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## Investigation of nuclear cluster phenomenology with the relativistic EDF approach

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**Summary.** — Relativistic energy density functionals (REDF) allowed to bring these last years a complementary understanding to the nuclear clustering phenomena. Three steps forwards are reported: i) localisation as a function of the nucleonic quantum numbers, indicating related cluster effects over the nuclear chart; ii) detailed comparisons with experimental data available on spectroscopy of light nuclei such as <sup>20</sup>Ne and <sup>12</sup>C and iii) investigation of pairing and quarteting effects within the REDF.

### 1. – Introduction

In recent years, nuclear clusterisation have been successfully studied using relativistic energy density functionals. A number of studies showed a relevant understanding of this phenomena, allowing to describe its occurrence as a function of the number of nucleons and the depth of the confining potentials [1, 2, 4, 3, 5]. On this purpose a localisation parameter was defined, and its Harmonic Oscillator (HO) counterpart leads to the following expression:

$$(1a) \quad \alpha_{loc} \simeq \frac{b}{r_0} = \frac{\sqrt{\hbar} A^{1/6}}{(2mV_0 r_0^2)^{1/4}}$$

where  $b$  is the dispersion of the nucleonic wave-function and  $r_0$  the typical internucleon distance. Several questions now emerge: how does the localisation parameter depend on each single-nucleonic state in a given nucleus? This will be addressed in section 2.

Another relevant question is the quantitative description of the available spectroscopic data on cluster nuclei: how well the present REDF approach do with this respect ? This is discussed in section 3. Finally, improvements in the method may also impact the results. This point addressed in section 4, where quarteting and pairing localisation effects are discussed.

## 2. – Localisation and nuclear clusterisation

The calculation of the dispersion of the wave function for each nucleon is undertaken, both with the microscopic REDF approach and the HO approximation [6]. In the spherical approximation, both approaches show that  $n=1$  nucleonic wave-functions are significantly more localised. Such a result is given on figure 1: the dispersion of the wave-function almost does not depend on the orbital angular momentum quantum number. Hence the generalisation of the localisation parameter could be analytically derived [6]:

$$(2a) \quad \alpha_{loc} \simeq \frac{b}{r_0} \sqrt{2n-1} = \frac{\sqrt{\hbar(2n-1)}}{(2mV_0r_0^2)^{1/4}} A^{1/6} .$$

The effect of the deformation is also microscopically investigated, showing that the levels originating from  $n=1$  states (in a Nilsson picture) are the more localised ones. This allows to understand that clusterisation is favored in light nuclei, not only because they have a smaller  $A$  and thus a small localisation parameter, but also because all their states behave as a  $n=1$  state. Conversely, in the case of heavy nuclei, those having a  $n=1$ -like valence state shall favor the formation of alpha clusters, such as in the  $^{212}\text{Po}$  case, and could also be linked to alpha radioactivity [6]. The present approach is therefore the first step towards a universal understanding of localisation properties at the root of to both clusterisation and alpha radioactivities.

## 3. – Comparison with the spectroscopic data

In order to validate the present REDF description of cluster phenomena, it is of critical importance to compare the corresponding predictions with the spectroscopic data of the rotational bands involving possible cluster states, such as in  $^{20}\text{Ne}$  and  $^{12}\text{C}$  cases. Such calculations based on the REDF using the DD-PC1 [7] functional, in the RHB framework are undertaken [8]. In order to compare with the data, it is necessary to perform projection and mixing of the states obtained throughout the potential energy surface maps. The states are projected on angular momentum and parity, with quadrupole+octupole degrees of freedom, in order to predict the states using the GCM approach. Such a state-of-the-art calculation requires some numerical details to be carefully taken into account [8]. The agreement with the  $^{20}\text{Ne}$  spectroscopic data (both on energies and transition probabilities) is of the same level than the one obtained by more restrictive models such as AMD. Promising results have also been obtained on the  $^{12}\text{C}$  case (with particle number projection) [9]. Its spectroscopic data is well described, and even more importantly, both the elastic and  $0^+$  towards the Hoyle state inelastic form factors are obtained with very good accuracy. It should be noted that in the present microscopic approach, no any specific parameter has been used: this is the regular REDF approach which therefore can handle both usual homogeneous quantum liquid nuclear states and cluster states.

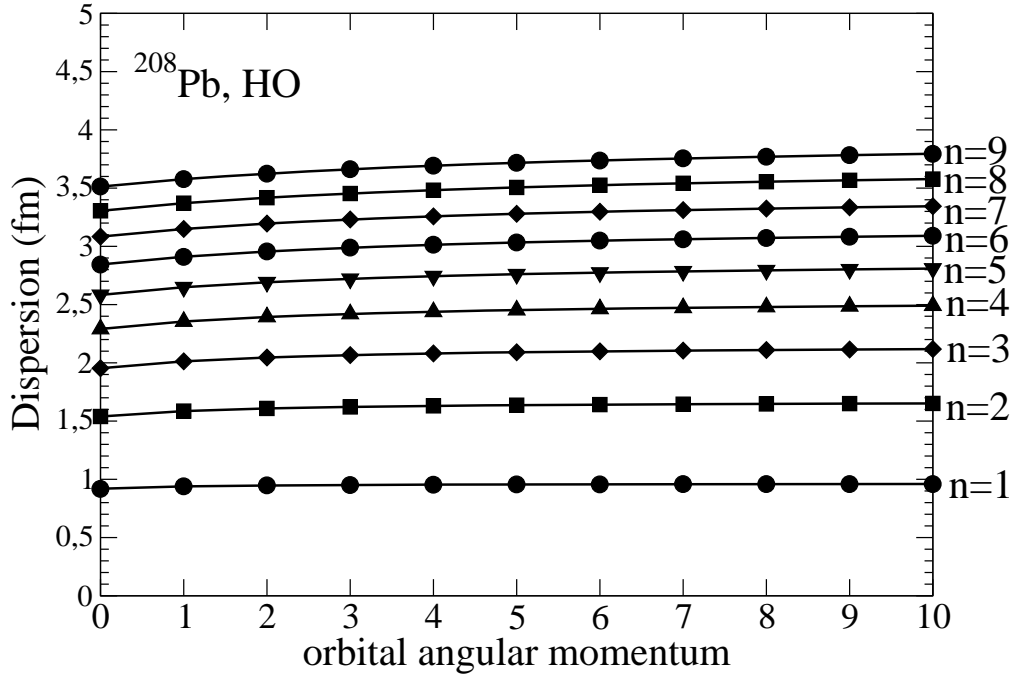


Fig. 1. – Dispersion of the single particle wave-function in  $^{208}\text{Pb}$  calculated with the 3D HO model.

#### 4. – Pairing and quarteting

It is also relevant to consider, for instance in the case of  $N=Z$  light nuclei, the effect of quarteting. This has already been studied in the non-relativistic framework [10, 11]. As a first step in the relativistic approach, we have analysed the spatial lengths related to pairing [12]. The RMF+BCS approach using the DD-ME2 [13] functional shows that similar conclusions can be reached compared to the non-relativistic framework. The effect of the projection on the particle number, on the spatial lengths related to pairing is also analysed for the first time in the relativistic framework [12]. The impact of pairing on the total density is usually acting against localised structures, due to the mixing of single-particle states triggered by the pairing effect. However, preliminary results obtained in the case of the quarteting show that localised structures of the total densities are preserved [14].

#### 5. – Conclusion

The study of nuclear clusterisation with relativistic energy density functionals is entering into a new era. A deeper understanding of the origin of clusterisation has been obtained in terms of nucleonic degree of freedom, allowing to set a bridge with cluster effects in heavy nuclei, such as in the case of alpha radioactivity. A consistent description of the low energy spectrum (energies, transition probabilities and form factors) are also obtained, paving the way for an interpretation of the data as well as predictions of relevant measurements. An inclusion of the triaxiality degree of freedom shall allow to

even increase the level of agreement with the data and to reach a full clusterised density in the case of the Hoyle state. Quarteting effects in the REDF approach and their impact on spatial localisation seem encouraging, and the calculation of the corresponding spatial lengths is on its way.

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