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Paar, Vladimir; Vorkapić, D.; Dieperink, A. E. L.

GOE-Type Energy Level Statistics and Regular Classical Dynamics for Rotational Nuclei in the Interacting Boson Model

V. Paar,[1] D. Vorkapić,[1] and A.E. L. Dierperink[2]
[1]*Department of Physics, Faculty of Science, University of Zagreb, 41000 Zagreb, Croatia*
[2]*Kernfysisch Versneller Instituut, Rijksuniversiteit, 9747 AA Groningen, The Netherlands*
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We study the fluctuation properties of 0^+ levels in rotational nuclei using the framework of SU(3) dynamical symmetry of the interacting boson model. Computations of Poincaré sections for SU(3) dynamical symmetry and its breaking confirm the expected relation between dynamical symmetry and classical chaos. On the other hand, the quantal energy level statistics in the realistic case of rotational nuclei is close to Gaussian-orthogonal-ensemble statistics, contrary to expectations. The conjectured relation between energy fluctuations and chaos is approached asymptotically. Several predictions for chaotic behavior of normally deformed and superdeformed rotational nuclei are given.

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Several recent papers have treated the intriguing problem of the connection between dynamical symmetry and quantum chaos [1,2], using coherent states based on a fixed chain of irreducible representations of a Lie group to define a dynamical symmetry through Casimir operators. In that approach, the existence of dynamical symmetry was taken to specify the absence of chaos. Conversely, the breaking of a dynamical symmetry was taken to be associated with chaos. These conclusions were illustrated [1] with some examples with the SU(3) dynamical group, such as the Elliott SU(3) model [3] for light nuclei and the quark model [4] in particle physics. Alhassid *et al.* have investigated [2] the chaotic properties of the interacting boson model [5] (IBM), which is characterized by the U(6) group structure. They have concluded [2] that near the SU(3) and O(6) dynamical symmetries of the IBM the fluctuation behavior of the states with angular momentum $L \geq 2$ is close to the Poisson statistics, changing gradually to the Gaussian-orthogonal-ensemble [6] (GOE) statistics as the interaction strength moves away from these dynamical symmetries. This was found to be consistent with the study [2] of the underlying classical motion using Monte Carlo calculations applied to boson condensates.

Here we investigate the collective states of zero angular momentum and positive parity ($J^\pi = 0^+$) in rotational nuclei, described in the framework of the interacting boson model [5] of Iachello and Arima, and we find some new chaotic features. In the IBM one considers a system of N bosons that occupy an s ($J=0$) level and a fivefold degenerate d_μ ($J=2$) level, interacting through a Hamiltonian that can be expressed in terms of 36 generators $s^\dagger s$, $d_\mu^\dagger s$, $s^\dagger d_\mu$, and $d_\mu^\dagger d_\nu$ of U(6). This model is of particular interest for the studies of regular and chaotic motion since it corresponds to a real physical system, provides a framework for algebraic description of states with three dynamical symmetries, has a microscopic basis, and successfully describes the low-lying nuclear phenomenology accounting for collectivity. In particular the SU(3) dynamical symmetry [5] of the IBM corresponds to rotational nuclei. The corresponding Hamiltonian can be presented in the form [7]

$$H^{\mathrm{SU}(3)} = -\frac{2\kappa_2}{N_B-1} Q^{(2)} \cdot Q^{(2)} + \frac{\kappa_1}{N_B-1} J^{(1)} \cdot J^{(1)} \qquad (1)$$

with the SU(3) quadrupole operator $Q_\mu^{(2)} = d_\mu^\dagger s + s^\dagger \tilde{d}_\mu \pm \frac{1}{2}\sqrt{7}(d^\dagger \tilde{d})_\mu^{(2)}$ and the angular momentum operator $J_\nu^{(1)} = \sqrt{10}(d^\dagger \tilde{d})_\nu$. Here, κ_2 and κ_1 are the interaction strengths, N_B is the total number of bosons, and the $\pm$ signs correspond to a prolate (oblate) shape.

The classical limit of the IBM Hamiltonian was constructed [7,8], in accordance with the general procedure [9], as an expectation value of the Hamiltonian, with the group realization for the coherent state. We limit our investigation to zero angular momentum and positive parity ($J^\pi = 0^+$), in analogy to the limitation to $L=0$ in the study of the hydrogen atom in a uniform magnetic field [10]. The classical Hamiltonian corresponding to the quantal Hamiltonian (1) for $J^\pi = 0^+$ is [7]

$$\begin{aligned} H_{\mathrm{cl}}^{\mathrm{SU}(3)} = \kappa_2 (& -P_\gamma^4/4\beta^4 + P_\gamma^2(5\beta^2 - P_\beta^2)/2\beta^2 - 4\beta^2 + 2\beta^2(\beta^2+P_\beta^2) - (\beta^2+P_\beta^2)^2/4 \\ & \pm 2[1 - P_\gamma^2/2\beta^2 - (\beta^2+P_\beta^2)/2]^{1/2}\{[P_\gamma^2/\beta - \beta(\beta^2+P_\beta^2)]\cos 3\gamma + \xi P_\beta P_\gamma \sin 3\gamma\}) \end{aligned} \qquad (2)$$

with $\xi = 2$. A simple way to break the SU(3) dynamical symmetry is to take $\xi \neq 2$.

The classical Hamiltonian (2) depends on two coordinates (shape parameters) β and γ and their conjugate momenta P_β and P_γ, respectively. The advantage of considering the classical Hamiltonian (2) is that the corresponding Poincaré sections are of dimension two and can be used to illustrate the onset of classical chaos. We compute the (β, P_β) Poincaré surfaces of section defined by $\gamma = 0$, $P_\gamma \geq 0$. Contrary to this case, in the study [2] by Alhassid *et al.* in a twelve-dimensional space, the

Poincaré sections have higher dimension and cannot be shown in any simple graphical way. Therefore, Alhassid *et al.* used [2] Monte Carlo techniques to calculate the fraction of chaotic volume, characterizing chaotic trajectories by a positive maximal Lyapunov exponent.

Let us first investigate the case when the SU(3) dynamical symmetry is broken ($\xi \neq 2$). Characteristic results are presented in Figs. 1(a)–1(d) for $\xi = 8$, showing a gradual change from regular behavior at low energies to chaotic at high energies. Such a gradual transition from regular to chaotic motion with increasing energy is a general feature, characteristic for the low-dimensional nonintegrable Hamiltonian systems like for example, the hydrogen atom in a uniform magnetic field [10], the Henon-Heiles system [11], and the planar motion [12] of a particle in an idealized nuclear potential.

On the other hand, in the case of SU(3) dynamical symmetry ($\xi = 2$), the Poincaré surfaces of section are regular at all energies [Figs. 1(e) and 1(f)]. These results are in accordance with the symmetry paradigm [1] and with the calculations [2] using the Monte Carlo technique.

Does the above regular classical behavior in the limit of SU(3) dynamical symmetry show up on the quantal properties of the model for $J^{\pi} = 0^+$ states? To answer that we have studied the corresponding quantal system (1) using two standard measures [13] of chaos: the nearest-neighbor level-spacing distribution $P(S)$ and the Dyson-Mehta statistics $\Delta_3(L)$. The energy level statistics are expected [13] to be the Poisson statistics or the GOE statistics when the corresponding classical system is regular or chaotic, respectively. This conjecture [13] on the relation between the quantal energy level statistics and the regularity or chaos of the corresponding classical dynamics was confirmed in a number of studies of low-dimensional systems [10,13–16], although some exceptions have been found [17]. There are also semiclassical arguments [18] that support this conjecture.

In the SU(3) scheme the IBM states are labeled [5] $|N,(\lambda,\mu),K,J,M\rangle$, where N is the total number of bosons, J the total angular momentum, and M the z projection of the angular momentum J. The labels λ,μ specify the SU(3)-irreducible representation; the allowed values of (λ,μ) are [5] given by $0 \leq \lambda = 2N - 2\mu - 6_m$, with m an

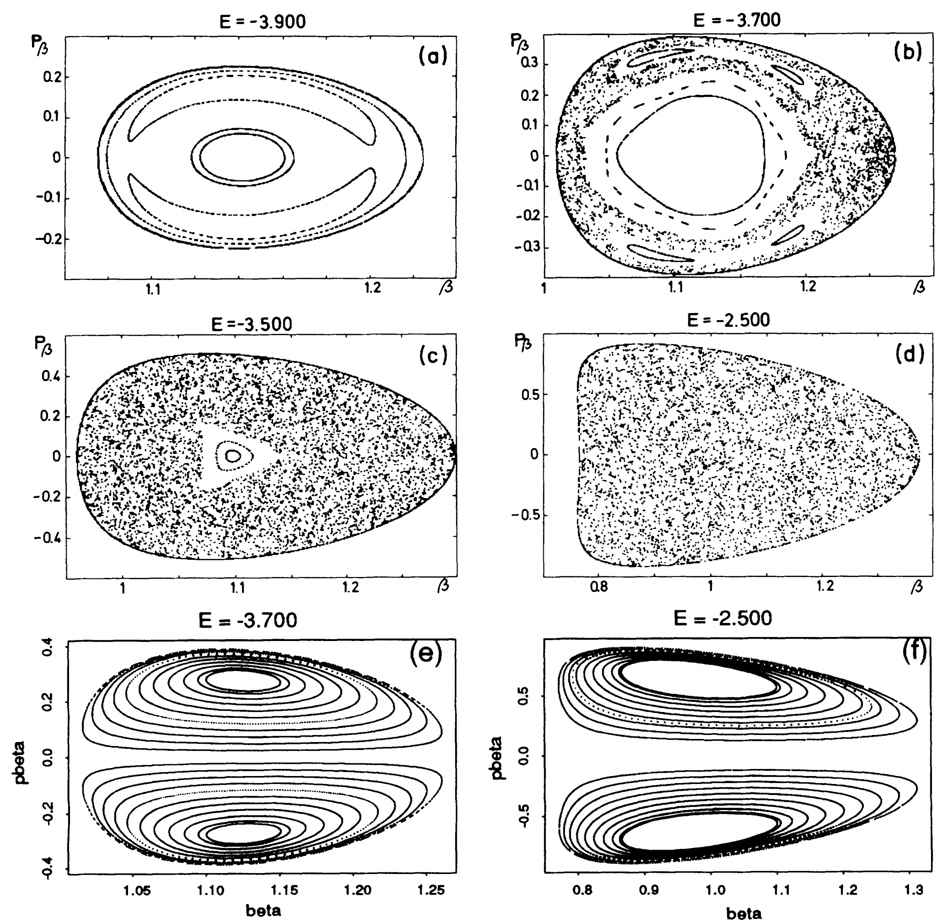

FIG. 1. (β, P_β) Poincaré surfaces of section for (a) $\xi = 8$, $E/\kappa_2 = -3.90$, (b) $\xi = 8$, $E/\kappa_2 = -3.70$, (c) $\xi = 8$, $E/\kappa_2 = -3.50$, (d) $\xi = 8$, $E/\kappa_2 = -2.50$, (e) $\xi = 2$, $E/\kappa_2 = -3.70$, and (f) $\xi = 2$, $E/\kappa_2 = -2.50$. Panels (e) and (f) correspond to the SU(3) dynamical symmetry and panels (a)–(d) to broken dynamical symmetry.

integer ≥ 0, μ an even interger ≥ 0. K is an extra label one needs to distinguish between different states with the same spin that can occur in a single SU(3) representation; it is closely related [5] to the K quantum number in the geometrical description where it measures the projection of angular momentum J along the symmetry axis. Within each (λ,μ) representation there is only one $J=0$ state; thus, the K quantum number is not needed for $J=0$ states.

In Figs. 2(a) and 2(b) we present the calculated nearest-neighbor spacing (NNS) distribution and the Dyson-Mehta Δ_3 statistics for a realistic case: The interaction strengths in the Hamiltonian (1) correspond to the ^{164}Er nucleus ($\kappa_1=-0.1978$, $\kappa_2=0.0452$) and the boson number $N_B=20$ is the same as used in Ref. [2]. Our calculation shows that the energy level statistics for the $J^\pi=0^+$ states with $N_B=20$ are close to GOE statistics. The calculated value of the Brody parameter for the NNS distribution is $\omega=0.80$ ($\omega=1$ for GOE and $\omega=0$ for Poisson). This result for the energy level statistics of 0^+ states appears in contrast to the conjecture on the relation between the quantal energy level statistics and the underlying classical dynamics, because the corresponding classical system (2) is regular. Furthermore, our results for the energy level statistics of 0^+ states, which are close to GOE predictions, appear in sharp contrast to the previous results obtained by Alhassid *et al.* [2] for the $J^\pi=2^+$, 4^+, 6^+, and 8^+ states. For these states the results were close to Poissonian, with the Brody parameter $\omega=0.03$.

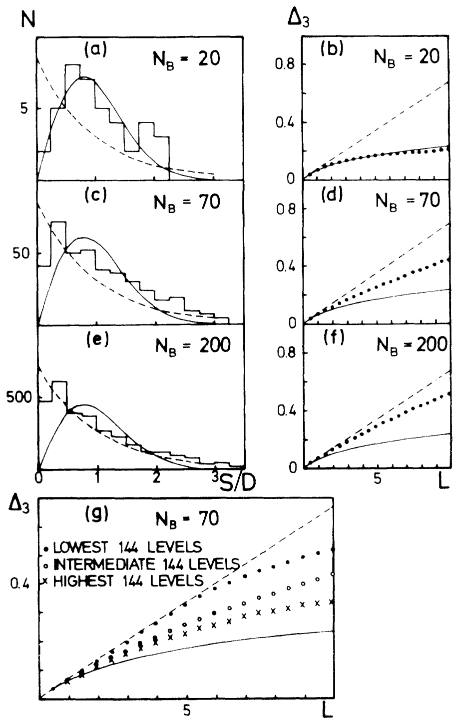

FIG. 2. Statistical fluctuations of the $J^\pi=0^+$ energy spectrum for the quantal IBM Hamiltonian (1). Shown are the nearest-neighbor level-spacing distributions [histograms in panels (a), (c), and (e)] and the Dyson-Mehta Δ_3 statistics [panels (b), (d), (f), and (g)]. The solid lines are the GOE limit and the dashed lines are the Poisson limit. The boson number is (a),(b) $N_B=20$, (c),(d),(g) $N_B=70$, and (e),(f) $N_B=200$. Panel (g) displays Δ_3 statistics for each of the three subsets of levels in the case $N_B=70$: for the 144 levels with lowest energies (dots), for the next 144 levels (open circles), and for the 144 levels with highest energies (crosses).

What causes a pronounced difference between the results for $J^\pi=0^+$ and $J^\pi\geq 2^+$ states? This is an effect of the previously mentioned additional quantum number K. For the $J^\pi=0^+$ states the K quantum number has only one value, $K=0$, and it does not influence the calculation of fluctuation measures. On the other hand, for $J^\pi\geq 2^+$ there are several possible values of K. Therefore, each set of all states of a given spin J and positive parity is a combination of several subsets of levels which differ in the value of the K quantum number. For example, the $J^\pi=2^+$ levels consist of two superposing sequences of levels, with ($J^\pi=2^+,K=0$) and ($J^\pi=2^+,K=2$). For each of these two sequences, taken separately [19], the energy level statistics are close to GOE, while for their superposition, i.e., for the set of all 2^+ levels, they are shifted towards Poissonian. Similarly, Alhassid *et al.* have obtained near the SU(3) limit of the IBM the near-Poissonian energy level statistics for the $J^\pi=8^+$ states because their calculation was performed for the superposition of sequences ($J^\pi=8^+,K=0$), ($J^\pi=8^+,K=2$), ..., ($J^\pi=8^+$, $K=8$). Thus the Poisson statistics found by Alhassid *et al.* is not a signature of chaos; instead, it is a consequence of superposing several sequences of levels, each having a single value of the additional quantum number K. We note that the problem of energy level statistics for combined sequences of levels with different values of an additional quantum number has been investigated in the case of the isospin quantum number [20,21].

Thus, it is seen that the $J^\pi=0^+$ states are particularly suitable for investigations of chaotic behavior. We note that the available experimental data on 0^+ states, although scarce, give some support [22] for the GOE-type of statistics.

As shown above, the 0^+ states in a realistic nuclear case present a counterexample to the symmetry paradigm [1,2] for the relation between energy level statistics and dynamical symmetry, and to the conjecture [13] on the relation between the energy level statistics and classical dynamics. However, the IBM framework provides an opportunity to study what happens with the energy level statistics of the model when the boson number N_B increases above the value which corresponds to the realistic case of rotational nuclei. Namely, the boson number N_B plays the role of a control parameter for the semiclassical approximation, with $N_B\to\infty$ corresponding to the classical limit [5,7]. This can be viewed in analogy to the three-level schematic shell model, investigated by Meredith, Koonin, and Zirnbauer [14] and Leboeuf and Saraceno [23], in which case the total number of fermions plays the role of $\hbar^{-1}$ in conventional quantum mechanics. The behavior of fluctuation measures with an increase of the boson number N_B beyond the value appropriate for rotational nuclei is illustrated in Figs. 2(c) and 2(d) for $N_B=70$ and in Figs. 2(e) and 2(f) for $N_B=200$. We see that the increase of N_B in the direction of the classical limit is accompanied by a change of the energy level statistics away from GOE in the direction of Poisson statistics. Only in the asymptotic limit of large N_B, far above the physical range of N_B for rotational nuclei (where $N_B \lesssim 20$), are the energy level statistics close to Poissonian.

There are three interesting predictions coming from the present IBM investigations, which should be tested in further studies: (i) The energy level statistics of 0^+ states in rotational nuclei should be close to GOE, although the underlying classical motion is regular. (ii) For superdeformed nuclei described [24] by the IBM (where N_B might be as large as $N_B \approx 50$), the energy level statistics should be shifted away from GOE towards Poissonian, in comparison to normally deformed nuclei (where $N_B \lesssim 20$). (iii) In the cases with sufficiently large values of N_B the energy level statistics should gradually change from the one which is closer to Poissonian at low energies toward GOE at higher energies.

Concluding, we have investigated the Poincaré sections for the 0^+ states described by the classical Hamiltonian (2) which corresponds to SU(3) dynamical symmetry of the interacting boson model for rotational nuclei. In accordance with the symmetry paradigm, the dynamical symmetry is associated with classical regularity and the breaking of dynamical symmetry with the onset of classical chaos. On the other hand, contrary to the conjecture on the relation between quantum fluctuations and classical dynamics, we find that the quantal energy level statistics of 0^+ states in the realistic case are near GOE, although the underlying classical motion is regular. With an increase of the value of the total boson number N_B above the realistic values, however, the quantal energy level statistics approach the Poisson statistics. Thus, the above-mentioned conjecture is satisfied in the asymptotic limit, which, however, lies outside of the realistic parameter range. Thus, the chaotic behavior of the interacting boson model is richer in phenomena than was previously thought, offering some interesting predictions for future studies.

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