatskom tržištu dionica. Performansa modela usporedena je sa performansom CAPM-a. Na temelju prethodnih pokušaja primjene trofaktorskog modela Fame i Frencha na tržišta u razvoju, odabrana su i komparirana tri različita postupka formiranja portfelja. Nadalje, proučavaju se i usporeduju implikacije upotrebe dvaju različitih proxyja za tržište i dva različita intervala povrata. Sveukupno, veličina kompanije i book-to-market faktor dodatno doprinose nivou objašnjenje varijacije presjeka prosječnih povrata iznad onog koji daje sam tržišni faktor. Fama-French trofaktorski model objašnjava presjek prosječnih povrata dionica na hrvatskom tržištu i objašnjava veći dio varijacije u povratima nego CAPM. Ipak, preostao je dio zajedničke varijacije u povratima dionica koji bi se mogao objasniti dodatnim faktorima koji odražavaju ključna obilježja tržišta u razvoju.

Životopis

Maja Koprčina (rođena 1. ožujka 1992.) pohađa osnovnu školu Milana Langa u Bregani. Nakon osnovne škole upisuje II. gimnaziju u Zagrebu. 2014. završava sveučilišni preddiplomski studij matematike na Matematičkom odsjeku Prirodoslovno-matematičkog fakulteta i upisuje diplomski sveučilišni studij Financijska i poslovna matematika na istom fakultetu. Nakon završene prve godine diplomskog studija, u sklopu Erasmus+ programa odlazi na studijsku razmjenu u trajanju od godine dana na sveučilište Bielefeld u Njemačkoj. 2016. nastavlja sa drugom godinom diplomskog studija u Zagrebu.