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Source / Izvornik: **Physical Review C, 2016, 94**

**Journal article, Published version**

**Rad u časopisu, Objavljena verzija rada (izdavačev PDF)**

<https://doi.org/10.1103/PhysRevC.94.024304>

Permanent link / Trajna poveznica: <https://um.nsk.hr/um:nbn:hr:217:730811>

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# Spin-orbit interaction in relativistic nuclear structure models

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(Received 15 April 2016; revised manuscript received 9 June 2016; published 3 August 2016)

Relativistic self-consistent mean-field (SCMF) models naturally account for the coupling of the nucleon spin to its orbital motion, whereas nonrelativistic SCMF methods necessitate a phenomenological ansatz for the effective spin-orbit potential. Recent experimental studies aim to explore the isospin properties of the effective spin-orbit interaction in nuclei. SCMF models are very useful in the interpretation of the corresponding data; however, standard relativistic mean-field and nonrelativistic Hartree-Fock models use effective spin-orbit potentials with different isovector properties, mainly because exchange contributions are not treated explicitly in the former. The impact of exchange terms on the effective spin-orbit potential in relativistic mean-field models is analyzed, and it is shown that it leads to an isovector structure similar to the one used in standard nonrelativistic Hartree-Fock models. Data on the isospin dependence of spin-orbit splittings in spherical nuclei could be used to constrain the isovector-scalar channel of relativistic mean-field models. The reproduction of the empirical kink in the isotope shifts of even Pb nuclei by relativistic effective interactions points to the occurrence of pseudospin symmetry in the single-neutron spectra in these nuclei.

DOI: [10.1103/PhysRevC.94.024304](https://doi.org/10.1103/PhysRevC.94.024304)

## I. INTRODUCTION

Self-consistent mean-field models based on relativistic energy density functionals (EDFs) with density-dependent strength parameters [1] have been successfully applied to studies of a broad variety of nuclear phenomena such as radii, masses, collective modes, fission, and shape coexistence (see, e.g., [2,3]). Remarkable results have been obtained both in the relativistic mean-field (RMF) framework [2,4] and more recently using the relativistic Hartree-Fock (RHF) scheme [5–7], even though RHF applications have mostly been restricted to spherical nuclei. One of the basic advantages of using functionals with manifest covariance is the natural inclusion of the nucleon spin degree of freedom, and the resulting nuclear spin-orbit potential which emerges automatically with the empirical strength in a covariant formulation [8].

Nonrelativistic EDFs such as, for instance, Skyrme [9] or the Gogny [10] functionals must, of course, also include a spin-orbit term. In this case, however, the strength of the phenomenological spin-orbit term has to be adjusted to data on the energy spacing between spin-orbit partner states. This approach has been extensively applied and refined over the last four decades [11–13], and it provides an effective description of spin-orbit effects in nuclei.

The omission of an explicit treatment of exchange terms in the RMF approach may have an impact on the description of the isovector channel, in particular for the energy gap between spin-orbit partner states when the ratio between neutrons and protons becomes very large. The modification of spin-orbit splittings predicted by RMF-based models differs from that obtained with nonrelativistic, e.g., Skyrme models [2,14]. Empirical constraints can be obtained by studying the changes in neutron spin-orbit splittings when the number of protons change and vice versa, as in the recent study of spectroscopic properties of <sup>35</sup>Si and <sup>37</sup>S [15]. This task is,

however, not straightforward because single-particle energies and occupation factors are not direct observables [16].

On the theoretical side, the difference between the isospin dependence of RMF and nonrelativistic spin-orbit interactions can be analyzed by performing a nonrelativistic reduction of the Dirac equation. Such a study was reported, for instance, by Sulaksono *et al.* [17] with the goal to compare in a global way the magnitude of spin-orbit terms in these two approaches. In this work we focus on the isospin dependence of the spin-orbit effect using relativistic EDFs with density-dependent strength parameters, and we evaluate the effect of explicit treatment of exchange terms in relativistic structure models.

## II. SPIN-ORBIT TERM IN RELATIVISTIC EFFECTIVE INTERACTIONS

### A. RMF case

Most SCMF models based on the relativistic mean-field approximation have been formulated using the finite-range meson-exchange representation, in which the nucleus is described as a system of Dirac nucleons coupled to meson fields through an effective Lagrangian. The isoscalar scalar  $\sigma$  meson, the isoscalar vector  $\omega$  meson, and the isovector-vector  $\rho$  meson build the minimal set of meson fields that, together with the electromagnetic field, is necessary for a description of bulk and single-particle nuclear properties. The corresponding Lagrangian density reads

$$\begin{aligned} \mathcal{L} = & \bar{\Psi}(i\gamma^\mu \partial_\mu - m)\Psi + \frac{1}{2}(\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2) \\ & - \frac{1}{2}(\frac{1}{2}\Omega^{\mu\nu}\Omega_{\mu\nu} - m_\omega^2 \omega_\mu \omega^\mu) \\ & - \frac{1}{2}(\frac{1}{2}\mathbf{R}_{\mu\nu} \cdot \mathbf{R}^{\mu\nu} - m_\rho^2 \boldsymbol{\rho}_\mu \cdot \boldsymbol{\rho}^\mu) - g_\sigma(\rho_B(\vec{r}))\bar{\Psi}\sigma\Psi \\ & - g_\omega(\rho_B(\vec{r}))\bar{\Psi}\gamma_\mu \omega^\mu \Psi - g_\rho(\rho_B(\vec{r}))\bar{\Psi}\gamma_\mu \boldsymbol{\rho}^\mu \cdot \boldsymbol{\tau}\Psi, \quad (1) \end{aligned}$$

where, for simplicity, we omit the Coulomb term which is not relevant to the present discussion.  $\Psi$  denotes the nucleon spinor,  $m$  is the nucleon bare mass, and  $m_\sigma$ ,  $m_\omega$ , and  $m_\rho$  denote the meson masses.  $\Omega^{\mu\nu} \equiv \partial^\mu \omega^\nu - \partial^\nu \omega^\mu$  and  $\mathbf{R}^{\mu\nu} \equiv \partial^\mu \rho^\nu - \partial^\nu \rho^\mu$  are the  $\omega$  and  $\rho$  meson field tensors. Boldface symbols denote vectors and tensors in isospin space. The meson-nucleon couplings are assumed to be functions of the nucleon density (time-like component of the nucleon 4-current)  $\rho_B(\vec{r})$ , and this density dependence in principle encodes all in-medium many-body correlations.

In the self-consistent RMF framework the dynamics of independent nucleons is determined by local scalar and vector self-energies. For simplicity, spherical nuclei are considered and time-reversal symmetry is assumed (pairwise occupied states with Kramers degeneracy), which ensures that the only nonvanishing components of the 4-vector fields are the time-like ones and thus there is no net contribution from nucleon currents. Because of charge conservation only the third component of the vectors in isospin space gives a nonvanishing contribution. The single-nucleon equation of motion is then the Dirac equation

$$[\vec{\alpha} \cdot \vec{p} + V + \beta(m + S)]\psi_i = E_i \psi_i, \quad (2)$$

where  $\vec{\alpha} = \gamma_0 \vec{\gamma}$ ,  $\beta = \gamma_0$ , with  $\gamma_0$  and  $\vec{\gamma}$  the Dirac matrices in Dirac representation, and  $\psi_i$  denotes the self-consistent solution for the  $i$ th Dirac state of energy  $E_i$

$$\begin{pmatrix} \phi_i \\ \chi_i \end{pmatrix} \quad (3)$$

with  $\phi_i$  and  $\chi_i$  denoting the large and small component, respectively. The scalar and time-like vector self-energies read

$$S(\vec{r}) = g_\sigma(\rho_B(\vec{r}))\sigma(\vec{r}), \quad (4)$$

$$V(\vec{r}) = g_\omega(\rho_B(\vec{r}))\omega(\vec{r}) + g_\rho(\rho_B(\vec{r}))\tau^3 \rho(\vec{r}) + \frac{dg_\sigma}{d\rho_B} \sum_i \bar{\psi}_i(\vec{r})\sigma(\vec{r})\psi_i(\vec{r}) \quad (5)$$

$$+ \frac{dg_\omega}{d\rho_B} \sum_i \bar{\psi}_i(\vec{r})\gamma_0\omega(\vec{r})\psi_i(\vec{r}) + \frac{dg_\rho}{d\rho_B} \sum_i \bar{\psi}_i(\vec{r})\gamma_0\rho(\vec{r})\tau^3\psi_i(\vec{r}). \quad (6)$$

The explicit dependence of the coupling functions on the baryon density  $\rho_B$  produces rearrangement contributions to the vector nucleon self-energy. The rearrangement terms result from the variation of the couplings with respect to the baryon density.

In applications to nuclear matter and finite nuclei, relativistic models are used in the *no-sea* approximation: the Dirac sea of states with negative energies does not contribute to the densities and currents. In the nuclear ground state,  $A$  nucleons occupy the lowest single-nucleon orbitals, determined self-consistently by the iterative solution of the Dirac equation (2). Expressing the single-nucleon energy as  $E_i = m + \varepsilon_i$ , where  $m$  is the nucleon mass, and rewriting the Dirac equation as a system of two equations for  $\phi_i$  and  $\chi_i$ , then, noticing that for bound states  $\varepsilon_i \ll m$ , the equation for the upper component

$\phi_i$  of the Dirac spinor reduces to the Schrödinger-like form [18–20]

$$\left[ \vec{p} \frac{1}{2\mathcal{M}(r)} \vec{p} + U(r) + V_{\text{so}}(r) \right] \phi_i = \varepsilon_i \phi_i \quad (7)$$

for a nucleon with effective mass

$$\mathcal{M}(r) \equiv m + \frac{1}{2}[S(r) - V(r)], \quad (8)$$

in the potential  $U(r) \equiv V(r) + S(r)$ . The resulting additional spin-orbit potential [18–20]

$$V_{\text{so}} = \frac{1}{2r\mathcal{M}^2(r)} \frac{d}{dr}(V - S)\vec{l} \cdot \vec{s} \quad (9)$$

plays a crucial role in reproducing the empirical nuclear magic numbers. The nonrelativistic limit corresponds to a  $\frac{1}{2\mathcal{M}(\vec{r})}$  expansion. In the lowest order the isoscalar density  $\rho_s(\vec{r})$  can be approximated by the nonrelativistic nucleon density  $\rho_B(\vec{r})$ . At the energy scale characteristic for nuclear binding, meson exchange ( $\sigma$ ,  $\omega$ ,  $\rho$ , ...) is just a convenient representation of the effective nuclear interaction. The exchange of heavy mesons is associated with short-distance dynamics that cannot be resolved at low energies, and therefore in each channel, meson exchange can be replaced by the corresponding local four-point (contact) interactions between nucleons. The relation between the two representations finite-range (meson exchange) and zero-range (point coupling), is straightforward in nuclear matter because of constant nucleon scalar and vector densities. The Klein-Gordon equations of the meson-exchange model with meson masses  $m_\phi$  and density-dependent couplings  $g_\phi(\rho)$  are replaced by the corresponding point-coupling interaction terms with strength parameters  $g_\phi^2/m_\phi^2$ . In finite nuclei, however, because of the radial dependence of the densities, the expansion of the meson propagator in terms of  $1/m_\phi^2$  leads to a series of gradient terms [21]. For the purpose of our discussion it suffices to consider only the lowest order, in which the self-energies  $S(\vec{r})$  [Eq. (4)] and  $V(\vec{r})$  [Eq. (6)] read

$$S(\vec{r}) = -\frac{g_\sigma^2(\rho_B(\vec{r}))}{m_\sigma^2} \rho_B(\vec{r}), \quad (10)$$

$$V(\vec{r}) = \frac{g_\omega^2}{m_\omega^2} \rho_B(\vec{r}) + \tau^3 \frac{g_\rho^2}{m_\rho^2} \rho_\tau(\vec{r}) - \frac{g_\sigma g'_\sigma}{m_\sigma^2} \rho_B^2(\vec{r}) + \frac{g_\omega g'_\omega}{m_\omega^2} \rho_B^2(\vec{r}) + \frac{g_\rho g'_\rho}{m_\rho^2} \rho_\tau^2(\vec{r}), \quad (11)$$

where  $\rho_B$  is the nucleon density and  $\rho_\tau = \rho_B^{(n)} - \rho_B^{(p)}$  is the isovector nucleon density. Introducing the notation

$$\alpha_i \equiv \frac{g_i^2}{m_i^2}, \quad (12)$$

$$\alpha'_i \equiv \frac{d\alpha_i}{d\rho_B} = \frac{2g_i g'_i}{m_i^2}, \quad (13)$$

$$\alpha''_i \equiv \frac{d^2\alpha_i}{d\rho_B^2} = 2 \frac{(g'_i)^2 + g_i g''_i}{m_i^2}, \quad (14)$$

for  $i = \{\sigma, \omega, \rho\}$ , and explicitly writing the neutron and proton contributions ( $q = \{n, p\}$ ) with  $\rho_B^{(q-q')} \equiv \rho_B^{(q)} - \rho_B^{(q')}$ , from Eqs. (9)–(11) one derives

$$V_{\text{so}}^{(q)} = \frac{\alpha_\sigma + \alpha_\omega + \alpha_\rho + 2\alpha'_\omega \rho_B + 2\alpha'_\rho \rho_B^{(q-q')} + \frac{-\alpha''_\sigma + \alpha''_\omega}{2} \rho_B^2 + \frac{\alpha''_\rho}{2} (\rho_B^{(q-q')})^2}{2r \left\{ m - \frac{1}{2} [(\alpha_\sigma + \alpha_\omega) \rho_B + \alpha_\rho \rho_B^{(q-q')} + \frac{-\alpha'_\sigma + \alpha'_\omega}{2} \rho_B^2 + \frac{\alpha'_\rho}{2} (\rho_B^{(q-q')})^2] \right\}^2} d\rho_B^{(q)} \vec{l} \cdot \vec{s} \\ + \frac{\alpha_\sigma + \alpha_\omega - \alpha_\rho + 2\alpha'_\omega \rho_B + \frac{-\alpha''_\sigma + \alpha''_\omega}{2} \rho_B^2 + \frac{\alpha''_\rho}{2} (\rho_B^{(q-q')})^2}{2r \left\{ m - \frac{1}{2} [(\alpha_\sigma + \alpha_\omega) \rho_B + \alpha_\rho \rho_B^{(q-q')} + \frac{-\alpha'_\sigma + \alpha'_\omega}{2} \rho_B^2 + \frac{\alpha'_\rho}{2} (\rho_B^{(q-q')})^2] \right\}^2} d\rho_B^{(q' \neq q)} \vec{l} \cdot \vec{s}. \quad (15)$$

This expression can be rewritten as

$$V_{\text{so}}^{(q)} = \left[ W_1 \frac{d\rho_B^{(q)}}{dr} + W_2 \frac{d\rho_B^{(q' \neq q)}}{dr} \right] \vec{l} \cdot \vec{s}, \quad (16)$$

and the relevant ratio that determines the isospin dependence of the spin-orbit potential reads

$$\frac{W_1^{(q)}}{W_2} (\alpha_\sigma, \alpha_\omega, \alpha_\rho) \equiv \frac{A^q(\alpha_\sigma, \alpha_\omega, \alpha_\rho) + B^q(\alpha_\rho, \rho_B^{(q-q')})}{A^q(\alpha_\sigma, \alpha_\omega, \alpha_\rho) - B^q(\alpha_\rho, 0)}, \quad (17)$$

with

$$A^q(\alpha_\sigma, \alpha_\omega, \alpha_\rho) \equiv \alpha_\sigma + \alpha_\omega + 2\alpha'_\omega \rho_B \\ + \frac{-\alpha''_\sigma + \alpha''_\omega}{2} \rho_B^2 + \frac{\alpha''_\rho}{2} (\rho_B^{(q-q')})^2 \quad (18)$$

and

$$B^q(\alpha_\rho, \rho_B^{(q-q')}) \equiv \alpha_\rho + 2\alpha'_\rho \rho_B^{(q-q')}. \quad (19)$$

Equation (17) shows that the ratio  $W_1/W_2$  differs from unity because of the isovector contribution (19) and is larger than unity for  $B^q > 0$ .

## B. RHF case

In the relativistic Hartree-Fock case in which exchange terms are treated explicitly, because of nonlocality it is not possible to derive a simple analytic expression for the nonrelativistic spin-orbit potential. For a direct comparison with the RMF case, one can first consider the point-coupling approximation to the meson-exchange RHF Lagrangian, and further perform a Fierz transformation to obtain a corresponding RMF Lagrangian [22]. The interacting part of the RHF Lagrangian reads

$$\mathcal{L}_{\text{int}} = -g_\sigma(\rho_B) \bar{\Psi} \sigma \Psi - g_\omega(\rho_B) \bar{\Psi} \gamma_\mu \omega^\mu \Psi \\ - g_\rho(\rho_B) \bar{\Psi} \gamma_\mu \rho^\mu \cdot \boldsymbol{\tau} \Psi. \quad (20)$$

In the lowest-order point-coupling approximation [23] the mesons fields can be expressed as

$$\sigma = -\frac{g_\sigma(\rho_B(\vec{r}))}{m_\sigma^2} \bar{\Psi} \Psi, \\ \omega^\mu = \frac{g_\omega(\rho_B(\vec{r}))}{m_\omega^2} \bar{\Psi} \gamma^\mu \Psi, \\ \rho^\mu = \frac{g_\rho(\rho_B(\vec{r}))}{m_\rho^2} \bar{\Psi} \gamma^\mu \boldsymbol{\tau} \Psi, \quad (21)$$

and for the equivalent Lagrangian in the point-coupling approximation

$$\mathcal{L}_{\text{int}}^{\text{PC}} = -\frac{1}{2} \alpha_\sigma (\bar{\Psi} \Psi) (\bar{\Psi} \Psi) - \frac{1}{2} \alpha_\omega (\bar{\Psi} \gamma_\mu \Psi) (\bar{\Psi} \gamma^\mu \Psi) \\ - \frac{1}{2} \alpha_\rho (\bar{\Psi} \gamma_\mu \boldsymbol{\tau} \Psi) \cdot (\bar{\Psi} \gamma^\mu \boldsymbol{\tau} \Psi), \quad (22)$$

one obtains the ground-state expectation value

$$\langle \mathcal{L}_{\text{int}}^{\text{PC}} \rangle = -\frac{1}{2} \alpha_\sigma \rho_s^2 - \frac{1}{2} \alpha_\omega \rho_B^2 - \frac{1}{2} \alpha_\rho \rho_\tau^2 + \frac{1}{2} \alpha_\sigma \rho_{s,\text{exch}}^2 \\ + \frac{1}{2} \alpha_\omega \rho_{v,\text{exch}}^2 + \frac{1}{2} \alpha_\rho \rho_{\tau,\text{exch}}^2. \quad (23)$$

Using the Fierz transformation, the couplings  $\tilde{\alpha}_i$  of the corresponding RMF Lagrangian are expressed in terms of those of  $\mathcal{L}_{\text{int}}^{\text{PC}}$  [22]:

$$\tilde{\alpha}_S = \frac{7}{8} \alpha_\sigma + \frac{1}{2} \alpha_\omega + \frac{3}{2} \alpha_\rho, \quad (24)$$

$$\tilde{\alpha}_V = \frac{1}{8} \alpha_\sigma + \frac{5}{4} \alpha_\omega + \frac{3}{4} \alpha_\rho, \quad (25)$$

$$\tilde{\alpha}_{tV} = \frac{1}{8} \alpha_\sigma + \frac{1}{4} \alpha_\omega + \frac{3}{4} \alpha_\rho, \quad (26)$$

$$\tilde{\alpha}_{tS} = -\frac{1}{8} \alpha_\sigma + \frac{1}{2} \alpha_\omega - \frac{1}{2} \alpha_\rho. \quad (27)$$

The resulting Fierz Lagrangian

$$\langle \mathcal{L}_{\text{int}}^{\text{Fierz}} \rangle = -\frac{1}{2} \tilde{\alpha}_S \rho_s^2 - \frac{1}{2} \tilde{\alpha}_V \rho_B^2 - \frac{1}{2} \tilde{\alpha}_{tV} \rho_\tau^2 - \frac{1}{2} \tilde{\alpha}_{tS} \rho_{tS}^2 \quad (28)$$

is then equivalent to the RMF Lagrangian of the previous section but, in addition, an isovector-scalar term appears because of the Fierz transformation. Additional terms in the pseudoscalar and pseudovector channels do not contribute to the self-consistent ground-state solution. Using this expression in Eq. (17) yields the ratio  $\frac{W_1^{(q)}}{W_2}$  for the Fierz Lagrangian

$$\frac{W_1^{(q)}}{W_2} \equiv \frac{W_1^{(q)}}{W_2} (\tilde{\alpha}_S, \tilde{\alpha}_V, \tilde{\alpha}_{tV} + \tilde{\alpha}_{tS}), \quad (29)$$

with the explicit functional dependence

$$\frac{W_1^{(q)}}{W_2} = \frac{\tilde{\alpha}_S + \tilde{\alpha}_V + \tilde{\alpha}_{tV} + \tilde{\alpha}_{tS} + 2\tilde{\alpha}'_V \rho_B + 2(\tilde{\alpha}'_{tV} + \tilde{\alpha}'_{tS}) \rho_B^{(q-q')}}{\tilde{\alpha}_S + \tilde{\alpha}_V - \tilde{\alpha}_{tV} - \tilde{\alpha}_{tS} + 2\tilde{\alpha}'_V \rho_B + \frac{-\tilde{\alpha}'_S + \tilde{\alpha}'_V}{2} \rho_B^2 + \frac{\tilde{\alpha}'_{tV} + \tilde{\alpha}'_{tS}}{2} (\rho_B^{(q-q')})^2} + \frac{\frac{-\tilde{\alpha}'_S + \tilde{\alpha}'_V}{2} \rho_B^2 + \frac{\tilde{\alpha}'_{tV} + \tilde{\alpha}'_{tS}}{2} (\rho_B^{(q-q')})^2}{\tilde{\alpha}_S + \tilde{\alpha}_V - \tilde{\alpha}_{tV} - \tilde{\alpha}_{tS} + 2\tilde{\alpha}'_V \rho_B + \frac{-\tilde{\alpha}'_S + \tilde{\alpha}'_V}{2} \rho_B^2 + \frac{\tilde{\alpha}'_{tV} + \tilde{\alpha}'_{tS}}{2} (\rho_B^{(q-q')})^2}. \quad (30)$$

The structure of Eq. (30) is similar to that of Eq. (17) but, in addition to the isovector vector, it contains also an isovector-scalar contribution but the strength parameter  $\tilde{\alpha}_{tS}$  of this channel is not independent. In the meson-exchange representation this channel corresponds to the exchange of a  $\delta$  meson. The isovector-scalar meson  $\delta$  can be, of course, explicitly included in the model Lagrangian but, as it has been often argued in the literature, it is difficult to determine its coupling strength from available data on finite nuclei. In the RMF meson-exchange model DD-ME $\delta$  developed and tested in Ref. [24], for instance, the isovector effective mass  $m_p^* - m_n^*$  derived from relativistic Brueckner theory was used to determine the coupling strength of the  $\delta$  meson and its density dependence. It was noted, however, that the explicit inclusion of the isovector-scalar meson does not improve the accuracy of calculated properties of finite nuclei such as masses and radii.

### III. RESULTS AND DISCUSSION

Conventional nonrelativistic Hartree-Fock mean-field calculations based on the Skyrme or Gogny force use a spin-orbit potential without explicit isospin dependence and with a constant strength parameter. The explicit treatment of the exchange term constrains the ratio of the resulting constants in the expression of Eq. (16) to  $W_1/W_2 = 2$  [11,14]. In some cases this choice is too restrictive, but it can be relaxed if the effective interaction is interpreted as resulting from an energy density functional in the sense of Kohn-Sham density functional theory [12,13]. In the relativistic mean-field approximation (cf. Sec. II A) a weak isospin dependence of the effective spin-orbit potential arises because of the  $\rho$ -meson contribution (in meson-exchange models) or the isovector-vector term of the interaction Lagrangian (in point-coupling models). Exchange terms are not computed explicitly and because of the way the spin-orbit potential Eq. (16) emerges in the nonrelativistic reduction of the single-nucleon Dirac equation, the ratio  $W_1/W_2$  in Eq. (17) explicitly depends on proton and neutron densities.

Figure 1 displays the radial dependence of the proton and neutron ratio  $W_1/W_2$  of parameters of the spin-orbit potential in Eq. (17), for the self-consistent ground states of  $^{16}\text{O}$ ,  $^{34}\text{Si}$ , and  $^{208}\text{Pb}$ , calculated with two of the most successful RMF effective interactions DD-ME2 [1] (meson-exchange) and DD-PC1 [23] (point-coupling). It should be noted that, in contrast to the value of the ratio  $W_1/W_2 = 2$  used in standard nonrelativistic Hartree-Fock calculations, in the RMF case the ratio  $W_1/W_2$  is close to unity. The absolute deviation from unity can be attributed to the contribution of the  $\rho$ -meson exchange, that is, the explicit contribution of

the isovector-vector channel: in the absence of the isovector degree of freedom in the interaction Lagrangian, Eq. (17) gives  $W_1 = W_2$ . The isovector contribution is, of course, also responsible for the difference between the effective proton and neutron single-particle potentials, while the radial (density) profiles depend on the shell structure of occupied orbitals in the self-consistent solution for a particular nucleus. In this respect, especially interesting is the case of  $^{34}\text{Si}$ , for which a possible central depletion of the proton density distribution has been analyzed using a variety of theoretical approaches [25,26], and experimental constraints on the strength of the two-body spin-orbit interaction have been reported [15,27]. For the effective interaction DD-ME $\delta$  that explicitly includes contributions from both  $\rho$ - and  $\delta$ -meson exchange in the direct term, the isovector channel of the spin-orbit potential is enhanced when compared to DD-ME2, although in both models the total isovector part of the spin-orbit potential is an order of magnitude weaker than the isoscalar contribution [24].

To illustrate the effect of the exchange terms in the RHF approximation on the single-nucleon spin-orbit potential, the radial dependence of the ratio  $W_1/W_2$  for protons and neutrons for the same nuclei are plotted in Fig. 2 using Eq. (30). This corresponds to using the RMF Lagrangian [Eq. (28)] obtained by performing the Fierz transformation on the interaction terms of the point-coupling RHF Lagrangian [Eq. (22)]. The effective RHF interaction is PKO2 [28] which includes the  $\sigma$ -,  $\omega$ -, and  $\rho$ -meson exchanges, but not the pion or the  $\delta$  meson. The most

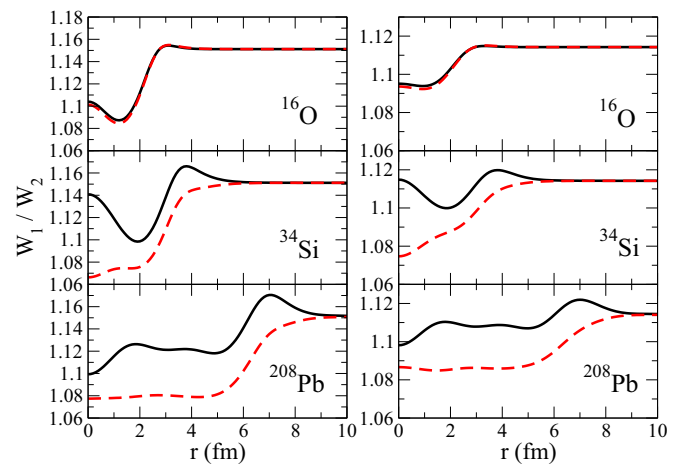


FIG. 1. Radial dependence of the proton (solid) and neutron (dashed) ratio ( $W_1/W_2$ ) of parameters of the spin-orbit potential [Eq. (17)], for the ground states of  $^{16}\text{O}$ ,  $^{34}\text{Si}$ , and  $^{208}\text{Pb}$ , calculated with the RMF effective interactions DD-ME2 (left) and DD-PC1 (right).

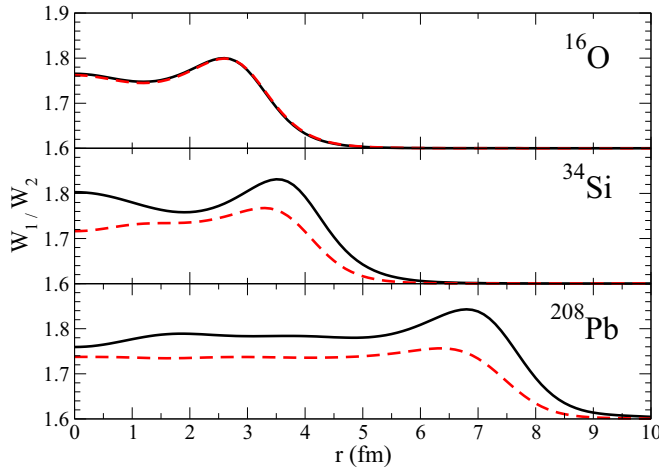


FIG. 2. Same as Fig. 1, but the ratio of the parameters is given by Eq. (30) for the case of the relativistic Hartree-Fock effective interaction PKO2.

important result is that in this case the overall value of the ratio  $W_1/W_2$  is around 1.8. This is significantly larger than in the simple RMF approach based on the Hartree approximation, and much closer to the value 2 which characterizes standard nonrelativistic HF calculations based on Skyrme forces. The difference with respect to the latter is due to the fact that there is already an isovector dependence of the effective spin-orbit potential for the Lagrangian PKO2 which arises because of the  $\rho$ -meson exchange contribution, and also due to the nonrelativistic reduction of the single-nucleon Dirac equation to the Schrödinger-like form of Eq. (7) that explicitly includes the effective spin-orbit potential.

An effect that has been attributed to the isospin dependence of the effective spin-orbit potential is the change (kink) of charge isotope shifts across the  $N = 126$  shell gap [29,30]. The charge isotope shift is the difference between the charge radius ( $r_{\text{ch}}^2$ ) of a given isotope with respect to the reference nucleus. The best known example is the kink in the isotope shifts of even Pb nuclei and, more recently, a similar effect has also been observed in polonium isotopes [30]. Numerous calculations over the last 20 years have shown that all relativistic mean-field effective interactions, both at the RMF level (without or with inclusion of the isovector-scalar  $\delta$  meson) and in the RHF approach, reproduce the empirical kink in the isotope shifts of even Pb isotopes [2,7,14,24,31]. This was explained by a relatively weak isospin dependence of the corresponding spin-orbit potentials. Conventional Skyrme HF parametrizations with  $W_1/W_2 = 2$  were unable to reproduce the kink and, therefore, in Ref. [31] the Skyrme framework was extended with an additional degree of freedom in the spin-orbit channel which allows us to modify the value of the ratio  $W_1/W_2$ . This simple modification of the Skyrme functional, in which the relative weights of the neutron and proton contributions to the spin-orbit potential can be freely adjusted, produces values for the isotope shifts of Pb in reasonable agreement with data.

In Fig. 3 we plot the experimental isotope shifts for even- $A$  Pb nuclei with respect to the reference nucleus  $^{208}\text{Pb}$ , in comparison with results obtained in the RMF calculation

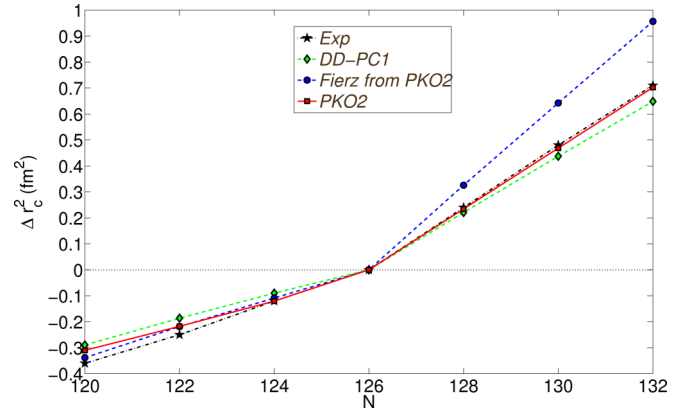


FIG. 3. Isotope shifts for even- $A$  Pb nuclei with respect to the reference nucleus  $^{208}\text{Pb}$ . Experimental values [29,30] are shown in comparison with theoretical results obtained in the RMF calculation with the effective interaction DD-PC1, using the relativistic Hartree-Fock effective interaction PKO2, and with the RMF model obtained by performing the Fierz transformation of the point-coupling approximation of PKO2.

with the effective interaction DD-PC1, using the relativistic Hartree-Fock effective interaction PKO2, and with the RMF model obtained by performing the Fierz transformation of the equivalent point-coupling approximation of PKO2. In all three cases the theoretical values reproduce the empirical kink at  $N = 126$  and, in particular, the kink is most pronounced in the RMF calculation with the Fierz-transformed effective interaction PKO2, even though  $W_1/W_2 \approx 1.8$  for this model. This result is consistent with a more recent interpretation of the change of charge isotope shifts across the  $N = 126$  shell gap [32], in which the kink is attributed to the occupation of the  $1i_{11/2}$  neutron orbital and the resulting overlap between neutron and proton orbitals with the same principal quantum number,  $n = 1$ . It was noted that effective forces for which the  $1i_{11/2}$  neutron orbital has a significant occupation above  $N = 126$  display an increase in the isotope shift of the  $n = 1$  proton states. This is because when neutrons gradually occupy the  $1i_{11/2}$  orbital, proton states expand to larger radii to maximally overlap with the additional neutrons [32]. In the present calculation, both for DD-PC1 and PKO2, the neutron orbitals  $1i_{11/2}$  and  $2g_{9/2}$  are almost degenerate above  $N = 126$ , and this leads to significant occupation of  $1i_{11/2}$  and the resulting sudden increase in the isotope shifts. In fact, the quasidegeneracy of  $2g_{9/2}$  and  $1i_{11/2}$  corresponds to an approximate realization of pseudospin symmetry of single-nucleon states with  $(n, l, j = l + 1/2)$  and  $(n - 1, l + 2, j = l + 3/2)$ . When the Fierz transformation is performed on the point-coupling approximation of PKO2, the equivalent RMF Lagrangian leads to the lowering of the orbital  $1i_{11/2}$  below  $2g_{9/2}$ . Although it cannot directly be compared to data [16], this discrepancy with the experimental spectra of  $^{209}\text{Pb}$  and  $^{211}\text{Pb}$  is probably caused by the fact that the parameters of the equivalent Lagrangian are not fine-tuned after performing the point-coupling approximation. Nevertheless, it leads to the pronounced kink shown in Fig. 3. In fact we note that the best agreement with the empirical kink is obtained with

those interactions for which pseudospin symmetry is realized in the single-neutron spectra (here DD-PC1 and PKO2, but also other relativistic interactions). If this symmetry is broken by further lowering  $1i_{11/2}$ , below  $2g_{9/2}$ , the kink in the isotope shifts becomes too strong compared to data (see Fig. 3). The presence of the kink in the isotope shifts and the relativistic models that reproduce the data thus provide evidence for the occurrence of pseudospin symmetry in neutron-rich Pb nuclei.

#### IV. CONCLUSION

In conclusion, we have analyzed the isospin dependence of the effective spin-orbit potential in standard relativistic meson-exchange or point-coupling (contact) effective interactions, when used in the mean-field (Hartree) or Hartree-Fock approximations. By performing a nonrelativistic reduction of the single-nucleon Dirac equation to a Schrödinger-like form that explicitly exhibits the spin-orbit potential, the corresponding isospin dependence can be directly compared to that of the non-relativistic Hartree-Fock models based on effective Skyrme forces. This isospin dependence can be characterized by the ratio  $W_1/W_2$  of the two parameters in the expression for the effective spin-orbit potential of Eq. (16). In conventional non-relativistic Hartree-Fock mean-field calculations based on the Skyrme force,  $W_1/W_2 = 2$ , whereas in standard RMF models this ratio is close to 1. The deviation from 1 arises because of the explicit isovector contribution to the spin-orbit potential. It should be noted that because of the medium dependence of the effective coupling parameters, either through an explicit density dependence or higher-order self-interaction terms, the ratio  $W_1/W_2$  is density dependent in the relativistic approach.

In the case of relativistic Hartree-Fock models, to evaluate the effect of exchange terms we have performed a Fierz transformation of the point-coupling RHF Lagrangian and derived an equivalent RMF Lagrangian that, in addition to the isovector-vector contribution of the original RHF Lagrangian ( $\rho$ -meson exchange), contains also an isovector-scalar term. As a result, the ratio  $W_1/W_2 \approx 1.8$  is much closer to the value that characterizes standard Skyrme Hartree-Fock models. This result is important in view of recent experimental efforts to explore the isospin dependence of spin-orbit forces in nuclei. It shows that, when comparing with results obtained using conventional Skyrme HF models, Fock terms should also be treated explicitly in relativistic mean-field models or, if one wants to preserve the advantage of local density functionals, the isovector-scalar channel has to be taken into account in addition to the usual isovector-vector contribution. Of course, this channel has been considered before in relativistic structure models; however, the standard data (masses, radii) could not be used to discern between the two isovector channels. Information on the isospin dependence of the energy spacings between spin-orbit partner states could thus be used to determine the isovector-scalar channel contribution. We have also shown that the reproduction of the empirical kink in the isotope shifts of even Pb nuclei by relativistic effective interactions points to the occurrence of pseudospin symmetry in the single-neutron spectra in these nuclei.

#### ACKNOWLEDGMENTS

The authors thank T. Nikšić and O. Sorlin for fruitful discussions. This work has been supported in part by the QuantiXLie Centre of Excellence.

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