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## Shape coexistence in the relativistic Hartree-Bogoliubov approach

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The phenomenon of shape coexistence is studied in the relativistic Hartree-Bogoliubov framework. Standard relativistic mean-field effective interactions do not reproduce the ground-state properties of neutron-deficient Pt-Hg-Pb isotopes. It is shown that, in order to consistently describe binding energies, radii, and ground-state deformations of these nuclei, effective interactions have to be constructed, which take into account the sizes of spherical shell gaps.

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### I. INTRODUCTION

Nuclear structure models based on the relativistic mean-field approximation have been successfully employed in the description of ground-state properties of nuclei all over the periodic table. By adjusting a minimal set of model parameters, meson masses and coupling constants, to binding energies and radii of few spherical closed shell nuclei, it has been possible to perform detailed structure calculation of a large number of spherical and deformed nuclei [1]. More recently, the relativistic mean-field framework has also been used in studies of nuclear structure phenomena in exotic nuclei far from the valley of  $\beta$  stability and of the physics of the drip lines. In particular, the relativistic Hartree-Bogoliubov model encloses a unified description of mean-field and pairing correlations, and we have used this model to calculate ground-state properties of exotic nuclei with extreme isospin values: the halo phenomenon in light nuclei [2], properties of light neutron-rich nuclei [3], the reduction of the effective single-nucleon spin-orbit potential in nuclei close to the drip lines [4], properties of neutron-rich Ni and Sn isotopes [5], the location of the proton drip line from  $Z = 31$  to  $Z = 73$  and the phenomenon of ground-state proton radioactivity [6–8].

On the other hand, it is still an open problem how far from stability can one extrapolate the predictions of standard relativistic mean-field effective interactions. The question is how well can effective interactions that are adjusted to global properties of spherical closed shell nuclei, describe the structure of nuclei far from stability, or predict new phenomena in nuclei with extreme isospin values. It is well known, for example, that various nonrelativistic and relativistic mean-field models differ significantly in the prediction of the exact location of the neutron drip line. This is, of course, related to different isovector properties of various effective interactions. In general, one cannot expect standard mean-field models to accurately describe the properties, which crucially depend on the proton and neutron single-particle spectra. An important example is the suppression of shell effects and the related phenomenon of deformation and shape coexistence.

A number of theoretical analyses of shell quenching and shape coexistence phenomena have been performed in the relativistic mean-field framework. In Ref. [9] we have stud-

ied the dissolution of the spherical  $N=28$  shell gap in neutron-rich nuclei. By performing constrained relativistic Hartree-Bogoliubov calculations, we have shown how the strong reduction of the gap between the neutron  $1f_{7/2}$  orbital and the  $2p_{3/2}$ ,  $2p_{1/2}$  levels results in deformed ground states and shape coexistence in neutron-rich Si, S, and Ar isotopes, in excellent agreement with experimental data. On the proton-rich side, the relativistic mean-field model has been employed in studies of shape coexistence in neutron-deficient Pt, Hg, and Pb isotopes [10–13]. However, as has been pointed out in Ref. [14], the results of these studies were at variance with experimental data, especially their predictions of deformed ground states in Pb isotopes, and of prolate and superdeformed ground states in Hg isotopes. In a very recent analysis of the  $Z=82$  shell closure in neutron-deficient Pb isotopes [15], relativistic mean-field calculations again predicted deformed ground states in some Pb isotopes, in contradiction with experiment.

In this work we employ the relativistic Hartree-Bogoliubov model in the analysis of shape coexistence phenomena in neutron-deficient Hg and Pb isotopes. Although standard relativistic mean-field forces do not reproduce the experimental data on ground-state properties, it is indeed possible to construct effective interactions that consistently describe binding energies, radii, and quadrupole deformations of neutron-deficient nuclei in this mass region. In addition to bulk properties, also the sizes of spherical gaps in the single-nucleon spectra have to be taken into account when adjusting the parameters of such an effective interaction. The spherical magic and semimagic gaps determine the relative excitation energies of coexisting minima based on different intruder configurations.

Section II contains an outline of the relativistic Hartree-Bogoliubov model. The problem of the relativistic effective interaction and shape coexistence in neutron-deficient Hg and Pb isotopes is discussed in Sec. III. The results are summarized in Sec. IV.

### II. OUTLINE OF THE RELATIVISTIC HARTREE-BOGOLIUBOV MODEL

In the relativistic framework a nucleus is described as a collection of nucleons that interact by the exchange of effec-

tive mesons. The lowest order of the quantum field theory is the *mean-field* approximation: the meson field operators are replaced by their expectation values. The sources of the meson fields are defined by the nucleon densities and currents. The nucleons move independently in the mean-field potential that originates from the nucleon-nucleon interaction. The ground state of a nucleus corresponds to the stationary self-consistent solution of the coupled system of Dirac and Klein-Gordon equations

$$\left\{ -i\boldsymbol{\alpha}\cdot\nabla + \beta(m + g_\sigma\sigma) + g_\omega\omega^0 + g_\rho\tau_3\rho_3^0 + e\frac{(1-\tau_3)}{2}A^0 \right\} \psi_i = \varepsilon_i\psi_i, \quad (1)$$

$$[-\Delta + m_\sigma^2]\sigma(\mathbf{r}) = -g_\sigma\rho_s(\mathbf{r}) - g_2\sigma^2(\mathbf{r}) - g_3\sigma^3(\mathbf{r}), \quad (2)$$

$$[-\Delta + m_\omega^2]\omega^0(\mathbf{r}) = g_\omega\rho_v(\mathbf{r}), \quad (3)$$

$$[-\Delta + m_\rho^2]\rho^0(\mathbf{r}) = g_\rho\rho_3(\mathbf{r}), \quad (4)$$

$$-\Delta A^0(\mathbf{r}) = e\rho_p(\mathbf{r}). \quad (5)$$

The single-nucleon dynamics is described by the Dirac equation (1).  $\sigma$ ,  $\omega$ , and  $\rho$  are the meson fields, and  $A$  denotes the electromagnetic potential.  $g_\sigma$ ,  $g_\omega$ , and  $g_\rho$  are the corresponding coupling constants for the mesons to the nucleon.  $g_2$  and  $g_3$  are the coefficients of the nonlinear  $\sigma$  terms, which introduce an effective density dependence in the potential. Due to charge conservation, only the 3rd component of the isovector  $\rho$  meson contributes. The source terms in Eqs. (2)–(5) are sums of bilinear products of baryon amplitudes with positive energy (*no-sea* approximation).

In addition to the self-consistent mean-field potential, pairing correlations have to be included in order to describe ground-state properties of open-shell nuclei. For spherical and deformed nuclei not too far from the stability line, pairing is often treated phenomenologically in the simple BCS approximation [1]. However, the BCS model presents only a poor approximation for exotic nuclei far from the valley of  $\beta$  stability [16]. The structure of weakly bound nuclei necessitates a unified and self-consistent treatment of mean-field and pairing correlations. In particular, the relativistic Hartree-Bogoliubov (RHB) model represents a relativistic extension of the Hartree-Fock-Bogoliubov framework. In the RHB model the ground state of a nucleus  $|\Phi\rangle$  is represented by the product of independent single-quasiparticle states. These states are eigenvectors of the generalized single-nucleon Hamiltonian that contains two average potentials: the self-consistent mean-field  $\hat{\Gamma}$ , which encloses all the long range particle-hole (*ph*) correlations, and a pairing field  $\hat{\Delta}$ , which sums up the particle-particle (*pp*) correlations. In the Hartree approximation for the self-consistent mean field, the relativistic Hartree-Bogoliubov equations read

$$\begin{pmatrix} \hat{h}_D - m - \lambda & \hat{\Delta} \\ -\hat{\Delta}^* & -\hat{h}_D + m + \lambda \end{pmatrix} \begin{pmatrix} U_k(\mathbf{r}) \\ V_k(\mathbf{r}) \end{pmatrix} = E_k \begin{pmatrix} U_k(\mathbf{r}) \\ V_k(\mathbf{r}) \end{pmatrix}, \quad (6)$$

where  $\hat{h}_D$  is the single-nucleon Dirac Hamiltonian (1), and  $m$  is the nucleon mass. The chemical potential  $\lambda$  has to be determined by the particle number subsidiary condition in order that the expectation value of the particle number operator in the ground state equals the number of nucleons.  $\hat{\Delta}$  is the pairing field. The column vectors denote the quasiparticle spinors and  $E_k$  are the quasiparticle energies. In most applications of the RHB model a phenomenological pairing interaction has been used: the pairing part of the Gogny force

$$V^{pp}(1,2) = \sum_{i=1,2} e^{-[r_1-r_2]/\mu_i} (W_i + B_i P^\sigma - H_i P^\tau - M_i P^\sigma P^\tau), \quad (7)$$

with the set DIS [17] for the parameters  $\mu_i$ ,  $W_i$ ,  $B_i$ ,  $H_i$ , and  $M_i$  ( $i=1,2$ ).

The RHB equations are solved self-consistently, with potentials determined in the mean-field approximation from solutions of Klein-Gordon equations for the meson fields. The Dirac-Hartree-Bogoliubov equations and the equations for the meson fields are solved by expanding the nucleon spinors  $U_k(\mathbf{r})$  and  $V_k(\mathbf{r})$ , and the meson fields in terms of the eigenfunctions of a deformed axially symmetric oscillator potential. A detailed description of the relativistic Hartree-Bogoliubov model for deformed nuclei can be found in Ref. [7].

### III. SHAPE COEXISTENCE IN NEUTRON-DEFICIENT Hg AND Pb ISOTOPES

The light isotopes of Hg and Pb exhibit a variety of co-existing shapes [18,19]. A systematic analysis of shape coexistence effects at  $I^\pi=0^+$  in these nuclei has been performed by Nazarewicz [20], using the shell correction approach with the axial, reflection asymmetric Woods-Saxon model. A common feature is the competition of low-lying prolate and oblate shapes. The ground states of Hg isotopes are weakly oblate deformed (two-proton hole states). In  $^{188}\text{Hg}$  the oblate ground-state band is crossed by the intruding deformed band, which corresponds to a prolate minimum ( $4p$ - $6h$  proton excitations into the  $h_{9/2}$  and  $f_{7/2}$  orbits). The excitation energy of the prolate band is lowered with decreasing neutron number, and reaches a minimum in  $^{182}\text{Hg}$ . For lighter Hg isotopes the excitation energy of the prolate minimum increases, and the oblate ground state evolves toward a spherical shape. Experimental data on energy spectra and charge radii show that the ground states of Pb isotopes are spherical, but both oblate ( $2p$ - $2h$  proton excitations) and prolate ( $4p$ - $4h$  proton excitations) low-lying minima are observed for  $N < 110$ . A beautiful example of oblate and prolate minima at almost identical excitation energies is found in  $^{186}\text{Pb}$  [21]. A recent review of experimental data on intruder states in neutron deficient Hg, Pb, and Po nuclei can be found in Ref. [19].

In Ref. [15] Bender *et al.* have used nonrelativistic and relativistic self-consistent mean-field models to analyze re-

cent experimental data that seem to indicate a strong reduction of the  $Z=82$  shell gap in neutron-deficient Pb isotopes. The systematics of differences between two-proton separation energies in adjacent even-even nuclei with the same neutron number (two-proton shell gaps) [22],  $Q_\alpha$  values and  $\alpha$  reduced widths [23] suggest a lessened magicity of the  $Z=82$  shell when neutrons are midway between  $N=82$  and  $N=126$ . However, in Ref. [15] it has been shown that the systematics of two-proton shell gaps can be described quantitatively in terms of deformed ground states of Hg and Po isotopes. The experimental  $Q_\alpha$  values also reflect deformation effects, and the systematics of  $\alpha$ -decay hindrance factors is consistent with the stability of the  $Z=82$  shell gap.

The structure of coexisting minima in neutron-deficient Hg and Pb nuclei has also been analyzed in the relativistic mean-field framework. In Refs. [10,11] the relativistic mean-field (RMF) model, with the NL1 [24] effective interaction, was used to calculate prolate, oblate, and spherical solutions for neutron-deficient Pt, Hg, and Pb isotopes. Pairing was treated in the BCS approximation with constant pairing gaps. Although the NL1 effective force strongly overbinds these nuclei, i.e., the calculated binding energies are 10–12 MeV larger than the empirical ones, nevertheless detailed predictions were made for ground-state deformations and relative positions of coexisting minima, including the excitation energies of superdeformed states. Most of these results, as pointed out in Ref. [14], contradict well established experimental data (wrong sign of the ground-state deformations for Hg isotopes, calculated deformed ground states for some Pb nuclei, superdeformed ground states, etc.). The calculations were repeated in Ref. [12] with the NL-SH effective interaction [25], and the values of the pairing gaps were varied by as much as 50%, but the results remained essentially the same. Eventually, deformed RMF+BCS calculations with the NL1 effective interaction reproduced the experimental ground-state oblate deformations of  $^{180-188}\text{Hg}$  [13]. The differences of the intrinsic energies of the lowest prolate and oblate states were also in agreement with experimental data. The gap parameters were not kept constant in each nucleus, rather they were determined in a self-consistent way from a monopole force with constant strength parameter  $G$ . This leads to deformation dependent pairing gaps. In particular, for the oblate ground states the proton pairing gaps vanish, while the neutron pairing gap is a factor 2 larger than the average value  $12/\sqrt{A}$ . Since the calculated ground-state properties are very sensitive to both pairing and deformation effects, it was argued [13] that a unified and self-consistent framework, i.e., the relativistic Hartree-Bogoliubov model, might be more appropriate for the study of neutron-deficient nuclei in this mass region. It is not only the treatment of pairing that makes it difficult to assess the conclusions of Ref. [13], but also the large deviations of the calculated binding energies from the experimental values. The relative positions of prolate and oblate minima differ by only  $\approx 0.5$  MeV in most cases, while the calculated binding energies are more than 10 MeV too large. This is, of course, caused by the well known fact that the parameter set NL1 fails to reproduce nuclear binding energies far from stability.

In this work we calculate ground-state properties of

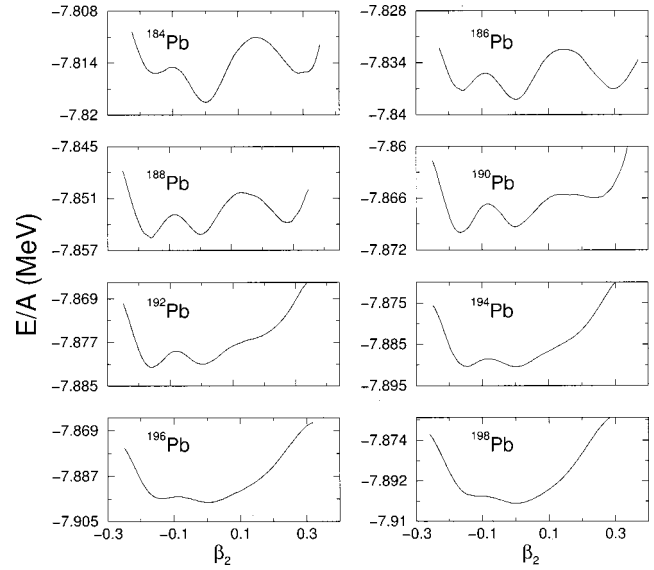


FIG. 1. Binding energy curves of even- $A$  Pb isotopes as functions of the quadrupole deformation. The curves correspond to RHB model solutions with constrained quadrupole deformation. The effective interaction is NL3 + Gogny D1S.

neutron-deficient Hg and Pb nuclei in the relativistic Hartree-Bogoliubov framework. In most applications of the RHB model we have used the NL3 effective interaction [26] in the particle-hole channel, and pairing correlations were described by the pairing part of the finite range Gogny interaction D1S [17]. This force has been very carefully adjusted to pairing properties of finite nuclei all over the periodic table. In particular, the basic advantage of the Gogny force is the finite range, which automatically guarantees a proper cutoff in momentum space. Properties calculated with NL3 indicate that this is probably the best effective relativistic interaction so far, both for nuclei at and away from the line of  $\beta$  stability. A recent systematic theoretical study of ground-state properties of more than 1300 even-even isotopes has shown very good agreement with experimental data [27]. However, in the analysis of the  $Z=82$  shell closure in neutron-deficient Pb isotopes [15], it was noted that RMF+BCS calculations with the NL3 interaction predict deformed ground states for several Pb nuclei.

In Fig. 1 we display the calculated binding energy curves of even- $A$   $^{184-198}\text{Pb}$  isotopes as functions of the quadrupole deformation. The curves correspond to axially deformed RHB model solutions with constrained quadrupole deformation. The effective interaction is NL3 + Gogny D1S.  $^{184}\text{Pb}$  and  $^{186}\text{Pb}$  have spherical ground states, and we find low-lying oblate and prolate minima, in qualitative agreement with experimental data [19]. With increasing neutron number, however, the oblate minimum is lowered in energy and the nuclei  $^{188-194}\text{Pb}$  have oblate ground states. This result, of course, contradicts experimental data. Only from  $A=196$  the calculated Pb ground states become again spherical, but both for  $^{196}\text{Pb}$  and  $^{198}\text{Pb}$  the potential curves display wide and flat minima on the oblate side.

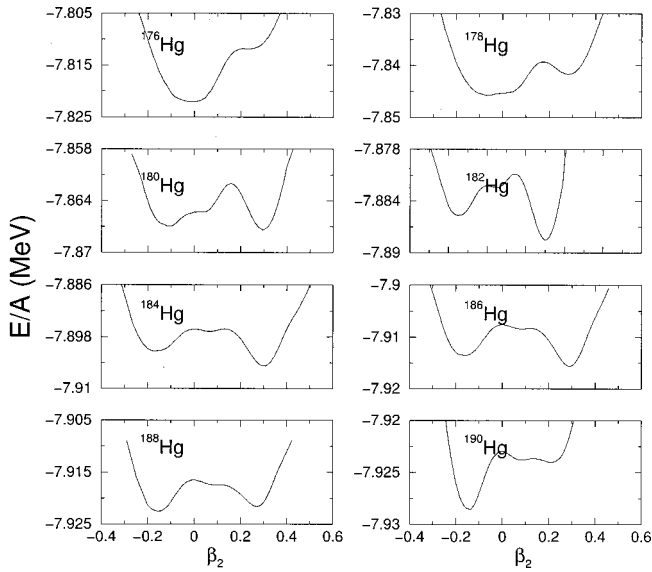
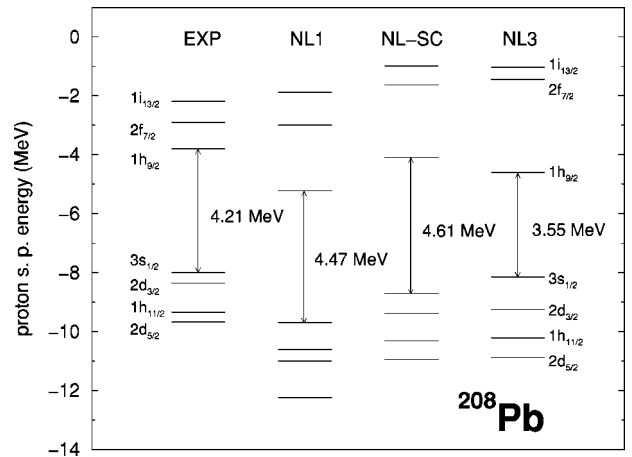


FIG. 2. Same as in Fig. 1, but for neutron-deficient Hg isotopes.

The RHB NL3+D1S binding energy curves of even- $A$   $^{176-190}\text{Hg}$  nuclei are shown in Fig. 2. Model calculations predict a spherical ground state for  $^{176}\text{Hg}$  and an almost spherical, but slightly oblate ground state for  $^{178}\text{Hg}$ . With increasing neutron number the spherical state evolves into an oblate minimum, and a pronounced minimum develops on the prolate side. The positions of the minima,  $\beta_2 \approx -0.15$  on the oblate side, and  $\beta_2 \approx 0.3$  for the prolate minimum, agree well with the values calculated by Nazarewicz [20] using the Nilsson-Strutinsky approach. The problem is, however, the relative excitation energies. For the NL3+D1S effective interaction the prolate minimum is the ground state for  $^{180,182,184,186}\text{Hg}$ . The calculated ground states are oblate for  $^{188,190}\text{Hg}$ . These results are at variance with experimental data: all Hg isotopes with  $A \geq 178$  have oblate ground states, the prolate minimum is estimated at 300–800 keV above the ground state [19]. If the NL1 effective interaction is used in the  $ph$  channel, we find that the oblate minima are always lower, and this is similar to the result obtained with the RMF+BCS calculation of Ref. [13]. The calculated binding energies, however, are on the average more than 10 MeV larger than the experimental ones.

Is it then possible to construct a relativistic effective interaction, which will consistently describe the ground-state properties of neutron-rich nuclei in this mass region? The motivation, of course, is not just to reproduce the experimental data. The real question is whether effective mean-field interactions can be accurately extrapolated from stable nuclei to isotopes with extreme isospin values and to the drip lines. In the particular example considered in the present study, the quantitative description of coexisting spherical, oblate, and prolate minima is, of course, beyond the mean-field approach. A detailed analysis of coexisting shapes necessitates the use of models, which can account for configuration mixing effects. However, a consistent description of ground states should be possible in the relativistic mean-field framework, even if their properties are affected by correlations not

FIG. 3. Proton single-particle states in  $^{208}\text{Pb}$ . The experimental levels in the first column are from Ref. [32]. The levels in the second, third, and fourth columns are calculated in the RHB model with the effective interactions NL1, NL-SC, and NL3, respectively.

explicitly included in the model. In a series of recent papers (see, for example, Refs. [28,29]), Furnstahl and Serot have argued that relativistic mean-field models should be considered as approximate implementations of the Kohn-Sham density functional theory, with local scalar and vector fields appearing in the role of relativistic Kohn-Sham potentials. The mean-field models approximate the exact energy functional of the ground-state density of a many-nucleon system, which includes all higher-order correlations, using powers of auxiliary meson fields or nucleon densities.

In the recent analysis of the phenomenon of shape coexistence within the nonrelativistic self-consistent Hartree-Fock method and the nuclear shell model [30], it was emphasized that the sizes of spherical magic and semimagic gaps in the single-nucleon spectrum determine the relative positions of many-particle many-hole intruder configurations with respect to the ground state. The spherical gaps are the main factor that determines the relative excitation energies of coexisting minima based on different intruder configurations. The relative distance between the individual shells also determines the effective strength of the quadrupole field [31]. The spherical proton  $Z=82$  shell closure is illustrated in Fig. 3, where we display the last four occupied and the first three unoccupied proton single-particle states in  $^{208}\text{Pb}$ . The experimental levels in the first column are from Ref. [32]. The levels in the second, third, and fourth columns are calculated in the RHB model with the effective interactions NL1, NL-SC (Shape Coexistence), and NL3, respectively. Comparing the magic gaps  $E(h_{9/2}) - E(s_{1/2})$ , one notices that the value calculated with the NL3 effective interaction is much smaller than the empirical gap. This explains why several neutron-deficient Pb isotopes, calculated with the NL3 interaction, have deformed ground states. The magic gap in the proton spectrum calculated with NL1, on the other hand, is somewhat larger than the empirical value. The spectrum of proton occupied states is, however, more bound and the total

binding energies calculated with NL1 do not compare well with the experimental masses of lighter Pb isotopes. We have tried, therefore, to construct a new effective interaction which, on one hand, preserves the good properties of NL3 (binding energies, charge radii, isotopic shifts), but at the same time takes into account the gap between the last occupied and the first unoccupied major spherical shells. The result is shown in the third column NL-SC of Fig. 3. There are several comments that should be made at this point.

First, the NL-SC interaction has been constructed by starting from the standard NL1 parametrization, and changing the parameters towards NL3. While the masses and coupling constants of the  $\sigma$  and  $\omega$  mesons are those of NL1, the parameters of the nonlinear  $\sigma$  terms and, especially important for the binding energies of isotopes with small  $N/Z$ , the coupling constant of the  $\rho$  meson, are very close to the NL3 parameter set. It should also be emphasized that, although in the construction of the NL-SC interaction we paid particular attention to the Pt-Hg-Pb region, the parameters were, as usual, checked to reproduce a set of ground-state data of about ten spherical nuclei from different mass regions. For nuclei with  $A > 100$  the calculated  $\chi^2$  per datum is comparable to the one obtained with the standard NL3 interaction. The ground-state properties of light nuclei calculated with NL-SC, on the other hand, are not as good as with NL3 and, therefore, we do not regard NL-SC as a new general effective relativistic mean-field interaction. NL-SC has been specifically tailored to illustrate the problem considered in the present analysis.

Second, the construction of the NL-SC interaction goes beyond the standard mean-field approach, in the sense that the parameters of the mean-field functional depend not only on the ground-state density, i.e., on the occupied states, but implicitly also on the relative position of the unoccupied spherical shells. In order to be consistent with the mean-field approach, the calculated spherical gap should be larger than the empirical value. It is well known that the coupling of single-particle states to collective vibrations increases the effective mass in the vicinity of the Fermi level, and this effect decreases the gap between occupied and unoccupied shells. We have recently shown how to include the particle-vibration coupling in the self-consistent relativistic mean-field framework [33], but the present analysis does not consider this effect. In Ref. [30] it was emphasized that the proper treatment of pairing and zero-point correlations, vibrational and rotational, is crucial for detailed predictions of shape coexistence effects. And while we treat pairing and mean-field correlations in the unified Hartree-Bogoliubov framework, no attempt is made to explicitly include zero-point energy corrections. Since in the analysis of Ref. [30] it was shown that the sum of rotational and vibrational zero-point energy corrections is approximately constant as function of deformation for a given nucleus, we assume that these corrections can be absorbed in the parameters of an effective interaction adjusted to reproduce binding energies, at least in a limited mass region. In any case, since the effective interaction NL-SC is adjusted to reproduce the spherical proton shell gap at  $Z=82$ , one cannot really expect that it accurately describes the ground-state shapes of nuclei in other  $(Z, N)$

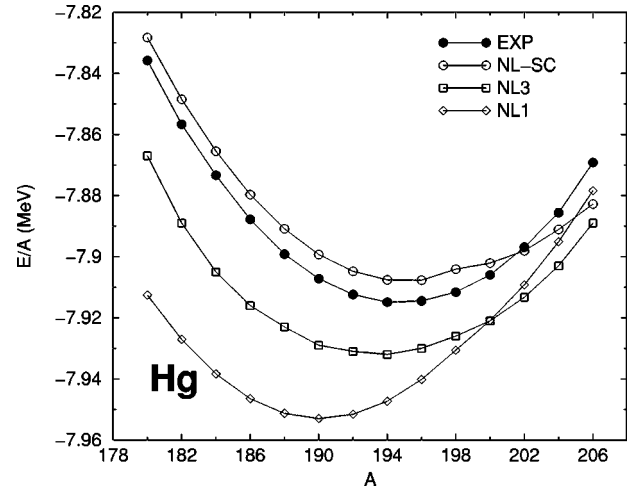


FIG. 4. Binding energies of even- $A$  Hg isotopes calculated in the RHB model with the mean-field effective interactions NL1, NL3, and NL-SC, and with the Gogny DIS interaction in the pairing channel. The theoretical binding energies are compared with the empirical values from Ref. [34].

regions characterized by shape coexistence.

In Figs. 4 and 5 we display the binding energies per nucleon of even- $A$  Hg and Pb isotopes, respectively, calculated in the RHB model with the mean-field effective interactions NL1, NL3, and NL-SC, and with the Gogny DIS interaction in the pairing channel. The theoretical binding energies are compared with the empirical values from Ref. [34]. Both the NL3 and the NL-SC interactions reproduce in detail the mass dependence of the binding energies of Pb isotopes, while the NL1 interaction strongly overbinds the nuclei below  $^{208}\text{Pb}$ . The reason is the much larger value of the  $\rho$  meson coupling constant. Essentially identical binding energies are calculated with all three forces for nuclei above  $^{208}\text{Pb}$ . A similar effect is observed for the Hg isotopes in Fig. 4. While NL3 and NL-SC reproduce much better the empirical data for the neutron-deficient Hg isotopes, comparable results are obtained with all three interactions for  $A \geq 100$ .

A remarkable success of relativistic mean-field models

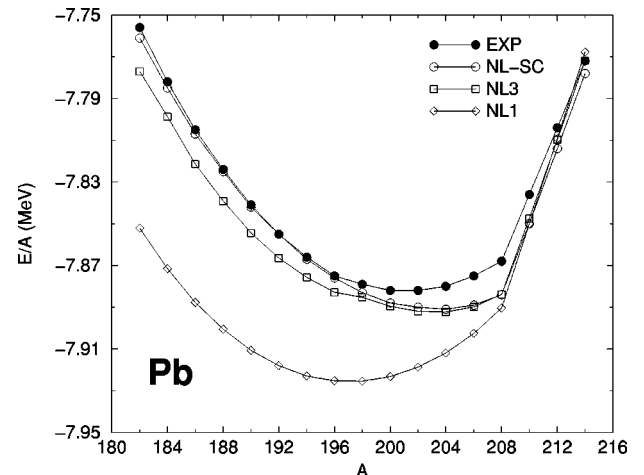


FIG. 5. Same as in Fig. 4, but for Pb isotopes.

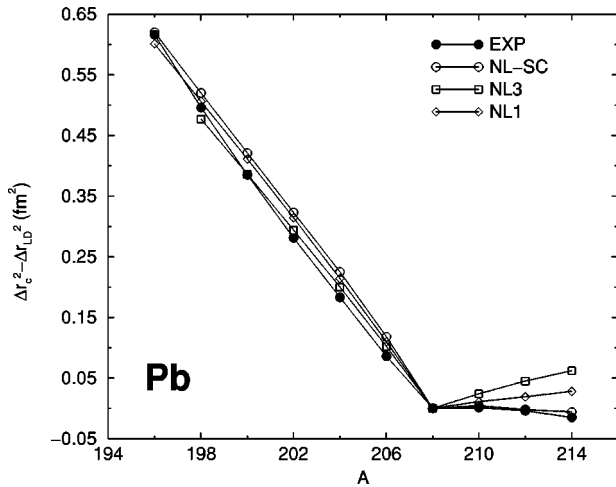


FIG. 6. Charge isotope shifts in even- $A$  Pb isotopes. The results of RHB calculations with the NL1, NL3, and NL-SC effective interactions, and with the Gogny D1S interaction in the pairing channel, are compared with experimental data from Ref. [36].

was the realization that, because of the intrinsic isospin dependence of the effective single-nucleon spin-orbit potential, they naturally reproduce the anomalous charge isotope shifts [35]. The well known example of the anomalous kink in the isotope shifts of Pb isotopes is shown in Fig. 6. The results of RHB calculations with the NL1, NL3, and NL-SC effective interactions, and with the Gogny D1S interaction in the pairing channel, are compared with experimental data from Ref. [36]. While all three forces reproduce the general trend of isotope shifts and the kink at  $^{208}\text{Pb}$ , this effect is more pronounced for NL1 and NL3. The experimental data are best reproduced by the NL-SC interaction. The RHB theoretical values for the charge isotope shifts of even- $A$  Hg isotopes are compared with the experimental data [36] in Fig. 7. The large discrepancy between data and the values calculated with NL3 for  $A < 188$ , simply reflects the fact that this interaction predicts the wrong sign for the ground-state deforma-

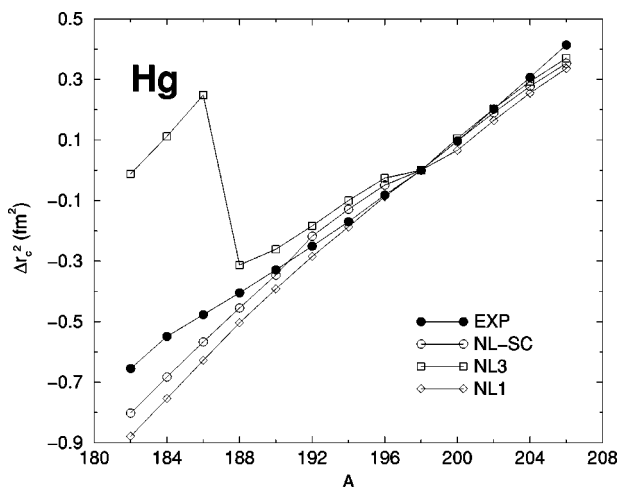


FIG. 7. The RHB theoretical values for the charge isotope shifts in even- $A$  Hg isotopes, compared with experimental data from Ref. [36].

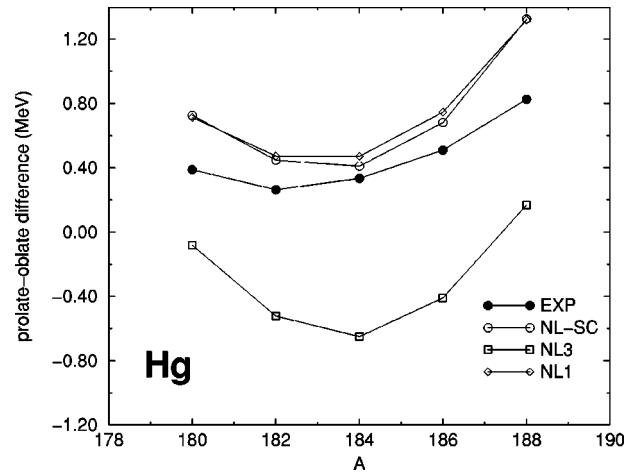


FIG. 8. The difference between binding energies of prolate and oblate states in even- $A$  Hg isotopes. The results of RHB calculations with the NL1, NL3, and NL-SC effective interactions, and with the Gogny D1S interaction in the pairing channel, are compared with the empirical data from Ref. [37].

tions of neutron-deficient Hg nuclei. The interactions NL1 and NL-SC reproduce the experimental data, although the agreement is not as good as in the case of Pb isotopes.

In Fig. 8 we plot the differences between the binding energies of prolate and oblate minima in even- $A$   $^{180-188}\text{Hg}$  isotopes. The values calculated with the NL1, NL3, and NL-SC effective interactions are shown in comparison with the empirical data [37]. As it was already shown in Fig. 2, prolate ground states are calculated with the NL3 interaction. The NL1 and NL-SC interactions, on the other hand, reproduce the relative positions of the prolate and oblate minima. One has to keep in mind, however, that the total binding energies calculated with NL1 are, on the average, more than 10 MeV too large. The RHB model with the NL-SC interaction in the  $ph$  channel and with the Gogny D1S interaction in the  $pp$  channel, provides a consistent description for the ground-state properties of neutron-deficient nuclei in the Pt-Hg-Pb region (binding energies, radii, ground-state deformations). We should emphasize that NL-SC was not adjusted to reproduce the relative excitation energies of the prolate minima in Hg nuclei. However, the fact that the calculated relative positions of prolate and oblate minima agree with the empirical values, might indicate that configuration mixing effects are not very important in these nuclei.

#### IV. CONCLUSIONS

This work presents an analysis of ground-state properties of neutron-deficient Hg and Pb isotopes in the framework of the RHB model. In the last couple of years this model has been very successfully applied in the description of nuclear structure phenomena in medium-heavy and heavy exotic nuclei far from the valley of  $\beta$  stability and of the physics of the drip lines. It is still very much an open problem, however, how far from stability one can apply relativistic effective interactions that have been adjusted to global properties of a small number of spherical closed shell nuclei. Can these

interactions accurately describe experimentally known properties and predict new structure phenomena in exotic nuclei far from stability?

One of the testing grounds of nuclear structure models is their ability to describe shape coexistence phenomena in soft nuclei. In the present study, in particular, we have analyzed the results of relativistic mean-field calculations in the region of neutron deficient Pt-Hg-Pb nuclei. Standard relativistic interactions, adjusted to ground-state properties of stable nuclei, do not reproduce the empirical ground-state shapes of neutron-deficient nuclei in this mass region. We have shown, however, that effective interactions can be constructed, which consistently describe binding energies, charge radii, ground-state quadrupole deformations and, at least qualitatively, the relative positions of coexisting minima in Hg and Pb isotopic chains. In adjusting the parameters of such an interaction, in addition to the usual experimental data on ground states of stable nuclei, also the sizes of spherical gaps in the single-nucleon spectra have to be taken into account. The spherical magic and semimagic gaps determine the relative excitation energies of coexisting minima based on different intruder configurations.

Although a quantitative analysis of shape coexistence phenomena goes beyond the mean-field approach, a consistent description of ground states is possible in the relativistic mean-field framework. We have shown that the RHB model with the NL-SC interaction in the  $ph$  channel, adjusted to the  $Z=82$  proton shell closure in  $^{208}\text{Pb}$ , and with the Gogny D1S interaction in the  $pp$  channel, reproduces in detail the empirical ground-state properties of neutron-deficient Hg and Pb nuclei.

The results of the present analysis suggest that, when constructing effective mean-field interactions to be used in regions of exotic nuclei far from stability, not only bulk properties of spherical nuclei, but also magic and semimagic gaps in the single nucleon spectra must be taken into account.

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