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The transformed harmonic oscillator basis (THO) is derived by a local scaling-point transformation of the spherical harmonic-oscillator radial wave functions. The unitary scaling transformation produces a basis with improved asymptotic properties. The THO basis is employed in the solution of the relativistic Hartree-Bogoliubov (RHB) equations in configurational space. The model is applied in the self-consistent mean-field approximation to the description of the neutron halo in Ne isotopes. It is shown that an expansion of nucleon spinors and mean-field potentials in the THO basis reproduces the asymptotic properties of neutron densities calculated by finite element discretization in the coordinate space. In the RHB description of neutron skins and halos, THO bases in two or three dimensions can be a useful alternative to technically complicated solutions on a mesh in coordinate space. [S0556-2813(98)03310-X]

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I. INTRODUCTION

Theoretical models based on the relativistic mean-field approximation have been successfully applied in the description of a variety of nuclear structure phenomena in spherical and deformed β-stable nuclei [1], and more recently in studies of exotic nuclei far from the valley of β stability [2–7]. Short-lived isotopes with extreme neutron-to-proton ratios display unique properties. On the neutron-rich side in particular, extremely diffuse and spatially extended neutron densities are found in nuclei close to the particle drip lines. Densities with large spatial extensions result from the extremely weak binding of the outermost nucleons and the coupling between bound states and the particle continuum. The theoretical description of the appearance of neutron skins, or eventually neutron halos, necessitates a careful treatment of the asymptotic part of the nucleonic densities. While calculations based on modern Monte Carlo shell-model techniques provide an accurate description of the structure of relatively light exotic nuclei, for medium-heavy and heavy systems the only viable approach at present are mean-field models.

In addition to the self-consistent mean-field potentials, pairing correlations have to be taken into account in order to describe ground-state properties of open shell nuclei. For strongly bound systems pairing can be included in the form of a simple BCS approximation in the valence shell. For drip line nuclei on the other hand, the separation energy of the last nucleons can become extremely small. The Fermi level is found close to the particle continuum, and the lowest particle-hole or particle-particle modes are embedded in the continuum. The BCS pairing model presents only a poor approximation for such systems. A correct description of the scattering of nucleonic pairs from bound states to the positive energy continuum, and therefore of the formation of spatially extended nucleonic densities, necessitates a unified treatment of mean-field and pairing correlations. In the nonrelativistic framework the structure of spherical exotic neutron-rich, as well as proton drip-line nuclei, has been described in the Hartree-Fock-Bogoliubov (HFB) theory in coordinate space [8,9]. Among models based on quantum hadrodynamics, the relativistic Hartree-Bogoliubov (RHB) model [3,10] has been very successfully employed in calculations of ground-state properties of spherical exotic nuclei on both sides of the valley of β stability. For β-stable nuclei the traditional Hartree-Fock models and the relativistic mean-field theory predict very similar ground-state densities. On the other hand, recent applications to the structure of drip-line nuclei have shown significant differences, especially in the isospin dependence of the spin-orbit potential.

In Ref. [3] it was emphasized that for a correct theoretical description of nuclei with large neutron excess, the model must include: (i) a self-consistent solution for the mean-field in order to obtain the correct increase of the surface diffuseness, (ii) the correct isospin dependence of the spin-orbit term in the mean-field potential, (iii) a unified self-consistent description of pairing correlations, (iv) coordinate space solutions for the equations that describe the coupling between bound and continuum states. However, the coordinate space representation of HFB or RHB equations still presents serious difficulties in the calculation of ground-state properties of deformed nuclei. For finite-range interactions the technical and numerical problems on a two-dimensional mesh are so involved that reliable self-consistent calculations in coordinate space should not be expected soon, and therefore one has to look for alternative solutions. In the configurational representation the HFB or RHB systems of partial differential equations have been solved by expanding the nucleon wave functions and mean-field potentials in a large oscillator basis. In many applications an expansion of the wave functions in an appropriate harmonic-oscillator (HO) basis of spherical or axial symmetry provides a satisfactory level of accuracy. For nuclei at the drip lines the expansion in the localized oscillator basis presents only a poor approximation for the continuum states. Oscillator expansions produce densities which decrease to steeply in the asymptotic region at

large distance from the center of the nucleus. The calculated rms radii cannot reproduce the experimental values, especially for halo nuclei.

In a recent work [11] a new configurational representation basis has been suggested, based on a unitary transformation of the spherical HO basis. The new basis is derived from the standard oscillator basis by performing a local-scaling point coordinate transformation [12]. The precise form of the transformation is determined by the desired asymptotic behavior of the densities. At the same time it preserves many useful properties of the HO wave functions. Using the new basis, characteristics of weakly bound orbitals for a square-well potential were analyzed, and the ground-state properties of some spherical nuclei were calculated in the framework of the energy density functional approach. It was shown that properties of spatially extended states can be described using a rather modest number of basis states. The new basis, it was suggested, might present an interesting alternative to the algorithms that are being developed for the coordinate space solutions of the HFB equations. In the present work we investigate a particular transformed HO basis which might be appropriate for the solution of the coupled system of Dirac-Hartree-Bogoliubov integrodifferential eigenvalue equations, and Klein-Gordon equations for the meson fields.

In Sec. II we present an outline of the relativistic Hartree-Bogoliubov theory and discuss the various approximations. The local-scaling transformed spherical HO basis is introduced in Sec. III, and results of calculations in the configurational representation for a series of Ne isotopes are compared with solutions obtained by discretization of the system of RHB equations on the finite element mesh in coordinate space [2,13]. A summary of our results is presented in Sec. IV.

II. RELATIVISTIC HARTREE-BOGOLIUBOV MODEL

The relativistic Hartree-Bogoliubov theory provides a unified description of mean-field and pairing correlations in nuclei. In this section we briefly outline the essential characteristics of the model that will be used in the calculation of ground-state properties. A detailed description of the model can be found in Refs. [3,13]. The ground state of a nucleus $|\Phi\rangle$ is described as vacuum with respect to independent quasiparticle operators. The generalized single-nucleon Hamiltonian contains two average potentials: the self-consistent mean-field $\hat{\Gamma}$ which encloses all the long-range particle-hole (ph) correlations, and a pairing field $\hat{\Delta}$ which sums up the particle-particle (pp) correlations. The variation of the energy functional with respect to the Hermitian density matrix ρ and the antisymmetric pairing tensor κ, produces the single quasiparticle eigenvalue equations. The model was formulated in Ref. [10]. In the Hartree approximation for the self-consistent mean field, the relativistic Hartree-Bogoliubov (RHB) equations read

$$\begin{pmatrix} \hat{h}_D - m - \lambda & \hat{\Delta} \\ -\hat{\Delta}^* & -\hat{h}_D + m + \lambda \end{pmatrix} \begin{pmatrix} U_k(\mathbf{r}) \\ V_k(\mathbf{r}) \end{pmatrix} = E_k \begin{pmatrix} U_k(\mathbf{r}) \\ V_k(\mathbf{r}) \end{pmatrix}, \tag{1}$$

where $\hat{h}_D$ is the single-nucleon Dirac Hamiltonian and m is the nucleon mass

$$\hat{h}_D = -i\boldsymbol{\alpha}\cdot\nabla + \beta[m + g_\sigma\sigma(r)] + g_\omega\tau_3\omega^0(r) + g_\rho\rho^0(r) + e\frac{(1-\tau_3)}{2}A^0(r). \tag{2}$$

The chemical potential λ has to be determined by the particle number subsidiary condition in order that the expectation value of the particle number operator in the ground state equals the number of nucleons. The column vectors denote the quasiparticle spinors and E_k are the quasiparticle energies. The Dirac Hamiltonian contains the mean-field potentials of the isoscalar scalar σ meson, the isoscalar vector ω meson, the isovector vector ρ meson, as well as the electrostatic potential. The RHB equations have to be solved self-consistently with potentials determined in the mean-field approximation from solutions of Klein-Gordon equations. The equation for the σ meson contains the nonlinear σ self-interaction terms [14]. Because of charge conservation only the three-component of the isovector ρ meson contributes. The source terms for the Klein-Gordon equations are calculated in the *no-sea* approximation.

The pairing field $\hat{\Delta}$ in Eq. (1) reads

$$\Delta_{ab}(\mathbf{r},\mathbf{r}') = \frac{1}{2}\sum_{c,d} V_{abcd}(\mathbf{r},\mathbf{r}')\boldsymbol{\kappa}_{cd}(\mathbf{r},\mathbf{r}'), \tag{3}$$

where a,b,c,d denote quantum numbers that specify the single-nucleon states, $V_{abcd}(\mathbf{r},\mathbf{r}')$ are matrix elements of a general two-body pairing interaction, and the pairing tensor is defined

$$\boldsymbol{\kappa}_{cd}(\mathbf{r},\mathbf{r}') = \sum_{E_k>0} U^*_{ck}(\mathbf{r})V_{dk}(\mathbf{r}'). \tag{4}$$

The matrix elements in the pairing channel V_{abcd} can, in principle, be derived as a one-meson exchange interaction by eliminating the mesonic degrees of freedom in the model Lagrangian [10]. However, the resulting pairing matrix elements are unrealistically large if for the coupling constants the standard parameter sets of the mean-field approximation are used. These parameters do not reproduce scattering data. On the other hand, since we are using effective forces, there is no fundamental reason to have the same interaction both in the particle-hole and particle-particle channel. Although encouraging results have been reported in applications to nuclear matter, a microscopic and fully relativistic derivation of the pairing force starting from the Lagrangian of quantum hadrodynamics still cannot be applied to realistic nuclei. Therefore, it was suggested in Ref. [15] that an appropriate approximation might be a phenomenological Gogny-type finite-range interaction

$$V^{pp}(1,2) = \sum_{i=1,2} e^{-[(\mathbf{r}_1-\mathbf{r}_2)/\mu_i]^2}(W_i + B_iP^\sigma - H_iP^\tau - M_iP^\sigma P^\tau), \tag{5}$$

with the parameters μ_i, W_i, B_i, H_i, and M_i $(i=1,2)$. The procedure requires no cutoff and provides a very reliable description of pairing properties in finite nuclei.

The eigensolutions of Eq. (1) form a set of orthogonal and normalized single quasiparticle states. The corresponding eigenvalues are the single quasiparticle energies. The self-consistent iteration procedure is performed in the basis of quasiparticle states. The resulting quasiparticle eigenspectrum is then transformed into the canonical basis of single-particle states, in which the RHB ground state takes the BCS form. The transformation determines the energies and occupation probabilities of the canonical states.

The self-consistent solution of the Dirac-Hartree-Bogoliubov integrodifferential eigenvalue equations and Klein-Gordon equations for the meson fields determines the nuclear ground state. In Refs. [2–5] we used finite element methods in the coordinate space discretization of the coupled system of equations. The model was applied to exotic neutron-rich nuclei. Very diffuse ground-state densities were described and the occurrence of neutron halo was investigated. In the description of not so exotic Ni and Sn isotopes [6], as well as for proton drip-line nuclei [7], we followed the prescription of Ref. [15]: the Dirac-Hartree-Bogoliubov equations and the equations for the meson fields were solved by expanding the nucleon spinors $U_k(\mathbf{r})$ and $V_k(\mathbf{r})$, and the meson fields in a basis of spherical harmonic oscillators for $N=20$ oscillator shells. For nuclei at the proton drip lines we verified the results by comparing calculations in configuration space with coordinate space solutions. Differences were negligible. As will be shown in the next section, things are very different on the neutron-rich side.

III. CALCULATIONS IN CONFIGURATIONAL REPRESENTATION

A. The transformed harmonic-oscillator basis

In the construction of the local-scaling transformed harmonic oscillator basis we will use the local-scaling transformation method [12]. A local-scaling point coordinate transformation (LST) is defined as

$$\mathbf{r}'=\mathbf{f}(\mathbf{r})\equiv\hat{\mathbf{r}}f(\mathbf{r}). \tag{6}$$

The transformed radius vector has the same direction $\hat{\mathbf{r}}\equiv\mathbf{r}/|\mathbf{r}|$, while its magnitude $r'=f(\mathbf{r})$ depends on the scalar LST function. $f(\mathbf{r})$ is assumed to be an increasing function of r, and $f(\mathbf{0})=0$. The set of invertible transformations of this type forms a LST group. The corresponding LST wave function can be expressed as

$$\Psi_f(\mathbf{r}_1,\mathbf{r}_2,\ldots,\mathbf{r}_A)=\left[\prod_{i=1}^{A}\frac{f^2(\mathbf{r}_i)}{r_i^2}\frac{\partial f(\mathbf{r}_i)}{\partial r_i}\right]^{1/2}\times\overline{\Psi}[\mathbf{f}(\mathbf{r}_1),\mathbf{f}(\mathbf{r}_2),\ldots,\mathbf{f}(\mathbf{r}_A)], \tag{7}$$

where $\overline{\Psi}(\mathbf{r}_1,\mathbf{r}_2,\ldots,\mathbf{r}_A)$ is a model A-particle wave function normalized to unity

$$\langle\overline{\Psi}|\overline{\Psi}\rangle=1. \tag{8}$$

The transformation (7) is unitary and therefore the LST wave function $\Psi_f(\mathbf{r}_1,\mathbf{r}_2,\ldots,\mathbf{r}_A)$ is also normalized to unity, independent of the choice of $f(\mathbf{r})$.

The local one-body density corresponding to an A-body wave function Ψ is

$$\rho(\mathbf{r})=A\int|\Psi(\mathbf{r},\mathbf{r}_2,\ldots,\mathbf{r}_A)|^2d\mathbf{r}_2,\ldots,d\mathbf{r}_A. \tag{9}$$

From expression (7) it follows that there exists a simple relation between the local density $\rho_f(\mathbf{r})$ associated with the LST function Ψ_f, and the density $\overline{\rho}(\mathbf{r})$ which corresponds to the prototypical model function $\overline{\Psi}$:

$$\rho_f(\mathbf{r})=\frac{f^2(\mathbf{r})}{r^2}\frac{\partial f(\mathbf{r})}{\partial r}\overline{\rho}(f(\mathbf{r})). \tag{10}$$

This relation is particularly useful when the form of the density $\rho_f(\mathbf{r})$ is known, for example from experimental data. In that case Eq. (10) becomes a first-order nonlinear differential equation for the LST function f. For a system with spherical symmetry, ρ_f, $\overline{\rho}$, and f depend only on $r=|\mathbf{r}|$, and Eq. (10) can be reduced to a nonlinear algebraic equation

$$\int_0^r\rho_f(u)u^2du=\int_0^{f(r)}\overline{\rho}(u)u^2du. \tag{11}$$

The solution is found subject to the boundary condition $f(0)=0$.

For shell-model or mean-field applications we have to consider the case when the model many-body wave function is a Slater determinant

$$\overline{\Psi}(\mathbf{r}_1,\mathbf{r}_2,\ldots,\mathbf{r}_A)=\frac{1}{\sqrt{A!}}\det|\overline{\phi}_i(\mathbf{r}_j)|. \tag{12}$$

The single-particle functions $\overline{\phi}_i(\mathbf{r})$ form a complete set. The LST wave function is defined by the transformation (7), and of course it is written as a product state

$$\Psi_f(\mathbf{r}_1,\mathbf{r}_2,\ldots,\mathbf{r}_A)=\frac{1}{\sqrt{A!}}\det|\phi_i(\mathbf{r}_j)|, \tag{13}$$

of the LST single-particle basis states

$$\phi_i(\mathbf{r})=\left[\frac{f^2(\mathbf{r})}{r^2}\frac{\partial f(\mathbf{r})}{\partial r}\right]^{1/2}\overline{\phi}_i(\mathbf{f}(\mathbf{r})). \tag{14}$$

In the present work we consider systems with spherical symmetry. The single-particle basis reads

$$\overline{\phi}_i(\mathbf{r},\sigma,\tau)=R_{nl}^{\mathrm{HO}}(r)\mathcal{J}_{jlm}(\Omega,\sigma)\chi_{1/2m_t(\tau)}. \tag{15}$$

The angular part describes the coupling of orbital angular momentum and spin

$$\mathcal{J}_i(\Omega,\sigma)\equiv J_{jlm_j}(\Omega,\sigma)=\sum_{m_lm_s}(lm_l1/2m_s|jm_j)Y_{lm_l}(\Omega)\chi_{1/2m_s(\sigma)}. \tag{16}$$

The harmonic-oscillator radial functions

$$R_{nl}^{\mathrm{HO}}(r)=\frac{2(n-1)!b^{-3/2}}{\Gamma(n+l+1/2)^3}(r/b)^{l+1}e^{-1/2\,(r/b)^2}L_{n-1}^{l+1/2}(r/b) \tag{17}$$

are characterized by a single parameter: the oscillator length $b=\sqrt{\hbar/m\omega}$.

If applied in the description of very weakly bound nuclei, the main drawback of HO wave functions is that, because of their Gaussian asymptotics, they cannot describe the density profiles far away from the center of the nucleus. In principle, one could try to resolve this problem by including a very large number of oscillator shells, but extremely large bases are impossible to handle in more microscopic applications. Therefore, by means of Eqs. (14) and (15) it might be useful to define a transformed harmonic-oscillator (THO) basis

$$\phi_i(\mathbf{r},\sigma,\tau)=R_{nl}^{\mathrm{THO}}(r)\mathcal{J}_{jlm}(\Omega,\sigma,\tau)\chi_{1/2m_t(\tau)}. \tag{18}$$

The angular part of course does not change, while the radial functions $R_{nl}^{\mathrm{THO}}(r)$ are defined by the local-scaling transformation

$$R_{nl}^{\mathrm{THO}}(r)=\left[\frac{f^2(r)}{r^2}\,\frac{df(r)}{dr}\right]^{1/2}R_{nl}^{\mathrm{HO}}(f(r)). \tag{19}$$

The choice of the LST function

$$f(r)=\begin{cases} r & \text{for } r\leqslant a \\ a(8r/a-8a/r+a^2/r^2-12\,\ln(r/a))^{1/2} & \text{for } r>a \end{cases} \tag{20}$$

is motivated by the desired asymptotic form of the nucleonic densities. For this particular parametrization the derivatives of $f(r)$ up to fourth order are continuous at the matching radius a, and $f(r)$ is a monotonically increasing function of r. The transformation (19) is unitary; the Jacobian $[f^2(r)/r^2][df(r)/dr]$ is positive for all values of r. This makes the THO basis complete, since it is unitary equivalent to the complete HO basis. The ansatz (20) for the LST function guarantees that all THO basis states (18) are localized in space. Asymptotically, the linear term in the expansion (20) dominates and $f(r\rightarrow\infty)\sim r^{1/2}$. Therefore, at large distances the radial functions decrease exponentially. The THO functions are continuous (up to fourth derivatives) and depend on two positive parameters: the oscillator parameter b and the matching radius a.

It should be emphasized that the simple parametrization (20) is motivated by our desire to keep the new basis as simple and practical as possible. In Ref. [16] it was shown that a three-parameter form of the LST function performs very well in actual calculations of ground-state properties of spherical nuclei. For light nuclei one of these parameters turns out to be very small. In the present application we choose it identically zero. The resulting matching conditions are satisfied analytically and the parametrization (20) is derived. Of course other parametrizations of $f(r)$ are possible. There are many ways to optimize the LST function, depending on the actual physical problem under consideration.

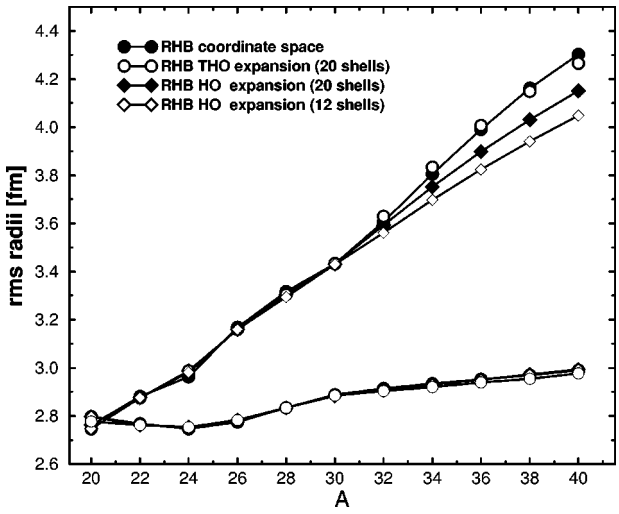

FIG. 1. Calculated proton and neutron rms radii for the Ne isotopes. Values obtained by finite element discretization on the coordinate mesh are compared with calculations in the configurational representation: expansion in the harmonic-oscillator (HO) and transformed harmonic-oscillator (THO) bases.

B. Neutron-rich Ne isotopes

In Ref. [2] we have applied the RHB model to the description of the neutron halo in the mass region above the s-d shell. Pairing correlations and the coupling to particle continuum states have been described by finite range two-body Gogny-type interaction. Finite element methods have been used in the coordinate space discretization of the coupled system of Dirac-Hartree-Bogoliubov and Klein-Gordon equations. As we have already emphasized, solutions in coordinate space appear to be essential for the correct description of the coupling between bound and continuum states. Using the parameter set NL3 [16] for the mean-field Lagrangian, and D1S [17] parameters for the Gogny interaction, we have found evidence for the occurrence of multineutron halos in heavier Ne isotopes. We have shown that the properties of the $1f$-$2p$ orbitals near the Fermi level and the neutron pairing interaction play a crucial role in the formation of the halo. In the present work we essentially repeat the calculations of Ref. [2], but in configurational representation with the THO basis defined by the LS transformation (18).

In Fig. 1 we plot the proton and neutron rms radii for the Ne isotopes as functions of the mass number. Results of both calculations in the coordinate space and in the configurational representation (HO and THO bases) are shown. Proton radii change very little and only display a slow increase. There is practically no difference between results of coordinate and configurational space calculations. Neutron radii of Ne isotopes follow the mean-field $N^{1/3}$ curve up to $N\approx 22$. For larger values of N a sharp increase of neutron radii is observed (calculations in coordinate space). The matter radii of course follow this change and the spatial extension of heavier isotopes is substantially enlarged. The sudden increase in neutron rms radii has been interpreted as evidence for the formation of a multiparticle halo [2]. We notice that neutron radii calculated with the expansion in the HO basis cannot reproduce the coordinate space results. Results calculated with 12 and 20 oscillator shells are compared, and in

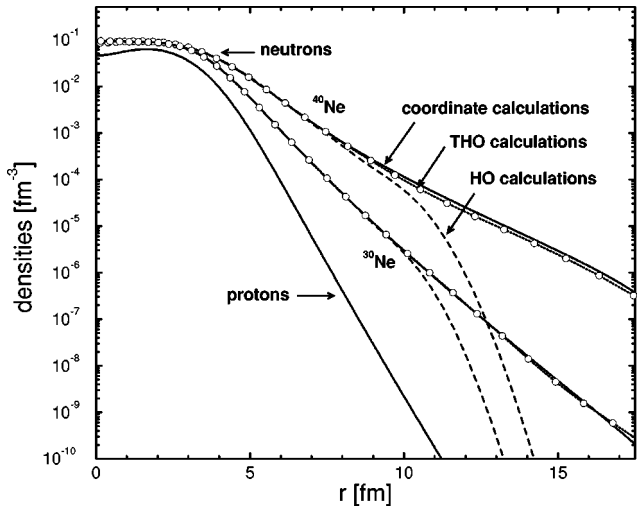

FIG. 2. Self-consistent RHB proton and neutron densities for the ground states of ^{30}Ne and ^{40}Ne. Density profiles calculated in configurational space are compared with results on the coordinate mesh.

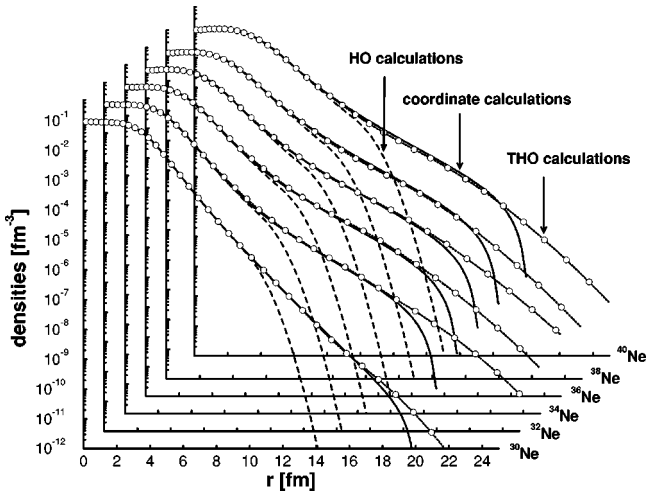

FIG. 3. Self-consistent neutron densities for the $^{30-40}$Ne isotopes in logarithmic scale. Distributions calculated by expansions in the HO and THO bases are displayed together with profiles obtained on a mesh in coordinate space.

both cases the radii follow the average mean-field curve. The neutron radii increase with the number of oscillator shells, but their values are still far from the results obtained in coordinate space. On the other hand, radii which result from the expansion in the THO basis almost exactly reproduce values of finite element discretization in coordinate space. All calculations with the THO basis have been performed with 20 oscillator shells. The same THO basis has been used both for protons and for neutrons.

In Fig. 2 we display the proton and neutron density profiles for ^{30}Ne and ^{40}Ne. In our calculation ^{40}Ne is the last bound isotope. The proton density distributions practically do not change with the mass number, but the neutron densities display an abrupt change between ^{30}Ne and ^{32}Ne. A long tail reveals the formation of a multiparticle halo. Again we notice that calculations in the HO basis do not reproduce the asymptotic behavior of the neutron densities. Except for the radii shown for comparison in Fig. 1, we have used a HO basis of 20 shells. While the results of HO calculations display a typical Gaussian dependence at large distances from the center of the nucleus, THO results with the same number of oscillator shells nicely reproduce the coordinate space calculations. More details are shown in Fig. 3, where neutron density profiles are plotted for the even isotopes $^{30-40}$Ne. While all three methods (coordinate space, HO and THO bases) produce almost identical density distributions for neutrons in the ''inner'' region $r \leqslant 10$ fm, results are very different for the asymptotic behavior. Not only is the THO basis obviously superior when compared with results of the HO calculations, but to some extent it could have also an advantage over calculations in coordinate space. Since these are performed in a box of finite dimension (20 fm in the present calculations), near the boundary of the box a typical, very steep decrease in the densities is observed. Coordinate space solutions in this region of course have nothing to do with physical densities, rather they are an artifact of the finite dimension of the mesh. THO solutions do not suffer from this drawback, and the corresponding asymptotic densities are more natural. Of course one can increase the coordinate mesh, but this might not always be feasible, especially in more than one dimension.

Calculations in the configurational space generally depend on the choice of parameters that determine the basis functions. In nonrelativistic self-consistent models the standard procedure is to minimize the energy of the ground state with respect to the basis parameters [18]. The minimization is performed in each step of the self-consistent iteration. This method however cannot be applied in relativistic models. Due to the presence of the Dirac sea, the energy functional does not display a minimum, but rather a saddle point in the ground state. The usual approach then is to find regions of values of basis parameters for which all calculated quantities saturate, i.e., properties of the ground state become independent on the specific choice of the basis. In particular, the spherical HO basis is characterized by the oscillator length parameter $b=\sqrt{\hbar/m\omega}$. Calculations along the β-stability line have shown that nuclear properties saturate for $\hbar\omega \simeq 41A^{1/3}$ [19]. The situation is different for nuclei at neutron drip lines. Properties associated with the halo states (as, e.g., neutron densities and rms radii) depend linearly on the oscillator length parameter. The new THO basis is determined by the two parameters: oscillator length b and the matching radius a. Our calculations have shown that quantities which characterize the ground-state saturate for those values of a and b for which the neutron densities decrease exponentially outside the nuclear volume. Therefore, the parameters of the basis are determined in such a way that the neutron densities decay exponentially in the asymptotic region (10–20 fm), where the nuclear interaction is negligible. It should be noted, however, that the rate of this exponential decay is not adjusted.

In Ref. [2] we have shown that the microscopic origin of the neutron halo can be found in a delicate balance between the self-consistent mean-field and pairing correlations. The RHB ground-state wave function can be written either in the quasiparticle basis as a product of independent quasiparticle states, or in the canonical basis as a highly correlated BCS state. The canonical states are eigenstates of the RHB density matrix, and the eigenvalues are the corresponding occupation

numbers. Since the density matrices in RHB are always localized, it follows that single-particle wave functions in the canonical basis are localized. The formation of the neutron halo is related to the quasidegeneracy of the triplet of states $1f_{7/2}$, $2p_{3/2}$, and $2p_{1/2}$. The pairing interaction promotes neutrons from the $1f_{7/2}$ orbital to the $2p$ levels. Due to their small centrifugal barrier, the $2p_{3/2}$ and $2p_{1/2}$ orbitals form the halo. In the present work we have also verified that calculations in the configurational representation with the THO basis reproduce the energies and occupation probabilities of canonical neutron states calculated on the mesh in coordinate space.

IV. SUMMARY

In the present work we have applied the relativistic Hartree-Bogoliubov theory in the description of ground-state properties of neutron-rich Ne isotopes. The model has been applied in the self-consistent mean-field approximation. Pairing correlations and the coupling to particle continuum states are described by the pairing part of the finite range Gogny interaction. In many applications of HFB and RHB it has been emphasized that, for a correct description of the asymptotic behavior of nucleonic densities, coordinate space solutions for the equations that describe the coupling between bound and continuum states are essential. This is especially important on the neutron-rich side of the valley of β stability, where extremely diffuse and spatially extended neutron densities are found at the drip lines. For these nuclei, solutions based on the expansion of wave functions and mean-field potentials in harmonic-oscillator bases result in ground-state densities with wrong asymptotic properties. Coordinate space calculations are relatively simple in one dimension, i.e., for spherical nuclei. In more than one dimension, numerical solutions on a coordinate mesh can become extremely involved, especially if finite range two-body interactions are considered. In future applications of the RHB theory to deformed exotic nuclei it would therefore be useful to have alternative methods for the solution of the coupled system of partial differential Dirac-Hartree-Bogoliubov and Klein-Gordon equations.

We have investigated properties of a new spherical basis in configuration space. This transformed harmonic-oscillator basis is derived by a local scaling-point transformation of the spherical harmonic-oscillator radial functions. The unitary scaling transformation produces a basis with improved asymptotic behavior, while it preserves useful properties of the HO wave functions. The THO basis has been employed in RHB calculations of ground-state properties of Ne isotopes. Results obtained in configurational representation have been compared with rms radii and densities calculated by finite element discretization in the coordinate space. It has been shown that densities calculated in the THO basis nicely reproduce results obtained on the coordinate mesh. In particular, using a rather small number of oscillator shells, we have been able to reproduce the sharp increase in neutron rms radii which had been interpreted as evidence for the formation of a multiparticle halo. Neutron densities calculated in the THO basis display the correct exponential decrease in the asymptotic region at large distances from the center of the potential. The present investigation has demonstrated that THO bases can be used in the description of the structure of spatially extended nuclei. This might be especially important in studies of deformed halos, where coordinate space discretization algorithms become technically very complicated.

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