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# Horizon conformal entropy in Gauss–Bonnet gravity

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## Abstract

We treat spherically symmetric black holes in Gauss–Bonnet gravity by imposing boundary conditions on fluctuating metric on the horizon. Obtained effective two-dimensional theory admits Virasoro algebra near the horizon. This enables, with the help of Cardy formula, evaluation of the number of states. Obtained results coincide with the known macroscopic expression for the entropy of black holes in Gauss–Bonnet gravity.

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## 1. Introduction

The well-known Bekenstein–Hawking formula [1] connects area of the black hole horizon with its entropy, i.e.,

$$S_{\text{BH}} = \frac{A}{4\hbar G}. \quad (1)$$

A considerable research effort in recent years was performed in order to understand microscopic interpretation of this relation. A particularly promising approach seems to be based on conformal field theory and Virasoro algebra. In fact, it was realized by Brown and Henneaux [2] that in  $2 + 1$  dimensions and after imposing asymptotic AdS3 symmetry one can iden-

tify two copies of Virasoro algebra and corresponding central charges. Further analysis [3] has reproduced Bekenstein–Hawking entropy for black holes in this theory. More recently, several papers addressed the problem of  $D$ -dimensional black holes. In particular, Solodukhin is treating [4] the spherically symmetric black holes with the metric

$$ds^2 = \gamma_{ab}(x) dx^a dx^b + r(x)^2 d\Omega_{D-2}, \quad (2)$$

where  $d\Omega_{D-2}$  is metric on  $(D - 2)$ -dimensional sphere of unit radius. In this approach one considers fluctuations of the field  $r(x)$  on a two-dimensional space–time with the metric  $\gamma_{ab}(x)$ . The author was able to identify a particular group of diffeomorphisms under which the horizon is invariant. The Einstein action reduces to a two-dimensional action of Liouville type. One is able to identify a Virasoro algebra. The aim is then to calculate the entropy from Cardy for-

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mula [5]

$$S_C = 2\pi \sqrt{\left(\frac{c}{6} - 4\Delta_g\right) \left(\Delta - \frac{c}{24}\right)}, \quad (3)$$

where  $\Delta$  is the eigenvalue of Virasoro generator  $L_0$  for the state we calculate the entropy and  $\Delta_g$  is the smallest eigenvalue. It was shown that the corresponding entropy reproduces BH result (1). Another approach is due to Carlip [6–10] where one requires in  $D$ -dimensional gravity a set of boundary conditions near horizon. That leads to central extension for the constraint algebra of general relativity. Due to assumed boundary conditions this algebra contains Virasoro algebra whose existence enables one to calculate conformal charge and via Cardy formula (3) the entropy. All these papers confirm that microscopic description via conformal theory reproduces the classical BH result for Einstein gravity. The question which we want to investigate in this Letter is if such description reproduces the classical result also for theories which differ from Einstein action by new terms written in terms of products of Riemann tensors and corresponding covariant derivatives. In fact, it is known that the classical entropy differs from the BH formula in these cases. Introduce, e.g., the (extended) Gauss–Bonnet (GB) densities

$$\mathcal{L}_m(g) = \frac{(-1)^m}{2^m} \delta^{\rho_1 \sigma_1 \dots \rho_m \sigma_m}_{\mu_1 \nu_1 \dots \mu_m \nu_m} \times R^{\mu_1 \nu_1}_{\rho_1 \sigma_1} \dots R^{\mu_m \nu_m}_{\rho_m \sigma_m}, \quad (4)$$

where  $R_{\mu\nu\rho\sigma}$  is Riemann tensor for metric  $g_{\mu\nu}$  and  $\delta^{\beta_1 \dots \beta_k}_{\alpha_1 \dots \alpha_k}$  is totally antisymmetric product of  $k$  Kronecker deltas, normalized to take values 0 and  $\pm 1$ . By definition, we take  $\mathcal{L}_0 = 1$  (cosmological constant term). Notice also that  $\mathcal{L}_1 = -R$ , i.e., ordinary Einstein action. General GB action (also known as Lovelock gravity [11]) is now given as

$$I_{\text{GB}} = - \sum_{m=0}^{[D/2]} \lambda_m \int d^D x \sqrt{-g} \mathcal{L}_m(g), \quad (5)$$

where  $g = \det(g_{\mu\nu})$  and  $[z]$  denotes integer part of  $z$ . Explicit expression for the entropy of general stationary black hole in GB theory is [12]

$$S_{\text{GB}} = \frac{4\pi}{\hbar} \sum_{m=1}^{[D/2]} m \lambda_m \oint d^{D-2} x \sqrt{\tilde{g}} \mathcal{L}_{m-1}(\tilde{g}_{ij}), \quad (6)$$

where the integration can be made on any  $(D-2)$ -dimensional spacelike slice of the Killing horizon and  $\tilde{g}_{ij}$  is the induced metric on it. In fact, classical expression for entropy in any generally covariant gravity theory have been suggested [13].

In this Letter we shall investigate in particular the Gauss–Bonnet action. This action has in fact many interesting properties:

- In  $D$ -dimensional space all terms for which  $m > D/2$  are identically equal to zero, because maximal rank of antisymmetric tensor in such space is  $D$ . It follows that there is always finite number of terms in the GB action (which we already included in the definition (5)). Term  $m = D/2$  is a topological term. In fact it is the original Gauss–Bonnet term which exists in even-dimensional spaces and which (with appropriate surface term added) is equal to the Euler character of that space. So, only terms for which  $m < D/2$  are contributing to equations of motion. It means that in  $D = 4$  GB action is (neglecting topological effects) just the Einstein action.
- Only GB terms have the property that resulting equations of motion contain no more than second derivative of metric [11]. They are also free of ghosts when expanded not only about flat space [14] but also about some Randall–Sundrum brane solutions in 5D [15].
- It has a good boundary value problem [16], in the sense that we can add surface terms such that the action can be extremized on space  $M$  while keeping only the metric fixed on the boundary  $\partial M$  (if non-GB terms are present in the action we have to also fix derivatives of components of the metric tensor on  $\partial M$ ).
- Analysis of spherically symmetric classical solutions in empty space is almost as simple as for pure Einstein case. But, unlike the Einstein case where there was unique solution (Schwarzschild), for general GB action there are black hole solutions having more complicated global topologies with multiple horizons and/or naked singularities [17].
- The entropy of GB black holes can be written (at least in stationary cases) as a sum of *intrinsic* curvature invariants integrated over a cross section of the horizon. As far as is known only GB actions

have this property. Interesting property that the entropy (6) has the same form as the action (5) can be described as dimensional continuation of the Gauss–Bonnet theorem.

- The entropy of GB black holes is negative for some region of parameter space. It is speculated that this is connected with the existence of a new type instability [18].
- It can be supersymmetrised.
- It is nonrenormalisable.

This properties suggest that GB action could be considered as a natural generalisation of Einstein action.

We shall investigate the entropy problem for this action with the Solodukhin method. We describe first the simpler case with only quadratic terms in Riemann tensor and consider spherically symmetric black holes with fixed boundary conditions for the fluctuating metric. We calculate the corresponding effective two-dimensional theory. It will be possible to find Virasoro algebra corresponding to the diffeomorphisms which preserve above boundary condition. Calculations of central charge and application of Cardy formula will determine entropy. We shall find that number of states obtained in such a way reproduces the classical result of Jacobson and Myers. In the Section 3 we generalise these results to the most general GB theory. In the last section we end with concluding remarks.

## 2. Effective CFT near the horizon

Now we turn our attention to particular microscopic derivation of “macroscopic” expression (6) for entropy of black holes in GB theory. For simplicity in this section we put  $\lambda_m = 0$  for  $m > 2$ . General action will be considered in the next section. We also take  $\lambda_0 = 0$  (cosmological constant), because we shall see that this term is irrelevant for our calculation. In this case action (5) becomes

$$I_{\text{GB}} = \int d^D y \sqrt{-g} [\lambda_1 R - \lambda_2 (R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4R_{\mu\nu} R^{\mu\nu} + R^2)]. \quad (7)$$

Coupling constant  $\lambda_1$  is related to more familiar  $D$ -dimensional Newton gravitational constant  $G_D$  through  $\lambda_1 = (16\pi G_D)^{-1}$ .

Following Solodukhin [4] we neglect matter and consider  $S$ -wave sector of the theory, i.e., we consider only radial fluctuations of the metric. It is easy to show that in this case (7) can be written in the form of an effective two-dimensional “higher-order Liouville theory” given with

$$I_{\text{GB}} = (D-2)(D-3)\Omega_{D-2} \int d^2 x \sqrt{-\tilde{\gamma}} \times \left\{ 2\lambda_2 (D-4)r^{D-5} (\nabla r)^2 \nabla^2 r + \lambda_2 (D-4)(D-5)r^{D-6} (\nabla r)^4 - [\lambda_1 r^{D-4} + 2\lambda_2 (D-4)(D-5)r^{D-6}] (\nabla r)^2 + \left[ \frac{\lambda_1 r^{D-2}}{(D-2)(D-3)} + 2\lambda_2 r^{D-4} \right] \mathcal{R} - [\lambda_1 r^{D-4} + \lambda_2 (D-4)(D-5)r^{D-6}] \right\}. \quad (8)$$

We now suppose that black hole with horizon *is existing* and we are interested in fluctuations (or better quantum states) near it. In the spherical geometry apparent horizon  $\mathcal{H}$  (a line in  $x$ -plane) can be defined by the condition [19]

$$(\nabla r)^2|_{\mathcal{H}} \equiv \gamma^{ab} \partial_a r \partial_b r|_{\mathcal{H}} = 0. \quad (9)$$

Notice that (9) is invariant under (regular) conformal rescalings of the effective two-dimensional metric  $\gamma_{ab}$ . Near the horizon (9) is approximately satisfied. It is easy to see that near the horizon first two terms in (8) are suppressed by a factor  $(\nabla r)^2$  relative to the third term (to see this just partially integrate latter and discard surface terms) and may be neglected.

If we make reparametrizations

$$\phi \equiv \frac{2\Phi^2}{q\Phi_h}, \quad \tilde{\gamma}_{ab} \equiv \frac{d\phi}{dr} \gamma_{ab}, \quad (10)$$

where

$$\Phi^2 = 2\Omega_{D-2} \times [\lambda_1 r^{D-2} + 2(D-2)(D-3)\lambda_2 r^{D-4}], \quad (11)$$

and  $q$  is arbitrary dimensionless parameter, the action (8) becomes

$$I_{\text{GB}} = \int d^2 x \sqrt{-\tilde{\gamma}} \left[ \frac{1}{4} q \Phi_h \phi \tilde{\mathcal{R}} - V(\phi) \right]. \quad (12)$$

This action can be put in more familiar form if we make additional conformal reparametrization:

$$\bar{\gamma}_{ab} \equiv \exp\left\{-\frac{2\phi}{q\Phi_h}\right\} \tilde{\gamma}_{ab}. \quad (13)$$

Now (12) takes the form

$$I_{\text{GB}} = - \int d^2x \sqrt{-\bar{\gamma}} \times \left[ \frac{1}{2} (\bar{\nabla}\phi)^2 - \frac{1}{4} q\Phi_h \phi \bar{\mathcal{R}} + U(\phi) \right], \quad (14)$$

which is similar to the Liouville action. The difference is that potential  $U(\phi)$  is not purely exponential but is given with

$$U(\phi) = (D-2)(D-3)\Omega_{D-2} \times [\lambda_1 r^{D-4} + \lambda_2 (D-4)(D-5)r^{D-6}] \times \frac{dr}{d\phi} \exp\left\{\frac{2\phi}{q\Phi_h}\right\}.$$

Action (14) is of the same form as that obtained from pure Einstein action. In [4] it was shown that if one imposes condition that the metric  $\bar{\gamma}_{ab}$  is *nondynamical* then the action (14) describes CFT *near the horizon*.<sup>1</sup> We therefore fix  $\bar{\gamma}_{ab}$  near the horizon and take it to be metric of static spherically symmetric black hole:

$$d\bar{s}_{(2)}^2 \equiv \bar{\gamma}_{ab} dx^a dx^b = -f(w) dt^2 + \frac{dw^2}{f(w)}, \quad (15)$$

where near the horizon  $f(w_h) = 0$  we have

$$f(w) = \frac{2}{\beta}(w - w_h) + O((w - w_h)^2). \quad (16)$$

We now make coordinate reparametrization  $w \rightarrow z$

$$z = \int^w \frac{dw}{f(w)} = \frac{\beta}{2} \ln \frac{w - w_h}{f_0} + O(w - w_h) \quad (17)$$

in which 2-dimensional metric has a simple form

$$d\bar{s}_{(2)}^2 = f(z)(-dt^2 + dz^2), \quad (18)$$

and the function  $f$  behaves near the horizon ( $z_h = -\infty$ ) as

$$f(z) \approx f_0 e^{2z/\beta}, \quad (19)$$

i.e., it *exponentially* vanishes. It is easy to show that equation of motion for  $\phi$  which follows from Eqs. (14), (18), (19) is

$$(-\partial_t^2 + \partial_z^2)\phi = \frac{1}{4} q\Phi_h \bar{\mathcal{R}} f + fU'(\phi) \approx O(e^{2z/\beta}), \quad (20)$$

and that the “flat” trace of the energy–momentum tensor is

$$-T_{00} + T_{zz} = \frac{1}{4} q\Phi_h (-\partial_t^2 + \partial_z^2)\phi - fU(\phi) \approx O(e^{2z/\beta}), \quad (21)$$

which is exponentially vanishing near the horizon.<sup>2</sup> From (20) and (21) follows that the theory of the scalar field  $\phi$  exponentially approaches CFT near the horizon.

Now, one can construct corresponding Virasoro algebra using standard procedure. Using light-cone coordinates  $z_{\pm} = t \pm z$  right-moving component of energy–momentum tensor near the horizon is approximately

$$T_{++} = (\partial_+ \phi)^2 - \frac{1}{2} q\Phi_h \partial_+^2 \phi + \frac{q\Phi_h}{2\beta} \partial_+ \phi. \quad (22)$$

It is important to notice that horizon condition (9) implies that  $r$  and  $\phi$  are (approximately) functions only of one light-cone coordinate (we take it to be  $z_+$ ), which means that only one set of modes (left or right) is contributing.

Virasoro generators are coefficients in the Fourier expansion of  $T_{++}$ :

$$T_n = \frac{\ell}{2\pi} \int_{-\ell/2}^{\ell/2} dz e^{i2\pi n z/\ell} T_{++}, \quad (23)$$

where we compactified  $z$ -coordinate on a circle of circumference  $\ell$ . Using canonical commutation relations it is easy to show that Poisson brackets of  $T_n$ 's are given with

$$i\{T_n, T_m\}_{\text{PB}} = (n-m)T_{n+m} + \frac{\pi}{4} q^2 \Phi_h^2 \times \left( n^3 + n \left( \frac{\ell}{2\pi\beta} \right)^2 \right) \delta_{n+m,0}. \quad (24)$$

<sup>1</sup> Carlip showed that above condition is indeed consistent boundary condition [20].

<sup>2</sup> Higher derivative terms in (8) make contribution to (21) proportional to  $f(\nabla\phi)^2 \approx o(\exp(2z/\beta))$ .

To obtain the algebra in quantum theory (at least in semiclassical approximation) one replaces Poisson brackets with commutators using  $[, ] = i\hbar\{, \}_{\text{PB}}$ , and divide generators by  $\hbar$ . From (24) it follows that “shifted” generators

$$L_n = \frac{T_n}{\hbar} + \frac{c}{24} \left( \left( \frac{\ell}{2\pi\beta} \right)^2 + 1 \right) \delta_{n,0}, \quad (25)$$

where

$$c = 3\pi q^2 \frac{\Phi_h^2}{\hbar}, \quad (26)$$

satisfy Virasoro algebra

$$[L_n, L_m] = (n - m)L_{n+m} + \frac{c}{12} (n^3 - n) \delta_{n+m,0} \quad (27)$$

with central charge  $c$  given in (26).

Outstanding (and unique, as far as is known) property of the Virasoro algebra is that in its representations a logarithm of the number of states (i.e., entropy) with the eigenvalue of  $L_0$  equal to  $\Delta$  is asymptotically given with Cardy formula (3). If we assume that in our case  $\Delta_g = 0$  in semiclassical approximation (more precisely,  $\Delta_g \ll c/24$ ), one can see that number of microstates (purely quantum quantity) is in leading approximation completely determined by (semi)classical values of  $c$  and  $L_0$ . Now it only remains to determine  $\Delta$ . In a classical black hole solution we have

$$r = w = w_h + (w - w_h) \approx r_h + f_0 e^{2z/\beta}, \quad (28)$$

so from (10) and (11) follows that near the horizon  $\phi \approx \phi_h$ . Using this configuration in (23) one obtains  $T_0 = 0$ , which plugged in (25) gives

$$\Delta = \frac{c}{24} \left( \left( \frac{\ell}{2\pi\beta} \right)^2 + 1 \right). \quad (29)$$

Finally, using (26) and (29) in Cardy formula (3) one obtains

$$S_C = \frac{c}{12} \frac{\ell}{\beta} = \frac{\pi}{4} q^2 \frac{\ell}{\beta} \frac{\Phi_h^2}{\hbar}. \quad (30)$$

Let us now compare (30) with classical formula (6), which in present case is

$$S_{\text{GB}} = \frac{4\pi}{\hbar} \oint d^{D-2}x \sqrt{\tilde{g}} (\lambda_1 - 2\lambda_2 R(\tilde{g}_{ij})), \quad (31)$$

where  $\tilde{g}_{ij}$  is induced metric on the horizon. In the spherically symmetric case horizon is a  $(D - 2)$ -dimensional sphere with radius  $r_h$  and  $R(\tilde{g}_{ij}) =$

$-(D - 2)(D - 3)/r_h^2$ , so (31) becomes

$$S_{\text{GB}} = \frac{4\pi}{\hbar} \Omega_{D-2} [\lambda_1 r_h^{D-2} + 2(D - 2)(D - 3)\lambda_2 r_h^{D-4}] = 2\pi \frac{\Phi_h^2}{\hbar}. \quad (32)$$

Using this our expression (30) can be written as

$$S_C = \frac{q^2}{8} \frac{\ell}{\beta} S_{\text{GB}}, \quad (33)$$

so it gives correct result apart from dimensionless coefficient, which can be determined in the same way as in pure Einstein case [20]. First, it is natural to set the compactification period  $\ell$  equal to period of Euclidean-rotated black hole,<sup>3</sup> i.e.,

$$\ell = 2\pi\beta. \quad (34)$$

The relation between eigenvalue  $\Delta$  of  $L_0$  and  $c$  then becomes

$$\Delta = \frac{c}{12}. \quad (35)$$

We shall see in the next section that this relation holds for general GB theory, i.e., for arbitrary values of coupling constants.<sup>4</sup> One could be tempted to expect this to be valid for larger class of black holes and interactions than those treated so far.

To determine dimensionless parameter  $q$  we note that our effective theory given with (14) depends on effective parameters  $\Phi_h$  and  $\beta$ , and thus one expects that  $q$  depends on coupling constants only through dimensionless combinations of them. Thus to determine  $q$  one may consider  $\lambda_2 = 0$  case and compare expression for central charge (26) with that obtained in [7], which is

$$c = \frac{3A_h}{2\pi\hbar G_D}, \quad (36)$$

where  $A_h = \Omega_{D-2} r_h^{D-2}$  is the area of horizon. One obtains that

$$q^2 = \frac{4}{\pi}. \quad (37)$$

One could also perform boundary analysis of Ref. [7] for GB gravity (see Appendix A). This procedure

<sup>3</sup> We note that our functions depend only on variable  $z_+$ , so the periodicity properties in time  $t$  are identical to those in  $z$ .

<sup>4</sup> For pure Einstein gravity this relation is implicitly given in [7].

gives  $\Delta = \Phi_h^2/\hbar$  which combined with (26) and (35) gives (37).

Using (34) and (37) one finally obtains desired result

$$S_C = S_{GB}. \quad (38)$$

### 3. General Gauss–Bonnet gravity

In  $D > 6$  Gauss–Bonnet action has additional terms and general action was given in (5). Using spherical symmetry one obtains effective two-dimensional action given now with

$$\begin{aligned} I_{GB} = & \Omega_{D-2} \sum_{m=0}^{[D/2]} \lambda_m \frac{(D-2)!}{(D-2m)!} \\ & \times \int d^2x \sqrt{-\gamma} r^{D-2m-2} [1 - (\nabla r)^2]^{m-2} \\ & \times \{ 2m(m-1)r^2 [(\nabla_a \nabla_b r)^2 - (\nabla^2 r)^2] \\ & + 2m(D-2m)r \nabla^2 r [1 - (\nabla r)^2] \\ & + m \mathcal{R} r^2 [1 - (\nabla r)^2] \\ & - (D-2m)(D-2m-1) [1 - (\nabla r)^2]^2 \}. \end{aligned} \quad (39)$$

After partial integration<sup>5</sup> and implementation of horizon condition  $(\nabla r)^2 \approx 0$ , (39) becomes near the horizon approximately

$$\begin{aligned} I_{GB} = & -\Omega_{D-2} \sum_{m=0}^{[D/2]} \lambda_m \frac{(D-2)!}{(D-2m-2)!} \\ & \times \int d^2x \sqrt{-\gamma} r^{D-2m-2} \left\{ m(\nabla r)^2 \right. \\ & \left. - \frac{m}{(D-2m)(D-2m-1)} \mathcal{R} r^2 + 1 \right\}. \end{aligned} \quad (40)$$

If we define

$$\Phi^2 \equiv 2\Omega_{D-2} \sum_{m=1}^{[D/2]} m \lambda_m \frac{(D-2)!}{(D-2m)!} r^{D-2m}, \quad (41)$$

<sup>5</sup> Notice that

$$\begin{aligned} 2r^n [(\nabla_a \nabla_b r)^2 - (\nabla^2 r)^2] \\ = 3nr^{n-1} \nabla^2 r (\nabla r)^2 + n(n-1)r^{n-2} (\nabla r)^4 \\ + \mathcal{R} r^n (\nabla r)^2 + \text{surface terms.} \end{aligned}$$

and make a reparametrization (10), the action (40) becomes

$$I_{GB} = \int d^2x \sqrt{-\tilde{\gamma}} \left[ \frac{1}{4} q \Phi_h \phi \tilde{\mathcal{R}} - V(\phi) \right], \quad (42)$$

which is of the same form as (12) (the only difference is the exact form of the potential which is unimportant in this calculation). Now one can repeat the analysis from (12) to (30) in previous section without a change and obtain for the entropy the expression (30), where  $\Phi_h$  is now given by (41) evaluated at a horizon. It only remained to show that also in the general case the entropy (6) and  $\Phi_h$  are related as in (32). For spherically symmetric metric (2) where horizon is a  $(D-2)$ -dimensional sphere with radius  $r_h$  one can show that (6) can be written as<sup>6</sup>

$$\begin{aligned} S_{GB} = & \frac{4\pi}{\hbar} \Omega_{D-2} \sum_{m=1}^{[D/2]} m \lambda_m \frac{(D-2)!}{(D-2m)!} r^{D-2m} \\ = & 2\pi \frac{\Phi_h^2}{\hbar} \end{aligned} \quad (43)$$

the same as in (32). Finally, using the same arguments as in previous section one obtains  $S_C = S_{GB}$ .

### 4. Conclusion

In this Letter we have calculated entropy of  $D$ -dimensional spherically symmetric black holes in Gauss–Bonnet gravity. The method used asymptotic conformal symmetry of the effective two-dimensional action near the horizon [4]. This makes it possible to find via Cardy formula number of microstates. The obtained relation for the entropy coincides with the macroscopic formula [12].

It would be desirable to investigate if this result pursues also in other interactions. It would be also of interest to treat a more general class of stationary black holes. Such questions maybe also addressed by Carlip methods [7]. This could also help to understand better

<sup>6</sup> In fact, it is obvious that the last term in (40) is minus  $m$ th Gauss–Bonnet density (4) for the  $(D-2)$ -dimensional sphere with radius  $r$ , i.e.,

$$\mathcal{L}_m = \frac{(D-2)!}{(D-2m-2)!} \Omega_{D-2} r^{D-2m-2}.$$

the relation of two methods and the question of their eventual equivalence (some progress in this direction was recently done in [8]). In fact, some of these questions will be addressed in a separate publication.

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## Appendix A

Eigenvalue  $\Delta$  of  $L_0$  can be calculated by boundary analysis previously applied for Einstein gravity by Carlip<sup>7</sup> [7]. It is a contribution to the boundary term of the Hamiltonian

$$H[\xi] = \int_{\mathcal{H}} Q[\xi] + \dots$$

Here  $\mathcal{H}$  denotes the  $(n-2)$ -dimensional intersection of the Cauchy surface with the horizon, and the  $(n-2)$ -form  $Q$  is equal to

$$Q_{a_3 \dots a_n}[\xi] = -\frac{\partial \mathcal{L}_{\text{GB}}}{\partial R_{abcd}} \eta_{ab} \nabla_{[c} \xi_{d]} \hat{\epsilon}_{a_3 \dots a_n},$$

$\eta_{ab}$  is the binormal to the  $\mathcal{H}$ , and  $\xi^a$  is the vector field to which corresponds generator of diffeomorphisms  $H[\xi]$ . Boundary and integrability conditions are fixing deformations to lie in “ $r$ - $t$ ” plane and  $\xi^a = K\rho^a + T\chi^a$ , where  $\chi^a$  is approximately Killing near the horizon (determined by  $\chi^2 = 0$ ), and  $\rho_a = -\nabla_a \chi^2 / 2\kappa$ . Scalars  $K, T$  are connected by  $K = \chi^2 \chi^a \nabla_a T / \kappa \rho^2 \equiv \dot{T} \chi^2 / \kappa \rho^2$ . Then one can calculate

$$\begin{aligned} J[\xi] &\equiv \int_{\mathcal{H}} Q[\xi] \\ &= \int_{\mathcal{H}} (\lambda_1 - 2\lambda_2^{(n-2)} R) \left( 2\kappa T - \frac{\ddot{T}}{\kappa} \right) \hat{\epsilon}_{a_3 \dots a_n}. \quad (\text{A.1}) \end{aligned}$$

One can show analogously to [7] that Fourier components  $T_m$  of  $T$  lead to generators  $J[T_m]$  whose Dirac brackets satisfy again Virasoro algebra. Eq. (A.1) then gives us

$$\begin{aligned} \hbar \Delta &= J[T_0] = \int_{\mathcal{H}} (2\lambda_1 - 4\lambda_2^{(n-2)} R) \hat{\epsilon}_{a_3 \dots a_n} \\ &= \Phi_{\hbar}^2. \end{aligned}$$

Comparison with (35) gives (37). All details will be given in a separate publication [21].

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<sup>7</sup> We use here the notation of Ref. [7] where possible.