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New dipole penguin contribution to $K \to \pi\pi$ decays

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A R T I C L E   I N F O

A B S T R A C T

We point out that the standard chromomagnetic penguin dipole operator has a counterpart corresponding to off-shell momenta for external quarks. By employing the chiral quark model, we show that this new dipole penguin operator has the same bosonisation as the standard Q_6 operator. Accordingly, this new operator enlarges by $\sim 5\%$ the referent Q_6 contribution, which gives the dominant contribution to the CP-violating ratio ε'/ε and also gives an important contribution to the $\Delta I = 1/2$ amplitude.

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1. Introduction

The physics of the $K \to 2\pi$ decay within the Standard Model has been a great challenge. First, the $\Delta I = 1/2$ rule is only vaguely understood, and similarly, the CP-violating quantity ε'/ε has been very difficult to estimate because of the inherent hadronic uncertainties. In electroweak decays of K-mesons one constructs an effective Lagrangian at the quark level. Thereby one uses the equations of motion. In QED it is known that the main part of the Lamb shift disappears if the equation of motion is used for the electron self-energy, warning us that it is a bound state, nonperturbative effect. What about the Lamb shift like effects in the QCD context of hadronic decays?

An early study by two of us [1,2] was undertaken in order to account for the off-shell effects in K-meson decays. In particular, in Ref. [1] we considered only the CP-conserving $K \to 2\pi$ amplitude, and the CP-violating off-shell part has been assigned to "the waiting list of pieces to be included in the re-evaluation of ε'/ε". At roughly the same time Bertolini et al. [3] included in such a

reevaluation the chromomagnetic Q_{11} penguin dipole contribution to ε'/ε, and in the present Letter we find as appropriate to consider its off-shell counterpart.

From the very beginning there has been a close relation between the gluonic penguin operators and the attempts to predict the direct CP-violation parameter ε'/ε of the $K \to 2\pi$ amplitude. At first, the gluonic penguin had pointed to a possibility of a sizable (direct) CP-violating $K \to 2\pi$ amplitude. However, there had been a turning point with the large value of the t-quark mass, that led to a substantial cancellation between the dominant (gluonic Q_6, and electroweak Q_8) penguin contributions. That called for investigation of other possible contributions, including the off-shell contribution at hand.

For a brief history of progress on evaluating ε'/ε and a more complete list of references on evaluation of ε'/ε we refer to the review in Ref. [4], and for later work to Ref. [5]. These references show the way in which ε'/ε is structured over the contributions from the operators belonging to, by now, standard operator basis [6]. The relatively large value for ε'/ε obtained by the Trieste–Oslo Collaboration [7] turned out to be a successful prediction for the outcome of the subsequent Fermilab [8] and CERN [9] measurements, that together with previous results from NA31 [10] and E731 [11] experiments reported the world average

$$\varepsilon'/\varepsilon = (16.7 \pm 1.6) \times 10^{-4}. \tag{1}$$

Basically, the prediction by the Trieste–Oslo group resides in the use of the chiral quark model (χQM) [12–14] which enabled to account for unavoidable nonperturbative QCD effects. Despite being a model approach, such a treatment has some unique features: Besides weakening the destructive interference between the matrix elements of Q_6 and Q_8 operators (which we explicate in the next section), it enables the evaluation of the matrix elements for all the relevant operators within a single framework. Let us stress that in the chiral quark model we could evaluate the mentioned off-shell effects in K-meson decays [1,2,15] and to account for the off-shell, off-diagonal self energy contribution to the CP-violating ratio ε'/ε [16]. We believe that the off-shell chromomagnetic effect, calculated in this Letter, presents an interesting new piece in illuminating the $K \to \pi\pi$ puzzle.

2. Effective Lagrangian for $\Delta S = 1$ decays

The amplitudes for $K \to 2\pi$ are described by an effective weak Lagrangian at quark level [6,17]

$$\mathcal{L}_W = \sum_i C_i(\mu) Q_i(\mu), \tag{2}$$

where all information on the short distance (SD) loop effects above a renormalisation scale μ is contained in Wilson coefficients C_i. These depend on the masses of the W, Z-bosons, the heavy quark masses ($m_q > \mu$), Λ_{QCD} and on the renormalisation scheme. The Q_i's are quark operators, typically containing products of two quark currents.

The standard basis (for $\mu < m_c$, relevant to kaon decays) includes ten 4-quark operators. We display the four most important for the $\Delta I = 1/2$ rule ($Q_{1,2,6}$) and $\varepsilon'/\varepsilon(Q_{6,8})$:

$$Q_1 = 4(\bar{s}_L \gamma^\mu d_L)(\bar{u}_L \gamma_\mu u_L),$$
$$Q_6 = -8 \sum_q (\bar{s}_L q_R)(\bar{q}_R d_L),$$
$$Q_2 = 4(\bar{s}_L \gamma^\mu u_L)(\bar{u}_L \gamma_\mu d_L),$$
$$Q_8 = -12 \sum_q \hat{e}_q (\bar{s}_L q_R)(\bar{q}_R d_L), \tag{3}$$

where $\hat{e}_q$ are the quark charges ($\hat{e}_u = 2/3$, $\hat{e}_d = \hat{e}_s = -1/3$), and $q_{L,R}$ are the left- and right-handed projections of the quark fields.

Some studies also include the standard chromomagnetic dipole operator [3,4,6,18] that can be written as

$$Q_{11} = \frac{g_s}{8\pi^2} \bar{d}[m_s R + m_d L]\sigma \cdot Gs + \text{h.c.}, \tag{4}$$

where $\sigma \cdot G \equiv \sigma^{\mu\nu} G^a_{\mu\nu} t^a$, $G^a_{\mu\nu}$ is the gluon field tensor, t^a are the $SU(3)$ generators normalised as $\text{Tr}(t^a t^b) = \delta^{ab}/2$, and $(L, R) = (1 \mp \gamma_5)/2$. However, we should stress that the operator for $s \to d + gluon(s)$ transition generated by loop diagrams [1,19] is not Q_{11} in Eq. (4), but is given by

$$\mathcal{L}(s \to dG) = B\epsilon^{\mu\nu\lambda\rho} G^a_{\mu\nu} (\bar{d}_L t^a i \overleftrightarrow{D}_\lambda \gamma_\rho s_L)$$
$$\to -\frac{1}{2} B\bar{d}_L [i\gamma \cdot D\sigma \cdot G + \sigma \cdot Gi\gamma \cdot D]s_L. \tag{5}$$

Here the second line is obtained by using simple algebra of Dirac matrices, and the coefficient $B \sim g_s G_F \lambda_{KM}$ depends on the loop integration. It is convenient to rewrite (5) as a sum of an off-shell term $\mathcal{L}_G$ and the chromomagnetic moment term $\mathcal{L}_\sigma$ [1]:

$$\mathcal{L}(s \to dG) = \mathcal{L}_G + \mathcal{L}_\sigma,$$
$$\mathcal{L}_G = C_G Q_G, \qquad \mathcal{L}_\sigma = C_{11} Q_{11}, \tag{6}$$

where we have introduced a counterpart of the standard dipole operator in Eq. (4):

$$Q_G = \frac{g_s}{8\pi^2} \bar{d}[(i\gamma \cdot D - m_d)\sigma \cdot GL + \sigma \cdot GR(i\gamma \cdot D - m_s)]s + \text{h.c.} \tag{7}$$

This operator vanishes by the QCD equation of motion for perturbatively interacting quark fields. The coefficients C_G and C_{11} above, being equal at the W-scale, evolve differently down to the scale ~ 1 GeV, where hadronic matrix elements are evaluated. Therefore, in the next section we consider SD QCD corrections to the Wilson coefficient C_G, that to our knowledge are not given in the literature.

In order to keep and calculate the contributions from Q_G, one needs a framework to incorporate the effects of off-shell quarks at low energies, or equivalently, a framework where the operator Q_G cannot be rotated away. An important point of this Letter is that the operator Q_G, to leading order, has the same bosonisation as the Q_6 operator. This fact will enable us a direct comparison of the off-shell contribution coming from the operator Q_G, and the leading CP-violating and CP-conserving contributions stemming from Q_6.

The Lagrangian (6) is just a part of the more complete effective weak Lagrangian at quark level relevant to K-decays

$$\mathcal{L}_W = \mathcal{L}_{4q} + \mathcal{L}_\sigma + \mathcal{L}_G + \mathcal{L}^R_{sd}, \tag{8}$$

where the additional terms are $\mathcal{L}_{4q}$ corresponding to standard four quark operators (3) and $\mathcal{L}^R_{sd}$, the renormalised off-diagonal self-energy [1,19]

$$\mathcal{L}^R_{sd} = -A\bar{d}(i\gamma \cdot D - m_d)(i\gamma \cdot DR + M_R R + M_L L)(i\gamma \cdot D - m_s)s. \tag{9}$$

The most important part of this last term for ε'/ε, the so-called self-penguin, was critically examined in Ref. [20] and was shown to have a considerable off-shell contribution in χQM framework of Ref. [16].

In the standard SD procedure [6,17], $\mathcal{L}_G$ and $\mathcal{L}^R_{ds}$ in $\mathcal{L}_W$, Eq. (8), would be absent when applying the equation of motion at quark level. Instead, a more appropriate procedure should be to transform these terms away by a field redefinition, introducing new quark fields

$$d' = d + B\sigma \cdot GLs$$
$$- \frac{1}{2}A(i\gamma \cdot DR + M_R R + M_L L)(i\gamma \cdot D - m_s)s,$$
$$s' = s + B^*\sigma \cdot GLd$$
$$- \frac{1}{2}A^*(i\gamma \cdot DR + M_R L + M_L R)(i\gamma \cdot D - m_s)d. \tag{10}$$

Then the parts $\mathcal{L}_G$ and $\mathcal{L}^R_{ds}$ involving the covariant derivatives are apparently removed, absorbed in the Dirac Lagrangian $\mathcal{L}_f(q)$ for $q = (u, d, s)$ [1]:

$$\mathcal{L}_f(q) + \mathcal{L}_G + \mathcal{L}^R_{ds} = \mathcal{L}_f(q'), \tag{11}$$

where $\mathcal{L}_f(q')$ is given later in Eq. (24), with q replaced by q'. In a strict SD treatment, primed and unprimed quark fields are equivalent. This means that $\mathcal{L}_G$ (and similarly $\mathcal{L}^R_{ds}$) does not contribute to $s \to d$ transitions for on-shell external quarks. In Section 4 we will show how the effects of the mentioned off-shell operators reappear when low-energy strong interactions are taken into account in terms of the χQM.

3. The Wilson coefficient C_G

It is convenient to distinguish the CP-conserving and CP-violating parts of the Wilson coefficients for the $\Delta S = 1$ quark operators in (2). At some scale μ they can be written as

$$C_i(\mu) = -\frac{G_F}{\sqrt{2}}[\lambda_u z_i(\mu) - \lambda_t y_i(\mu)], \tag{12}$$

where G_F is the Fermi coupling, the functions $z_i(\mu)$ and $y_i(\mu)$ are the CP-conserving and CP-violating parts of the coefficients, respectively, and $\lambda_q = V_{qd} V_{qs}^*$ (for $q = u, t$) are the CKM factors. The numerical values of z_i and y_i are in the range of order one down to 10^{-4}, and can be found in the literature [6] for operators up to Q_{11}. In what follows we calculate the corresponding values for the operator Q_G, restricted to the truncated basis given by $Q_\pm = (Q_2 \pm Q_1)/2$ and Q_G operators. Thereby we denote by $a_\pm$ the Wilson coefficients of the 4-quark operators $Q_\pm$ with diagonal anomalous dimension matrix. In the logarithmic approximation, for $\mu \leqslant m_c$ they are

$$a_\pm(\mu^2) = \left[\frac{\alpha_s(m_c^2)}{\alpha_s(\mu^2)}\right]^{\frac{d_\pm}{b(3)}} \left[\frac{\alpha_s(m_b^2)}{\alpha_s(m_c^2)}\right]^{\frac{d_\pm}{b(4)}} \left[\frac{\alpha_s(M_W^2)}{\alpha_s(m_b^2)}\right]^{\frac{d_\pm}{b(5)}}, \quad (13)$$

where $d_+ = +2$ and $d_- = -4$ are the anomalous dimensions and $b(N_f) = 11 N_c/3 - 2 N_f/3$, where N_f is the number of active flavours. The coefficient $a_\pm$ in the equation above, will become either the function $z(\mu)$ for the CP conserving part, or the function $y(\mu)$ for the CP violating part. For these cases, the values of μ will be $\mu \simeq 1$ GeV or $\mu = m_c$, respectively.

The anomalous dimension matrix for the truncated basis of the three operators $Q_\pm$ and Q_n has, for $n = 6, 11, G$ the form:

$$\gamma = \begin{bmatrix} d_+ & 0 & X_+ \\ 0 & d_- & X_- \\ 0 & 0 & Y_n \end{bmatrix}, \quad (14)$$

where

$$X_\pm = \begin{cases} \frac{11 N_c}{18} - \frac{29}{18 N_c} \pm \frac{3}{2} & \text{for } n = 11, G, \\ \frac{1}{6} & \text{for } n = 6, \end{cases} \quad (15)$$

$$Y_n = \begin{cases} -6 \frac{N_c^2 - 1}{2 N_c} + \frac{N_f}{3} & \text{for } n = 6, \\ -2 N_c - \frac{4}{N_c} & \text{for } n = 11, \\ 0 & \text{for } n = G. \end{cases} \quad (16)$$

One should note that Q_G, being an operator that vanishes by the perturbative QCD equation of motion, has zero anomalous dimension, $Y_G = 0$, in contrast to Q_6 and Q_{11}.

For handling the leading QCD corrections, there is a suitable prescription introduced in Refs. [21,22] and applied by others [23–26]. Using this prescription, one can write the amplitude as an integral over virtual quark loop momenta. The QCD-corrected coefficients z_G and y_G can be expressed in the integral form, which for CP-conserving case reads

$$z_G = \int_{\mu^2}^{m_c^2} \frac{dp^2}{p^2} \frac{\alpha_s(p^2)}{4\pi} \left[X_+ a_+(p^2) + X_- a_-(p^2) \right]/2. \quad (17)$$

For the CP-violating case we have

$$y_G = (F_2^c - F_2^t) + \int_{m_c^2}^{M_W^2} \frac{dp^2}{p^2} \frac{\alpha_s(p^2)}{4\pi} \left[X_+ a_+(p^2) + X_- a_-(p^2) \right]/2, \quad (18)$$

where $F_2^i \equiv F_2(m_i^2/M_W^2)$ are well-known Inami–Lim functions [27]. Similarly, repeating the standard renormalisation group procedure of Refs. [6,28], gives for the Wilson coefficient of Q_G, for the CP-conserving case,

$$z_G(\mu) = \frac{X_+}{d_+} (\eta^{\hat{d}_+} - 1) a_+(m_c) + \frac{X_-}{d_-} (\eta^{\hat{d}_-} - 1) a_-(m_c), \quad (19)$$

where $\eta = \alpha_s(m_c)/\alpha_s(\mu)$, and $\hat{d}_\pm \equiv d_\pm/b(3)$. For the coefficient relevant for CP violation one obtains

Table 1
Wilson coefficients of relevant operators obtained by leading order renormalisation group evolution from scale $\mu = M_W$, with flavour number thresholds at $m_b = 4.4$ GeV and $m_c = 1.3$ GeV, using $\alpha_s(M_Z) = 0.117$.

	Q_6	Q_{11}	Q_G
$z(\mu = 1$ GeV$)$	−0.009	−0.033	−0.035
$y(\mu = m_c)$	−0.083	−0.318	−0.407

$$y_G(\mu = m_c) = (F_2^c - F_2^t)$$
$$+ \frac{X_+}{d_+} (a_+(m_c) - 1) + \frac{X_-}{d_-} (a_-(m_c) - 1). \quad (20)$$

These expressions lead us to values for Wilson coefficient of Q_G displayed in Table 1. In this table we also give our values for coefficients of Q_6 and Q_{11} that conform to those given in [6].

4. Bosonisation in the chiral quark model

For light pseudoscalar mesons there is a well defined effective theory, chiral perturbation theory (χPT), having the basic symmetries of QCD. One can try to match χPT to the weak Lagrangian at quark level, Eq. (2), by *bosonizing* the quark operators Q_i:

$$Q_i \to \sum_j F_{ij} \hat{\mathcal{L}}_j, \quad (21)$$

where the $\hat{\mathcal{L}}_j$'s are chiral Lagrangian terms having the symmetry of Q_i, and F_{ij} are quantities to be calculated with non-perturbative methods (including quark models). Knowing the bosonization in (21), we could calculate the various K-decay amplitudes from a $\Delta S = 1$ chiral Lagrangian

$$\mathcal{L}_W(\chi PT) = \sum_j G_j \hat{\mathcal{L}}_j, \quad G_j = \sum_i C_i F_{ij}. \quad (22)$$

The idea of such an approach is that the coefficients should be calculated (and matched) at the border of the SD and LD regimes. Thereby the factorized form of the coefficients G_j in Eq. (22) explicates the separation of SD contributions sitting in the C_i's, and LD contributions residing in the F_{ij}'s.

In order to bosonise our relevant operators we employ the chiral quark model (χQM), that has been advocated by many authors [12–14] as an effective low-energy QCD. In this model, chiral-symmetry breaking is taken into account by adding a term to ordinary QCD:

$$\mathcal{L}_{QCD} \to \mathcal{L}_{QCD\chi} = \mathcal{L}_{QCD} + \mathcal{L}_\chi, \quad (23)$$

where $\mathcal{L}_{QCD}$ in addition to the pure gluonic part contains the fermionic part

$$\mathcal{L}_f(q) = \bar{q}(i\gamma \cdot D - \mathcal{M}_q)q, \quad q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}. \quad (24)$$

Here $\mathcal{M}_q = \text{diag}(m_u, m_d, m_s)$ is the *current* quark mass-matrix, whereas a non-perturbative term in (23),

$$\mathcal{L}_\chi = -m(\bar{q}_L \Sigma^\dagger q_R + \bar{q}_R \Sigma q_L), \quad (25)$$

contains the parameter m that is interpreted as the *constituent* quark mass (~ 200–250 MeV). Note that the constituent and current masses are tied to different terms in the Lagrangian, with different transformation properties. Here q is the $SU(3)$ flavour triplet quark field, and Σ contains the Goldstone-octet fields π^a:

$$\Sigma = \exp\left(i \sum_a \lambda^a \pi^a / f_\pi\right), \quad (26)$$

where λ^a are the Gell-Mann matrices. The term $\mathcal{L}_\chi$ contains meson-quark couplings. This means that the quarks can be integrated out and the coefficients of the various terms in the chiral Lagrangian are calculable from $\mathcal{L}_{\text{QCD}\chi}$ in Eq. (23).

Our procedure based on Eqs. (23), (24), (25) has to be understood in the following way: At scales above the cut-off Λ_χ, the total Lagrangian is the sum of the standard $\mathcal{L}_{\text{QCD}}$ and the weak effective Lagrangian $\mathcal{L}_W$ in Eq. (8), given in terms of the physical u, d, s-fields. Then at scales below Λ_χ, the term $\mathcal{L}_\chi$ in Eq. (25) is turned on, so that the matrix elements of $\mathcal{L}_W$ between mesonic states can be calculated owing to the meson-quark couplings.

The model has a "rotated" picture, where the term $\mathcal{L}_\chi$ in (25) is transformed into a pure mass term $-m\bar{\chi}\chi$ for flavour rotated "constituent quark" fields $\chi_{L,R}$:

$$q_L \rightarrow \chi_L = \xi q_L \quad \text{and} \quad q_R \rightarrow \chi_R = \xi^\dagger q_R, \tag{27}$$

where $\xi \cdot \xi = \Sigma$. The meson–quark couplings in this rotated picture arise from the kinetic (Dirac) part of the constituent quark Lagrangian. These interactions can be described in terms of vector and axial vector fields coupled to constituent quark fields $\chi = \chi_R + \chi_L$. The sum of $\mathcal{L}_f$ in (24) and $\mathcal{L}_\chi$ in (25) are transformed into the equivalent form

$$\mathcal{L}_{\chi QM} = \bar{\chi}\big[\gamma^\mu(iD_\mu + \mathcal{V}_\mu + \gamma_5\mathcal{A}_\mu) - m\big]\chi - \bar{\chi}\tilde{M}_q\chi, \tag{28}$$

where

$$\mathcal{V}_\mu = \frac{1}{2}\big[\xi^\dagger(i\partial_\mu\xi) + \xi(i\partial_\mu\xi^\dagger)\big],$$
$$\mathcal{A}_\mu = \frac{1}{2}\big[\xi^\dagger(i\partial_\mu\xi) - \xi(i\partial_\mu\xi^\dagger)\big], \tag{29}$$

and the current quark masses are residing in

$$\tilde{M}_q = \tilde{M}_q^V + \tilde{M}_q^A\gamma_5, \quad \text{with } \tilde{M}_q^{V(A)} = \frac{1}{2}\big(\xi^\dagger\mathcal{M}_q\xi^\dagger \pm \xi\mathcal{M}_q^\dagger\xi\big). \tag{30}$$

Now, having quarks that are exposed to strong interactions, and are described by the chiral quark model at hand, we have to use the inverse of the transformation of Eq. (10) into Eq. (25). In this way the apparently removed terms are effectively reappearing as a new term in $\mathcal{L}_\chi$:

$$\mathcal{L}_\chi(q) = \mathcal{L}_\chi(q') + \Delta\mathcal{L}_\chi(q') + O(G_F^2), \tag{31}$$

where the new term $\Delta\mathcal{L}_\chi(q') \sim G_F$ introduces new vertices which compensate for those in $\mathcal{L}_G$ and $\mathcal{L}_{ds}^R$. When the fields q' are integrated out, the result for a physical amplitude at mesonic level will be the same as if $\mathcal{L}_G$ and $\mathcal{L}_{ds}^R$ were applied without the field redefinition.

In general, each term in $\mathcal{L}_W$ which contains at least one power of $(i\gamma \cdot D - m_q)$ may be removed by a transformation like (10). However, for each term which is removed from $\mathcal{L}_W$, there will be a corresponding term appearing in $\mathcal{L}_\chi$. For the dipole operator Q_G, we obtain a contribution proportional to the constituent mass m

$$\Delta\mathcal{L}_\chi(q')_G = \frac{C_G}{8\pi^2}m\big(\bar{q}'_L\lambda_-\sigma \cdot G\Sigma q'_R + \bar{q}'_R\Sigma^\dagger\sigma \cdot G\lambda_-^\dagger q'_L\big), \tag{32}$$

where $\lambda_- = (\lambda_6 - i\lambda_7)/2$ is the combination of Gell-Mann matrices which transforms an s-quark into a d-quark. Employing the flavour rotation from Eq. (27) we obtain the simple expression

$$\Delta\mathcal{L}_\chi(q')_G = \frac{C_G}{8\pi^2}m\bar{\chi}'F_{(-)}\sigma \cdot G\chi' + \text{h.c.}, \tag{33}$$

where

$$F_{(-)} = \xi\lambda_-\xi^\dagger. \tag{34}$$

The expression (33) is ideal for bosonisation in terms of quark loops, whereas the analogous term for the self-energy still contains two derivatives, and can be calculated in a different way, as done in Ref. [16].

Using (28), the strong chiral Lagrangian $O(p^2)$ can be understood as two axial currents $\mathcal{A}_\mu$ attached to a quark loop, leading to

$$\mathcal{L}_s^{(2)} \sim \text{Tr}\big[\mathcal{A}_\mu\mathcal{A}^\mu\big]. \tag{35}$$

Using the relations

$$2i\mathcal{A}_\mu = -\xi^\dagger(D^\mu\Sigma)\xi^\dagger = \xi(D_\mu\Sigma^\dagger)\xi, \tag{36}$$

one obtains the leading strong chiral Lagrangian

$$\mathcal{L}_s^{(2)} = \frac{f^2}{4}\text{Tr}\big(D^\mu\Sigma^\dagger D_\mu\Sigma\big), \tag{37}$$

where D_μ is the covariant derivative. Note that $\mathcal{A}_\mu$ is invariant under local chiral transformations [12,13], in agreement with the invariance of $\mathcal{L}_s^{(2)}$. In contrast, the vector field $\mathcal{V}_\mu$ transforms as a gauge field. Attaching in addition to two $\mathcal{A}_\mu$'s also the mass term structures in (30), we will obtain the well-known L_5 term which enters the matrix element of Q_6.

In addition to the Q_6 operator, the referent object to which we compare our new off-shell dipole penguin is the chromomagnetic dipole operator (4). It can be written in a chiral $SU(3)$ invariant form, as a first step in its bosonisation procedure:

$$Q_{11} = \frac{g_s}{8\pi^2}\big[\bar{q}_R\mathcal{M}_q\lambda_-\sigma \cdot Gq_L + \bar{q}_L\sigma \cdot G\lambda_-\mathcal{M}_q^\dagger q_R\big], \tag{38}$$

where $q = (u, d, s)$. Note that this operator transforms as $(\underline{8}_L, \underline{1}_R)$ under the chiral $SU(3)_L \times SU(3)_R$ symmetry if the current quark matrix is taken to transform as $\mathcal{M}_q \rightarrow V_R\mathcal{M}_qV_L^\dagger$, where V_R and V_L are the chiral $SU(3)$ transformation matrices.

In the next step, we write Q_{11} in the flavour rotated picture:

$$Q_{11} = \frac{g_s}{8\pi^2}\bar{\chi}\big[F_{(-)}^V + F_{(-)}^A\gamma_5\big]\sigma \cdot G\chi, \tag{39}$$

where $F_{(-)}^{V,A} = (F_{(-)}^R \pm F_{(-)}^L)/2$ are expressed in terms of

$$F_{(-)}^L = \xi^\dagger\mathcal{M}_q\lambda_-\xi^\dagger \quad \text{and} \quad F_{(-)}^R = \xi\lambda_-\mathcal{M}_q^\dagger\xi. \tag{40}$$

This operator is understood in terms of a quark loop. Let us first stress that its lowest-order contribution is $\text{Tr}(F_{(-)}^V)$, which is cancelled according to the FKW theorem [29]. To the NLO, we obtain a term corresponding to an interaction of $F_{(-)}^V$ and two axial fields attached to a quark loop, shown in Fig. 1:

$$\mathcal{L}^{(4)}(Q_{11}) \sim \text{Tr}\big[F_{(-)}^V\mathcal{A}_\mu\mathcal{A}^\mu\big]. \tag{41}$$

Note that $F_{(-)}^A$ is not contributing, because there must be an even number of γ_5's in the quark loop. Using (36) and (40), we find that $\mathcal{L}^{(4)}(Q_{11})$ can be written in the final bosonised form

$$\mathcal{L}^{(4)}(Q_{11}) = G_{\underline{8}}^{(4)}(Q_{11})\text{Tr}\big[\big(\Sigma^\dagger\mathcal{M}_q\lambda_- + \lambda_-\mathcal{M}_q^\dagger\Sigma\big)D^\mu\Sigma^\dagger D_\mu\Sigma\big]. \tag{42}$$

The coefficient $G_{\underline{8}}^{(4)}(Q_{11})$ was calculated in [18] to be $\sim \langle\frac{\alpha_s}{\pi}G^2\rangle C_{11}/(8\pi^2)$, where the two-gluon condensate is a model dependent quantity in our approach. Finally, one can deduce [18] the $K \rightarrow 2\pi$ amplitude from the chiral structure of $\mathcal{L}^{(4)}(Q_{11})$:

$$\mathcal{A}\big(K^0 \rightarrow \pi^+\pi^-; Q_{11}\big) = \frac{\sqrt{2}}{f^3}(m_s - m_d)m_\pi^2 G_{\underline{8}}^{(4)}(Q_{11}). \tag{43}$$
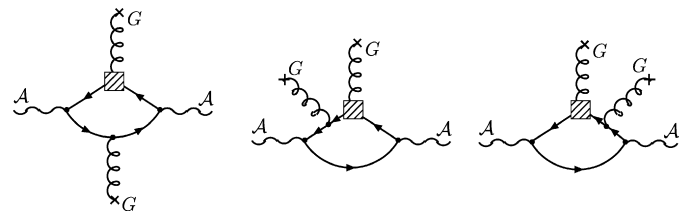

Fig. 1. Bosonisation of the operators Q_{11} and Q_G, where shaded squares represent appropriate insertions from Eqs. (39) and (33), respectively.

Even though the coefficient in $\mathcal{L}^{(4)}(Q_{11})$ is large, the Q_{11} contribution to $K \to 2\pi$ is small because of the factor m_π^2 (in place of m_K^2 obtained for Q_6). Therefore, the modest role played by Q_{11} in ε'/ε is due to this kinematical suppression rather than due to being NLO in the chiral expansion.

The bosonisation of Q_6 follows basically the same line as for Q_{11} above. The standard expression for Q_6 obtained by Fierz transformation and displayed in (3) can be rewritten in the rotated picture as

$$Q_6 = -8(F_{(-)})_{\alpha\beta}(\bar{\chi}_L)_\alpha(\chi_R)_\delta(\bar{\chi}_R)_\delta(\chi_L)_\beta, \tag{44}$$

where $F_{(-)} = \xi\lambda_-\xi^\dagger$, and Greek letters represent the flavour indices. Thus, likewise to $\mathcal{L}^{(4)}(Q_{11})$, the chiral representation of Q_6 to leading order can be written as

$$\mathcal{L}^{(2)}(Q_6) \sim \mathrm{Tr}\big[F_{(-)}\mathcal{A}_\mu\mathcal{A}^\mu\big]. \tag{45}$$

By means of Eq. (36) this can be written in the same form as other familiar $\Delta S = 1$ octet operators $O(p^2)$ [30]

$$\mathcal{L}^{(2)}(Q_6) = G_{\underline{8}}^{(2)}(Q_6)\,\mathrm{Tr}\big(\lambda_- D^\mu \Sigma^\dagger D_\mu \Sigma\big). \tag{46}$$

This term gives rise to the $K \to 2\pi$ amplitude

$$\mathcal{A}\big(K^0 \to \pi^+\pi^-; Q_6\big) = \frac{\sqrt{2}}{f^3}\big[m_K^2 - m_\pi^2\big]G_{\underline{8}}^{(2)}(Q_6). \tag{47}$$

The relevant coefficient in this expression has been calculated [4] to be

$$G_{\underline{8}}^{(2)}(Q_6) = -16C_6\frac{L_5}{f_\pi^2}\big|\langle\bar{q}q\rangle\big|^2, \tag{48}$$

where $L_5 \simeq 1.4 \times 10^{-3}$ is the coefficient of the $\mathcal{O}(p^4)$ chiral strong Lagrangian mentioned at the beginning of this section [4,7,18].

Finally, as indicated in Eq. (33), the bosonisation of the operator Q_G proceeds by inserting the expression (33) as the shaded squares in Fig. 1. The corresponding loop evaluation gives

$$G_{\underline{8}}(Q_G) = -\frac{C_G}{8\pi^2}\frac{1}{24}\Big\langle\frac{\alpha_s}{\pi}G^2\Big\rangle. \tag{49}$$

For the gluon condensate in (49) we take the value $\langle\frac{\alpha_s}{\pi}GG\rangle^{1/4} = 310$ MeV, in agreement, within uncertainties, with the lattice results and values used in Refs. [4,7,31].

Both expressions (48) and (49) are concrete examples of the general separation of SD and LD effects, contained in the factorized form of the G_j's in Eq. (22) for our procedure of bosonisation. By performing the steps above, we have all ingredients that are necessary to estimate the new contribution of our dipole penguin operator Q_G on the same footing as the previously calculated contributions.

5. Results and discussion

In the present Letter we are investigating a new dipole penguin operator that is the off-shell partner of the standard chromomagnetic penguin dipole operator Q_{11}. Namely, in addition to the chromomagnetic penguin dipole operator (4) phrased [6] as the mass insertions on external quark lines, there is, as explained in Eqs. (5) and (6), an additional dipole operator Q_G displayed in Eq. (7) corresponding to off-shell momenta for external quarks confined in hadrons. Such Lamb-shift like effects in strong interactions are in another context very recently discussed in Ref. [32].

The new dipole penguin operator Q_G studied here has several attractive features. We have shown that it has the same bosonisation as the standard Q_6 operator. Accordingly, Q_G dominates over Q_{11} which is higher order in chiral expansion. Indeed, the bosonised form $\mathcal{L}^{(4)}(Q_{11})$ results in the suppressed $K \to 2\pi$ amplitude $\mathcal{A}(K^0 \to \pi^+\pi^-; Q_{11})$ presented in Eq. (43). Thereby, the

bosonised form $\mathcal{L}^{(2)}(Q_6)$ leads to the referent $K \to 2\pi$ amplitude $\mathcal{A}(K^0 \to \pi^+\pi^-; Q_6)$ in Eq. (47), and the two operators, Q_G and Q_6, differ only in their respective coefficients, $G_{\underline{8}}(Q_G)$ in Eq. (49) and $G_{\underline{8}}^{(2)}(Q_6)$ in Eq. (48).

The ratio between Q_G and Q_6 contributions can now be read from the LD hadronic factors and the SD Wilson coefficients contained in the coefficients $G_{\underline{8}}(Q_G)$ and $G_{\underline{8}}^{(2)}(Q_6)$:

$$\rho \equiv \frac{\mathcal{A}(K^0 \to \pi^+\pi^-; Q_G)}{\mathcal{A}(K^0 \to \pi^+\pi^-; Q_6)} = \frac{C_G}{C_6}h, \tag{50}$$

where the hadronic factor h denotes the ratio of the respective LD pieces,

$$h = \frac{f_\pi^2\langle\frac{\alpha_s}{\pi}G^2\rangle}{24 \cdot 8\pi^2 \cdot 16L_5|\langle\bar{q}q\rangle|^2}. \tag{51}$$

By substituting the numerical values, including $\langle\frac{\alpha_s}{\pi}GG\rangle^{1/4} = 310$ MeV and $L_5 \simeq 1.4 \times 10^{-3}$, we obtain the hadronic factor $h \simeq 0.011$.

Finally, by employing the appropriate Wilson coefficients for the operators Q_G and Q_6 given in Table 1, we obtain for the CP-violating and the CP-conserving parts of the ratio in Eq. (50):

$$\rho_{\text{CP-violating}} = \frac{y_G}{y_6}h \simeq 0.05, \tag{52}$$

$$\rho_{\text{CP-conserving}} = \frac{z_G}{z_6}h \simeq 0.04. \tag{53}$$

This represents $\sim 5\%$ of the referent Q_6 contribution, to which it adds both in CP-violating and CP-conserving parts. In particular, there is a net coherent contribution from the CP-violating off-shell amplitudes, the one from the new dipole operator considered here, and the previously calculated off-shell self energy contribution of $\sim 15\%$ to ε'/ε in [16]. In conclusion, within the chiral quark model approach, we obtain in total an increase of $\sim 20\%$ with respect to the leading Q_6 contribution to the CP-violating ratio ε'/ε from off-shell operators. This result is still within the uncertainty of the theoretical value of Trieste-Oslo group, and although slightly higher than the world average (1), it is closer to the new preliminary KTeV result $\varepsilon'/\varepsilon = (19.2 \pm 1.1 \pm 1.8) \times 10^{-4}$ [33].

Note added

After the first submission of this Letter we noticed another Letter [32] addressing the effects of Lamb shift type in QCD. We also became aware of the new KTeV result based on doubling of the statistics and an improved control of systematics [33].

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