AN OVERVIEW OF THE INTERPLAY OF WEAK AND HADRONIC SKYRMIONS

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We review a case of cross-fertilization of two distinct areas of physics, namely, hadronic and electroweak theory. In hadronic physics, concepts like solitons, nonperturbative and nonlinear phenomena, and anomalies have found many applications in the Skyrme model of baryons. This has in turn given a large additional impetus to the studies of such concepts and phenomena in electroweak physics too. To close the circle, these studies give us clues how to represent the other kind of hadrons, namely mesons, as solitons of the Skyrme type.

1. Introduction

In the Skyrme model[1,2], baryons are pictured as topological solitons appearing in purely mesonic effective chiral theories, or, more precisely, in various variants of the nonlinear chiral sigma-model. This idea has attracted considerable interest in the past few years both in the elementary particle and nuclear physics community. Long-standing concepts about nonlinear phenomena, solitons, anomalies, etc., find a concrete realization in the Skyrme model, whereby also many fundamental issues in hadronic physics are seen from a new angle.

In context of electroweak interactions, nonperturbative phenomena may be observed when accelerator energies reach and surpass the electroweak symmetry-breaking scale. This is obvious from the relation $M_H^2 = 2\lambda \langle h \rangle^2$ between the Higgs-boson mass M_H, its coupling λ, and the Higgs vacuum expectation value $\langle h \rangle_{exp} \approx 0.2$ TeV, since M_H close to the TeV range and beyond dictates so large a coupling λ that $\lambda/4\pi$ cannot be used as a perturbative expansion parameter any more. Several authors have therefore considered[3-8] the possibility of soliton solutions to the classical equations of motion. Namely, when the Higgs-boson mass M_H in the Weinberg-Salam model tends to a »sufficiently large« value (≈ 1 TeV and beyond), the model becomes equivalent to a gauged chiral model, so in this limit there is an immediate possibility of Skyrme-type solitonic excitations of gauge and Higgs bosons.

Here we want to review one such possible interplay between Skyrme and Weinberg-Salam solitons, i. e., analogies and parallelisms between solitons in hadronic and electroweak physics.

We shall elaborate this interesting interplay on the example of the solution obtained in Ref. 8. This was the first electroweak Skyrme-type soliton with gauge couplings fully included, whereas earlier analyses did not fully take into account the effects of nonzero gauge couplings.

We recapitulate this in the second section. In the third section we show how electroweak solitons can be translated to the hadronic context, as was first done in Ref. 9. This reference is the second of the basic articles (in addition of Ref. 8) on which the bulk of this overview is based.

2. *Skyrmion solutions to the Weinberg-Salam model*

2.1. *Infinite-Higgs-mass, or strong-coupling theory*

As in Ref. 8, we assume the standard Weinberg-Salam model with a single Higgs doublet $\Phi = (\varphi_1, \varphi_2)$. The Lagrangian describing the Higgs-boson sector is

$$\mathscr{L}_\varphi = (\mathscr{D}_\mu \Phi)^+ (\mathscr{D}^\mu \Phi) + M_H^2 \Phi^+ \Phi - \lambda (\Phi^+ \Phi)^2. \tag{2.1}$$

$\mathscr{D}_\mu$ is the $\mathrm{SU}(2)_\mathrm{L} \otimes \mathrm{U}(1)$ covariant derivative, $\mathscr{D}_\mu = \partial_\mu - (ig/2) A_\mu - (ig'/2) B_\mu$, $A_\mu = \vec{A}_\mu \cdot \vec{\tau}$. For convenience, we define the matrix

$$M = \begin{bmatrix} \varphi_2^* & \varphi_1 \\ -\varphi_1^* & \varphi_2 \end{bmatrix}, \tag{2.2}$$

and then perform the polar decomposition $M = hU$, where $U \in \mathrm{SU}(2)$ represents the unphysical Goldstone bosons, while $h \equiv (\Phi^+\Phi)^{1/2}$ is real and represents the physical Higgs. We make two assumptions: (i) $g' = 0$ and (ii) h is »frozen« in its vacuum expectation value $\langle h \rangle^2 = M_H^2/2\lambda > 0$. Assumption (i) is equivalent to assuming $\sin^2 \Theta_W = 0$. As Klinkhamer and Manton[4] pointed out, it is necessary for finding a spherically symmetric solution. Since $\sin^2 \Theta_W$ is small, we hope that we are doing a reasonable approximation. Assumption (i) leads to the global $\mathrm{SU}(2)_\mathrm{V}$ symmetry $U \to VUV^+$, $A_\mu \to VA_\mu V^+$, $V \in \mathrm{SU}(2)$. Assumption (ii) is equivalent to having an infinite Higgs-boson mass M_H (and infinitely strong Higgs coupling

λ) in the tree-level approximation. Assumptions (i) and (ii) and the Yang-Mills kinetic-energy term $\mathscr{L}_A$ gives us

$$\mathscr{L}_\varphi + \mathscr{L}_A = -\frac{\langle h \rangle^2}{2} \operatorname{Tr}\,[(D_\mu U)^\dagger\,(D^\mu U)] - \frac{1}{8} \operatorname{Tr} F^{\mu\nu} F_{\mu\nu}, \tag{2.3a}$$

$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu - \frac{\mathrm{i}g}{2}\,[A_\mu, A_\nu], \tag{2.3b}$$

$$D_\mu U \equiv \partial_\mu U - \frac{\mathrm{i}g}{2} A_\mu U. \tag{2.3c}$$

Equations (2.3) define the classical Lagrangian for the gauged SU $(2)_{\mathrm{L}} \otimes$ SU $(2)_{\mathrm{R}}$ chiral model (where SU $(2)_{\mathrm{L}}$ is the gauge group). Chiral models have an identically conserved current

$$K^\mu = \frac{1}{24\pi^2}\,\varepsilon^{\mu\nu\lambda\varrho} \operatorname{Tr}\,[R_\nu R_\lambda R_\varrho], \tag{2.4a}$$

where

$$R_\mu = (D_\mu U)\,U^+. \tag{2.4b}$$

In this context, K^μ is not meaningful, since it is *not* gauge invariant. Admittedly, one can define a gauge-invariant current $\widetilde{K}^\mu$,

$$\widetilde{K}^\mu = \frac{1}{24\pi^2}\,\varepsilon^{\mu\nu\lambda\varrho} \operatorname{Tr}\,[R_\nu R_\lambda R_\varrho + \frac{3}{4} g R_\nu F_{\lambda\varrho}]. \tag{2.5a}$$

However, it is not conserved:

$$\partial_\mu \widetilde{K}^\mu = -\frac{g^2}{128\pi^2}\,\varepsilon^{\mu\nu\lambda\varrho} \operatorname{Tr}\,[F_{\mu\nu} F_{\lambda\varrho}]. \tag{2.5b}$$

The charge $Q \equiv \int K^0 \mathrm{d}^3 x$ is an integer in the ungauged chiral model. To keep the energy finite, $U(\vec{x})$ must tend to a constant at spatial infinity. Space is therefore compactified to a three-sphere which is mapped by $U(\vec{x})$ to the SU (2) manifold. The integers Q are associated with classes of the homotopy group π_3 (SU (2)) $=$ $= Z$. However, when SU $(2)_{\mathrm{L}}$ is gauged, finite energy requires only the condition $D_\mu U \to 0$ at spatial infinity. We can, of course, choose $U \to 1$ at $|\vec{x}| \to \infty$, yielding an integer-valued Q, but one should not claim to have thereby a topological configuration since this procedure has no gauge-invariant meaning. Of course, the charge $\widetilde{Q} = \int \widetilde{K}^0 \mathrm{d}^3 x$ *is* gauge invariant, but it has no topological meaning. In general, $\widetilde{Q}$ is *not* an integer. Below we show that there exist static localized solutions to the equations of motion, with $\widetilde{Q}$ not an integer.

Note that the Lagrangian (2.3) is just the Lagrangian of the gauged nonlinear σ-model. Skyrme[1] was able to obtain his stable soliton solutions after the added a fourth-order stabilizing term to the standard (ungauged) nonlinear σ-model, since this term scaled differently from the usual chiral Lagrangian. We do not need the stabilizing term for this purpose since already the gauge-boson kinetic-energy term scales differently from the Higgs-boson Lagrangian, allowing for the possibility of a localized solution. (Recall that in some versions of the Skyrme model, the Skyrme term is replaced by an interaction with vector mesons, allowing for a description of baryons[10,11,2] via static solutions to such models.) Below we shall be able to obtain a finite-energy and a finite-size static soliton solution to (2.3). However, it is unstable under small perturbations of classical fields, i. e., it corresponds to a saddle point and not a minimum of energy. (Note that the term »stability« has been used quite sloppily by many authors. For some authors, classical stability refers merely to the existence of a solution! For us, classically stable solutions are those which correspond to local minima of energy.)

We can recover stability by including fourth-order terms like the Skyrme term in the effective Lagrangian. Such terms appear owing to quantum corrections to the model[12]. We will obtain solutions with the gauged Skyrme term ($\mathscr{L}_{gSky}$) added, and conjecture that they may correspond to real particles. So, we add

$$\mathscr{L}_{gSky} \equiv \frac{1}{32e^2} \operatorname{Tr} [R_\mu, R_\nu]^2 \tag{2.6}$$

to (2.3a) to obtain the complete Lagrangian

$$\mathscr{L} = \mathscr{L}_\varphi + \mathscr{L}_{gSky} + \mathscr{L}_A. \tag{2.7}$$

2.2. *Solutions for spherically symmetric ansatze*

To make the problem tractable, only spherically symmetric solutions of (2.7) were considered[8,13,9], i. e., in addition to Skyrme's »hedgehog« ansatz for the static, classical Goldstone-boson field,

$$U(x) \to U^{cl}(\vec{r}) = \cos\Theta(r) + i\vec{\tau}\cdot\hat{r}\sin\Theta(r), \tag{2.8}$$

we choose the spherically symmetric ansatz for A_μ:

$$-\frac{g}{2} A_i^{cl} = \frac{\alpha(r) - \frac{1}{2}}{r} (\hat{r} \times \vec{\tau})_i + \frac{\beta(r)}{r} \tau_i + \frac{\delta(r) - \beta(r)}{r} (\vec{\tau}\cdot\hat{r})\, \tau_i, \tag{2.9}$$

where the time component of A_μ is eliminated by the gauge choice $A_0 = 0$. The remaining $U(1)$ gauge degree of freedom in (2.9) is eliminated by the additional gauge condition $\beta(r) = 0$. The static energy functional is then

$$M_0^{WS}[U^{cl}, A_i^{cl}] \equiv \frac{8\pi \langle h \rangle}{g} \widetilde{M}_0 [U^{cl}, A_i^{cl}], \tag{2.10}$$

$$\widetilde{M}_0 \, [U^{cl}, A_i^{cl}] = \int_0^\infty d\varrho \left\{ \frac{2}{\varrho^2} \left(a^2 - \frac{1}{4} \right)^2 + a'^2 + 4\sigma^2 a^2 + \varrho^2 (\Theta' + \sigma)^2 + \right.$$

$$\left. + F \left[1 + \frac{(g/e)^2}{32\varrho^2} (F + 4\varrho^2 (\Theta' + \sigma)^2) \right] \right\}, \tag{2.11}$$

where $F \equiv 2 \left(a + \sin^2 \Theta - \frac{1}{2} \right)^2 + \frac{1}{2} \sin^2 2\Theta$ and

$$\sigma \equiv \frac{\delta}{\varrho} = -\Theta' \left[\frac{\varrho^2 + F(g/e)^2/8}{\varrho^2 + 4a^2 + F(g/e)^2/8} \right]. \tag{2.12}$$

$\varrho \equiv g \langle h \rangle r/2$ is a dimensionless variable and the prime denotes differentiation with respect to ϱ. Note that σ (or equivalently δ) appears in (2.11) with no derivatives and hence is an auxiliary variable, expressed by (2.12). Therefore, only two variables, Θ and a, are dynamical degrees of freedom and their solutions specify the spherically symmetric soliton configurations appearing in the gauged σ-model defined by (2.7). These solutions were obtained in Ref. 8, and, except for the pure Weinberg-Salam case $1/e = 0$ (i. e., $\mathscr{L} = \mathscr{L}_\varphi + \mathscr{L}_A$), independently also in Ref. 7, and especially extensively discussed in Ref. 13. Here we review just the most important points. First, what we call the pure Weinberg-Salam ($1/e = 0$, i. e., $\mathscr{L}_{gSky} = 0$) classical solution: Its energy is $E = (8\pi \langle h \rangle / g)\,(1.79)$. Using $M_W \simeq 83$ GeV, $g \simeq 0.67$, and $\langle h \rangle = \sqrt{2}\, M_W/g \approx 175$ GeV, it is $E \approx 11.6$ TeV. Its energy density is localized within the length of roughly 0.01 fm. If a solution is stable under variations of classical fields, it is justified to assume that the existence of a corresponding state in quantum theory is very probable. However, our classical solution to (2.3), i. e., the pure Weinberg-Salam solution, is not stable. It is easy to find a variation $\delta\Theta(r)$ which lowers the energy of the solution, so that we do not have a local minimum in the energy. However, in quantum theory, higher-order terms appear in the effective action[13]. An example of such a term is the gauged Skyrme term $\mathscr{L}_{gSky}$, Eq. (2.6), which reduces to the usual Skyrme term in the limit $g \to 0$. By continuous variation of the parameters g and e, one can deform the pure Weinberg-Salam solution discussed above (corresponding to the limit $1/e = 0$) to the Skyrme soliton solution ($g \to 0$) which is classically stable. In traversing a path in the g-e plane from the pure Weinberg-Salam solution to the pure Skyrme solution, we should therefore find a transition from a classically unstable solution to a stable one. Indeed, after adding the gauged Skyrme term, solving the equations of motion for $0 < (g/e)^2 \lesssim 0.39$ yields two solutions for each g/e. (For $(g/e)^2 > 0.39$ there are no solutions.) One is classically unstable and goes into the pure Weinberg-Salam classical solution as $1/e \to 0$. The other goes into a classically stable solution of the ungauged Skyrme model and is therefore most probably also classically stable. (The rigorous proof of stability is of course very difficult.) Fig. 1 shows the energies for both branches of solutions along with values of $\widetilde{Q}$ for several values of $(g/e)^2 \cdot \widetilde{Q} \to 1$ only on the lower branch as $g \to 0$, i. e., $\widetilde{Q} \to Q$ as the solution goes over into the Skyrme solution.

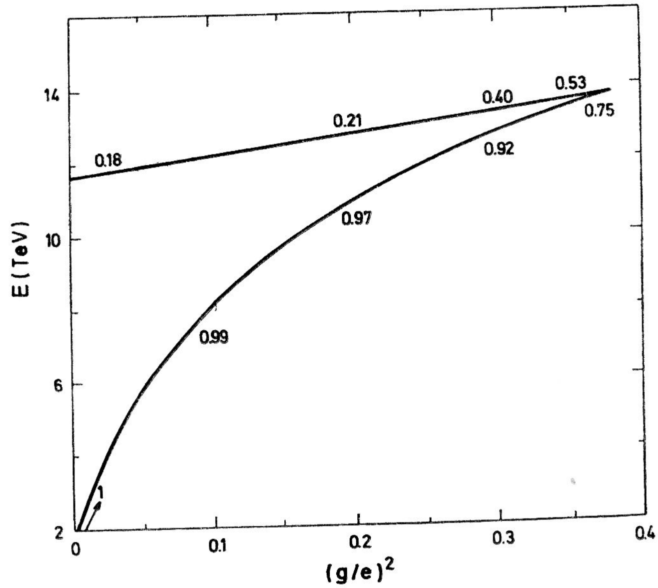

Fig. 1. The energy in teraelectronvolts of the solutions of the model given by the theory (2.7) and the ansatze (2.8), (2.9), as a function of $(g/e)^2$. The weak coupling is taken to be $g = 0.67$. $1/e$ dictates the strength of the Skyrme term. (Ref. 8)

The physical interpretation of the solutions on the upper unstable branch is not clear, but may be related to the other saddle-point solutions discovered in the Weinberg-Salam model, notably Klinkhamer's and Manton's sphaleron[4].

The presumably stable lower branch may correspond to real particles. We call them »weak skyrmions«, analogous to hadronic skyrmions, but on the mass scale of several TeV. This allows for the possibility of directly observing weak skyrmions in future accelerators, for example through W-boson fusion.

3. *Mesons as nontopological solitions of a gauged σ-model*

3.1. *Motivating, defining, and justifying the model inspired by electroweak skyrmions*

One of the appealing features of chiral solitons representing baryons in the Skyrme model[1,2] is that, in this picture, baryons are objects of finite size, as all hadrons should of course be. However, what about the finite size of the other kind of hadrons — mesons? The Skyrme model seems to be unable almost by construction to explain mesons: Baryons are extended »kinks« appearing in some effective field theories of mesons (especially pions). Mesons themselves are treated as point-like, »elementary« fields of the effective Lagrangian. Indeed, the pion is very special among mesons, and well suited for the role of a quasi-elementary chiral particle. However, for other mesons their composite nature is more manifest. Thus it would be very desirable to develop a description that would reflect their composite nature also within the Skyrme model; I will here propose a possible way how to represent at least one meson, a_1, as an extended object, namely, a *nontopological* soliton of a chiral meson Lagrangian, i. e., a solution of the baryon number zero sector of the gauged σ-model.

»Mesonlike« soliton configurations in the Skyrme model were also considered in Ref. 14. However, these were obtained in quite a different way, through a new, rather complicated ansatz, and corresponded to combinations of two-baryon — two-antibaryon configurations. These solitons consequently have very large masses (the lowest one is 4234 MeV) and their connection with physical mesons is not clear.

The approach I explain below was first proposed in my Ref. 9 and was inspired by the work on nontopological solitons in electroweak gauge theory[8,13]. Here I review a slightly shortened treatment of Ref. 9.

Various authors[3–8] conjectured about the existence of solitonic excitations of gauge and Higgs bosons at energies around and beyond the electroweak symmetry-breaking scale. Such considerations in electroweak theory were further stimulated and inspired by the soliton picture of baryons In fact, the Weinberg-Salam model becomes equivalent to a gauged chiral model when the Higgs-boson mass M_H becomes sufficiently large The possibility[5,6] of solutions to electroweak theory which are analogous to those for the skyrmion is therefore immediate. Their researchers[6,15], however, did not pay sufficient attention to the effects of gauge couplings.

In the Skyrme model, the topological index is known to be equal to the baryon number. For analogous solutions in electroweak theory, similar identifications[5] were made — but erroneously, since for a nonvanishing gauge coupling it is impossible to define a gauge-invariant topological index. Indeed, Refs. 7 and 8 obtained a soliton solution to the Weinberg-Salam model that is nontopological in nature precisely because of the presence of massless gauge fields, as discussed in the preceding section. Furthermore, Ref. 13 showed that these electroweak skyrmions should be quantized as bosons.

This finally gives us the complete picture of this fascinating interplay: Hadronic skyrmions inspired electroweak skyrmions, and they in two suggested us how we can try to picture certain mesons with the Skyrme model on an essential equal footing with baryons, namely, as *nontopological* solitons, which are the extended configurations of background effective meson fields of spin 0, i. e., pions. (Note that the pion field $\vec{\pi} = F_\pi \hat{r} \Theta(r)$ out of which the Skyrme baryon is made is not the *physical* pion. The physical, dynamical particle can be pictured as a time-dependent fluctuation on top of the soliton solution for $\vec{\pi} = F_\pi \hat{r} \Theta(r)$, which in turn, by its nontrivial configuration, serves to represent baryons.)

3.2. *Defining the model*

As in Section 2, we consider the Lagrangian of the gauged and Skyrme-stabilized σ-model,

$$\mathscr{L} = \mathscr{L}_\sigma + \mathscr{L}_{g\,Sky} + \mathscr{L}_A, \tag{3.1}$$

where

$$\mathscr{L}_\sigma = -\frac{F_\pi^2}{16} \mathrm{Tr}\, R_\mu R^\mu . \tag{3.2}$$

R_μ is defined by (2.4b) as in the preceding section, but U is now connected to the pion field π:

$$R_\mu = (D_\mu U)\, U^+, \qquad U = e^{i\vec{\pi}\cdot\vec{\tau}/F_\pi}. \tag{3.3}$$

Similarly, $\mathscr{L}_A$, $\mathscr{L}_{gSky}$ and $F_{\mu\nu}$ and $D_\mu U$ in Eqs. (3.1)—(3.3) are defined by (2.3a), (2.6), (2.3b) and (2.3c), respectively, except that A^a_μ are now (auxiliary) vector *mesons* and that their coupling constant g is now simply a free parameter, whereas in Section 2, g was the electroweak coupling in the Weinberg-Salam model. Moreover, $\mathscr{L}_\sigma$ in (3.2) is identical to $\mathscr{L}_\varphi$ after assumptions (i) and (ii), i. e., to $\mathscr{L}_\varphi$ in (2.3a), except for the overall energy scale in front ($-\langle h\rangle^2/2 \to -F_\pi^2/16$) which is now given by the pion decay constant F_π instead of the Higgs vacuum expectation value $\langle h\rangle$. Thus, modulo different energy scales and different g, (3.1) defines the same model as (2.3). This means that, just as electroweak skyrmions were obtained by translating the background pion into the Higgs field, we can translate it back, whereby weak gauge bosons A^a_μ are translated into vector mesons. Note that there is no contradiction in introducing this A^a_μ as a truly massless gauge particle since A^a_μ is also an auxiliary background meson field (just like the hedgehog pion $\vec{\pi} = F_\pi \hat{r}\Theta(r)$), while the physical and, of course, massive-vector mesons should be obtained as rotational excitations through the standard semiclassical quantization introduced in this context by Adkins, Nappi and Witten[16]. In this context, the quantization must be applied to the »Skyrme meson«, i. e., to the classical, static spherically symmetric configuration of the pion π^a and the gauge-meson A^a_μ, just as the nucleon and Δ are $J = 1/2$ and $3/2$ excitations of the Skyrme baryon. Note that a problem appears here, since mesons must be quantized as bosons, with an integer spin. This means that this treatment is not so firmly rooted in QCD as the treatment of baryons, but it is more ad hoc. Namely, to get baryons in the hadronic Skyrme model, one goes to a three-flavoured chiral model and includes the Wess-Zumino term which contains N_c, the number of colours in QCD. The odd $N^c = 3$ uniquely fixes[17,2] the quantization scheme to be fermionic. Thus, to quantize our nontopological skyrmions as bosons, we have to give up this beautiful and deep connection with QCD. In order to model mesons as solitons, we must settle for less and take the SU (2) chiral model as a completely phenomenological model whose stable nontopological solitons we quantize as bosons ad hoc, at least at this point. However, developments which may reestablish an a posteriori justification from a more fundamental level are possible, for examples, along the lines recently discussed by Kaplan[18]. In the first place, his model connects QCD and a nonrelativistic constituent quark model, but it also amounts in a way to a synthesis of the constituent-quark model and the Skyrme model at a deeper level. In Kaplan's view, a constituent, dressed quark Q is a nontrivial configuration of the light-quark condensate $\langle\bar{q}q\rangle$, caused by the »seed« light bare quark q. (Q carries all discrete quantum numbers of q.) In this way, not baryons but constituent quarks are topological solitons. Light flavour mesons, $\bar{\mathrm{Q}}\mathrm{Q}$ objects, would then be bound soliton-antisoliton configurations, with topological index zero. If such a configuration indeed gets constructed and solved, it is very likely that it will look similar to meson skyrmions described here.

On the other hand, this issue of the foundation of the Skyrme model in QCD provides an additional, albeit somewhat negativistic motivation for such an ad hoc treatment of mesons for which no justifications are given from the level of QCD as the fundamental theory: — In addition to arousing enthusiasm, the Skyrme model has also been subject to much criticism. A relatively recent and quite comprehensive critique has been given by Ball[19]. He concludes that there are no stable solitons in the meson (and glueball) sector of QCD, so that baryons cannot be skyrmions. If this is correct, i. e., if, contrary to the analysis of Witten and others[17], baryon skyrmions do not have a basis in QCD after all, one is obliged to understand how skyrmions can have so many successes. The study of meson skyrmions, proposed in this paper, for which no fundamental justification from the level of QCD itself has been attempted at all, can then help to clarify whether also the successes of the Skyrme baryons can be explained simply by having the correct symmetries and the correct energy scale built into the model.

Note that there is no contradiction of this model with the work done in those variants of the Skyrme model of baryons which include vector mesons in the Lagrangian[2]. In some of them the ϱ, ω and a_1 meson fields were also included via the gauge principle, but in such treatments the fields were gauge bosons of a *broken* gauge theory, i. e., they had Lagrangian masses from the very start. Evidently such massive fields must vanish at infinity and cannot play any role in the boundary condition at $|\vec{r}| \to \infty$ so it remains the standard, strong one: $U \to$ const as $|\vec{r}| \to \infty$, compactifying the domain of $U(r)$ into S^3 and causing $U(\vec{r})$ to be a topologically nontrivial map. In contrast to that, our A^a_μ is massless, so that in the chiral model gauged with it, the finite energy implies a condition on the covariant derivative of the chiral field: $D_\mu U \to 0$ as $|\vec{r}| \to \infty$. This is precisely the reason for the non topological nature of the gauged solitons, as explained in Section 2 for the electroweak skyrmion.

3.2. *Simplifying ansätze, semiclassical quantization, and results for the a_1 meson*

The same simplifying ansätze are used here as in Refs. 8 and 13, and in Section 2 of this work. In addition to the spherically symmetric »hedgehog« for a static, classical pion field, $\vec{\pi} = F_\pi \hat{r} \Theta(\vec{r})$, causing $U^{cl}(\vec{r})$ to be again given by (2.8), we once more choose the spherically symmetric ansatz (2.9) for A_μ and gauge choices $A_0 = 0$ and $\beta(r) = 0$. The static energy functional therefore differs from the one in Section 2 just by a scale prefactor,

$$M_0 = [U^{cl}, A_i^{cl}] \equiv \frac{\sqrt{8}\,\pi F_\pi}{g} \widetilde{M}_0 [U^{cl}, A_i^{cl}], \tag{3.4}$$

where, as before, dimensionless quantity $\widetilde{M}_0 [U^{cl}, A_i^{cl}]$ is defined by (2.11), σ by (2.12) and $F \equiv 2\left(\alpha+\sin^2\Theta-\frac{1}{2}\right)^2 + \frac{1}{2}\sin^2 2\Theta$. We stress again that unlike in

Section 2 where g was the experimental weak coupling (so that the energy depended only on one parameter, the ratio g/e), in this section the coupling g is in principle also a free parameter. (Below we shall see that physical considerations in fact constrain it somewhat.) The only other differences are that the overall energy scale is now determined by the pion decay constant F_π and that the dimensionless variable is accordingly defined as $\varrho = rgF_\pi/2\sqrt{8}$. Thus, the dimensionless equations of motion and solutions for $\Theta(\varrho)$ and $\alpha(\varrho)$ are completely identical as the ones obtained in the electroweak case of Refs. 8 and 13 and discussed in Section 2. There are no solutions for $(g/e)^2 \gtrsim 0.39$, whereas below this value we have two branches of solutions, as shown in Fig. 1. Again, only the branch lower in energy belongs to the presumably stable soliton solutions, whereas the upper branch corresponds to the classically unstable solutions. In this paper we consider only the stable solutions, although the unstable ones might still turn out to be useful, for example, for describing so quickly decaying objects as meson resonances.

The meson we try to describe by a soliton is a_1 (1260). The reason is that the hedgehog ansatz (2.8), i. e., our classical skyrmion with mass M_0 (Eq. (3.4)) carries the spin-isospin parity assignment $K^\pi \equiv (|\vec{J} + \vec{I}|)^\pi = 0^+$, i. e., it can be viewed as a mixture of states with $J = I$ and positive parity. Therefore, upon the standard collective-coordinate quantization with $J = I = 1$, the excitation should be identified with the axial-vector meson a_1 because of its (+) parity. We are not sure if we can get a physical rho, i. e., massive vector mesons with $I = J = 1$, since it has *negative* parity. Still, there is a possibility that the solution of this parity problem is very simple, since it may be simply a matter of convention, which we can see in the following way: Since the parity is defined through[2)]

$$\hat{\pi}_{op} U(\vec{x}, t)\, \hat{\pi}_{op}^{-1} = U^+(-\vec{x}, t)$$

(where $\hat{\pi}_{op}$ is the parity operator), we conclude that the hedgehog ansatz (2.8) is parity invariant, i. e., its parity is (+). However, since in the Lagrangian (3.1) only the combinations $R_\mu = (\partial_\mu U)\, U^+$ occur, (3.1) is invariant under the substitution $U \to \tilde{U} \equiv \mathrm{i}U$. Then (2.8) is changed like this: $U^{cl}(\vec{x}) \to \tilde{U}^{cl}(\vec{x}) = \mathrm{i}U^{cl}(\vec{x})$ and, obviously, $\mathrm{i}U^+(-\vec{x}, t) = -\tilde{U}^+(-\vec{x}, t)$. Thus, the parity is switched to (−) by a trivial multiplication by i, leaving the energy functional and the equations of motion unchanged. If this interpretation is correct, $U^{cl}(\vec{x})$ describes ϱ rather than a_1 mesons. (In this case, however, we would have to introduce an additional term to break ϱ and a_1 degeneracy.)

The physics constrains the parameters of the model more tightly than it looks at first sight. The solutions as functions of the dimensionless coordinate $\varrho = rgF_\pi/2\sqrt{8}$ and therefore their sizes depend only on the ratio g/e in the dimensionless ϱ-space. The size in the usual coordinate r-space for a given g/e is therefore inversely proportional to the coupling g. For $(g/e)^2 = 0.0145$, 0.06 and 0.386, the energy-density-weighted[13)] r. m. s. radii $\langle r \rangle_E$ are, respectively, $\langle r \rangle_E = 0.367/g$ fm, $0.75/g$ fm and $2.44/g$ fm. Since the size of the a_1 meson as a hadronic system should not differ very much from, say, the size of the proton, we can get upper and lower limits on g. Assuming that $0.3\ \mathrm{fm} \leq \langle r \rangle_E \leq 2\ \mathrm{fm}$, it follows that $g \approx 1.2$—8 around $(g/e)^2 \approx 0.386$, $g \approx 0.4$—2.5 around $(g/e)^2 \approx 0.06$, and $g \approx 0.2$—1.2 in the region around $(g/e)^2 \approx 0.0145$. Obviously, the regimes $(g/e)^2 \approx 0.386$ and $(g/e)^2 \approx 0.0145$ can hardly be compatible. Thus it is not surprising that a detailed calculation of the energy of the quantized soliton further below will further restrict the possible ranges of both g and $(g/e)^2$. Actually, a tighter lower bound on g can already be obtained from the static energy M_0 (Eq. (10)) or rather from the product $g \cdot M_0$, which depends only on g/e. Since the a_1 mass, m_{a_1}, is just M_0 plus the rotational energy after quantization, m_{a_1} cannot be smaller than M_0. The minimal value of g is thus $gM_0/(m_{a_1})_{exp}$. (See Fig. 2).

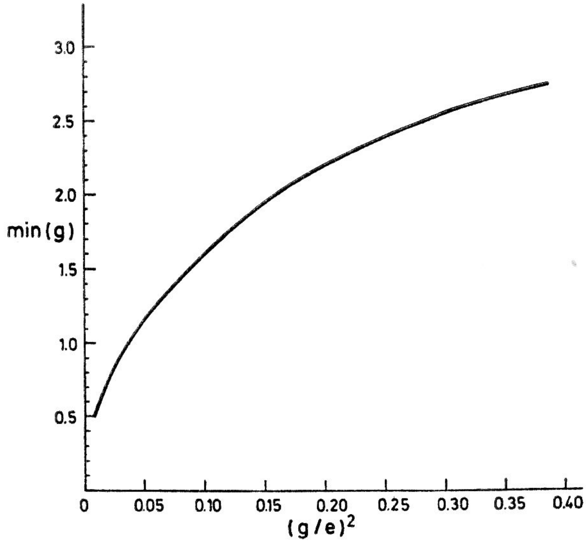

Fig. 2. min (g), i. e., the minimal allowed values of the coupling parameter g for various values of $(g/e)^2$, the squared product of the gauge coupling and the strength of the stabilizing term. (Ref. 9)

The semiclassical quantization is performed in the standard way[13,16]:

$$U^{cl}(\vec{r}) \to U(\vec{r}, t) = V(t)\, U^{cl}(\vec{r})\, V^{+}(t), \tag{3.5}$$

$$A_k^{cl}(\vec{r}) \to A_k(\vec{r}, t) = V(t)\, A_k^{cl}(\vec{r})\, V^{+}(t), \tag{3.6}$$

$$V \in \mathrm{SU}(2),$$

that is, using the time-dependent isospin rotations $V(t)$. Now we arrive again at an important issue already pointed out above: In order to choose the quantization scheme to be bosonic, we must give up the connection which the Skyrme model has with QCD. We sacrifice this connection in order to at least obtain a completely phenomenological solitonic description of mesons, and follow the bosonic quantization scheme of Ref. 13. The mass of our Skyrme meson a_1 is then

$$M_{a_1} = M_0\,[U^{cl}, A_i^{cl}] + \frac{2l\,(l+1)}{a\,[U^{cl}, A_i^{cl}]}, \tag{3.7}$$

where the spin of a_1 is $l = 1$ and

$$a\,[U^{cl}, A_i^{cl}] \equiv \frac{512\sqrt{2}}{3}\frac{1}{g^3 F_\pi}\,\tilde{a}\,[U^{cl}, A_i^{cl}], \tag{3.8}$$

$$\tilde{a}\,[U^{cl}, A_i^{cl}] = \int\limits_0^\infty \mathrm{d}\varrho\,\varrho^2 \left\{\sin^2\Theta\left[1 + \frac{1}{8}\left(\frac{g}{e}\right)^2\left(\Theta'^2 + \frac{\sin^2\Theta}{\varrho^2}\right) + \right.\right.$$

$$\left.\left. + \left(\frac{\alpha - \frac{1}{2}}{\varrho}\right)^2 + \frac{\sigma^2}{2}\right]\right\}. \tag{3.9}$$

Putting (3.4), (3.7) and (3.8) together:

$$M_{a_1} = \frac{1}{g}\,2\sqrt{2}\,\pi F_\pi\,\tilde{M}_0\,[U^{cl}, A_i^{cl}] + g^3\frac{3F_\pi}{128\sqrt{2}}\frac{1}{\tilde{a}\,[U^{cl}, A_i^{cl}]}. \tag{3.10}$$

The dependence of M_{a_1} on $(g/e)^2$ and, parametrically, on g is shown in Fig. 3 and compared with the experimental mass of the a_1 meson m_{a_1}.

The dependence of M_{a_1} is very peculiar, being able to fit m_{a_1} only for quite a restricted range of $(g/e)^2$ and g. As $(g/e)^2 \to 0$, the moment of inertia $a \to 0$ and the rotational energy diverges. The rotational energy varies quickly also with finite $(g/e)^2$. From finite but very small $(g/e)^2$ to $(g/e)^2 = 0.39$ it varies over three orders of magnitude. As (g/e) grows, the rotational energy falls very quickly with $1/a$, but then the static contribution M_0 becomes dominant, so that curves for M_{a_1} rise again if g is not too large ($g \lesssim 2$). In such cases, M_{a_1} can never be lowered to the experimental value. However, since $M_0 \sim 1/g$, M_0 will be sufficiently suppressed for sufficiently high g. At $g = 2.5$ the curve does not rise but flattens out, and for $g \approx 3$, the M_{a_1} curves start intersecting the experimental m_{a_1}. This happens at the values of $(g/e)^2$ which are already quite close to the limiting value $(g/e)^2 \approx 0.39$ beyond which there are no more solutions.

However, the rotational energy grows as g^3, so for higher g's it contributes considerably even for relatively high $(g/e)^2$. This is the reason that we cannot reach m_{a_1} for g above $g \approx 5.5$. For instance, for $g = 6$, M_{a_1} falls sharply, but cannot get as low as m_{a_1} before $(g/e)^2 \approx 0.39$, beyond which no solutions exist. Fig. 3 shows that the model can reproduce the a_1 mass for $(g/e)^2$ between 0.34 (for $g \approx 3.5$) and 0.39 (for $g \approx 5.5$ but also for $g \approx 3$). The coefficient of the stabilizing term, e, is thus constrained to be between roughly 4.8 and 9. Interestingly, this is consistent with the value $e \approx 5$ from the baryon Skyrme model. The value $e \approx 5$ indicates that compatibility with the standard baryon Skyrme model would favour the value of the coupling $g \approx 3$—3.5.

3.3. *Further possibilities*

As shown in Fig. 3, the model cannot yield masses below 1.1 GeV. Does this mean that already because of the possible masses, it is not possible to describe, for instance, vector mesons, even if the other problems — like parity assignments — are solved? Of course not! The present model has been chosen in order to exploit to the fullest what had already been learnt about electroweak skyrmions[8,13]. The dynamics can, however, be modified. For example, one can try out various other forms for the stabilization term and use the ones which permit solutions of lower energy than in the present model, making it possible to describe also mesons lighter than a_1.

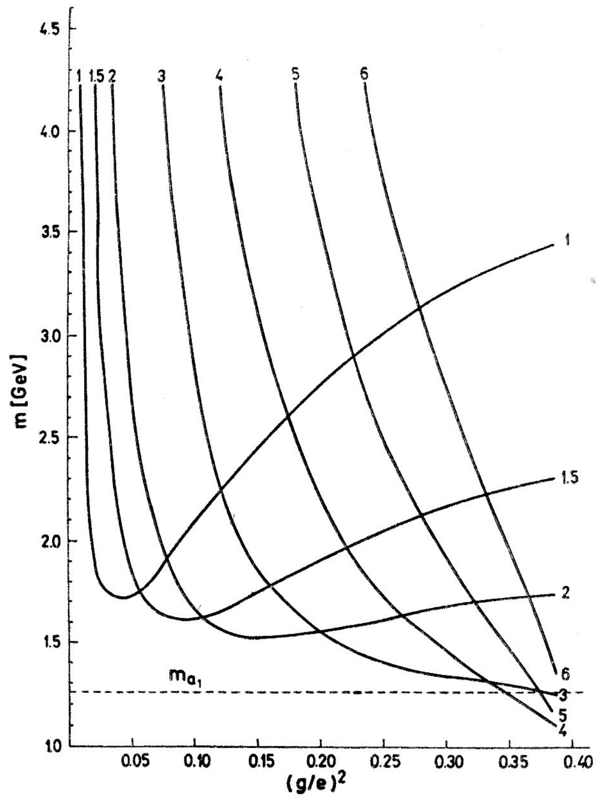

Fig. 3. The dependence of the model a_1 energy $m = M_{a1}$ on $(g/e)^2$ for various values of the coupling parameter g. The dashed line m_{a1} is the experimental mass of the a_1 meson. (Ref. 9)

This ability to encompass many different mesons will open many interesting questions. For example, while for the spin $l = 1$ we obtain isovector $(|\vec{I}| = 1)$ mesons (either ϱ or a_1, and hopefully it will be possible to incorporate both), a careful analysis should show whether the ground state with $l = 0$ can be identified in some sense with the physical η $(I = J = 0)$ meson or whether it is similar to the ordinary skyrmion in that it is not a physical state.

Incorporating more mesons will not only help to pin down their couplings g even more precisely, but, more importantly, it may also enable us to clarify its connection with the empirical meson coupling constants. On the one hand, the couplings g will have to be fine-tuned to reproduce the meson. On the other hand, if they are to be identified with the couplings of physical ϱ, a_1, ... particles, they will have to satisfy empirical successful relations, such as $g_\varrho = m_\varrho^2/f_\varrho$, i. e., the universality for the ϱ-coupling, or the KSFR relation[20].

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PREGLED MEĐUIGRE SLABIH I HADRONSKIH SKYRMIONA

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Dan je pregled jednog slučaja unakrsne fertilizacije dvaju različitih područja fizike, naime hadronske i elektroslabe teorije. U hadronskoj fizici, pojmovi kao solitoni, neperturbativne i nelinearne pojave i anomalije mnogo su primjenjivani u Skyrmeovom modelu bariona. To je pak dalo veliki dodatni podsticaj proučavanjima takvih pojmova i pojava u fizici elektroslabih procesa. Krug se zatvara time što nam ta proučavanja kažu kako prikazati drugu vrstu hadrona, naime mezone, kao solitone Skyrmeovog tipa.