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# Instantons and baryon mass splittings in the MIT bag model

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The contribution of instanton-induced effective interquark interactions to the baryon mass splittings is considered in the bag model. It is found that results are different from those obtained in the constituent quark model where the instanton effects are like those from one-gluon exchange. This is because, in the context of the bag model calculation, the one-body instanton-induced interaction has to be included.

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## I. INTRODUCTION AND MOTIVATION

QCD instantons are supposed to have interesting consequences for the structure of hadrons. Nevertheless, their role is difficult to disentangle from the other effects, and the ways of embedding the instanton-related effects in the description of the hadronic structure can still be somewhat ambiguous.

As an example, let us consider the role of instantons in the baryon mass splittings. Shuryak and Rosner [1] studied them in the constituent quark model and concluded that an effective instanton two-body interaction provides as satisfactory a description of mass splittings, in the octet and decuplet baryons, as the more conventional picture based on hyperfine interaction due to one-gluon exchange. So, in the nonrelativistic constituent quark model, instantons and one-gluon exchange are doing essentially the same thing as far as the mass spectrum is concerned. While this makes it problematic to disentangle which part in the mass shifts comes from one-gluon exchange and which from instanton effects, it also opens some attractive possibilities of refining the models of hadronic structure.

To illustrate the above statement, let us consider the MIT quark bag [2]. This relativistic model is, in some sense, complementary to the constituent quark model, and it is therefore important to see if the effects of instantons in these two models are mutually consistent. Moreover, if instanton effects in the bag model turn out to be similar to the effects of one-gluon exchange, a possibility opens for a much needed refinement of the bag model, i.e., the reduction of the strong-coupling constant to a value which would be truly consistent with perturbation theory. Namely, the bag model is a marriage of two opposite regimes. It captures the long-distance confining effects of QCD by postulating a confining boundary. On the other hand, inside this boundary, quarks are supposed to interact by perturbative QCD (supposedly saturated by one-gluon exchange), which describes the physics of small interquark separations. Thus it is, in fact, not surprising that MIT bag model fits always require a too-large strong-coupling constant $\alpha_c$, since this "perturbative" one-gluon exchange is forced to account for all nonconfining quark interactions inside the cavity (the confinement part is summarized by an impregnable boundary), where interquark separations can be as high as $2R_{\text{bag}} \simeq 2$ fm. (The situation is somewhat different in some more elaborate bag models, such as in some variants of the chiral bag model [3]. For example, in the chiral little bag model [4], the inner quark core is squeezed to $R_{\text{bag}} \simeq 0.5$ fm by a meson soliton outside, and such a configuration requires smaller $\alpha_c$ inside the bag. However, in this model the meson soliton pretty much sums up the long- and intermediate-range nonperturbative gluon effects, and in this work we want to see if, and to what extent, these effects can be described by instantons. That is why we stick with the simplest MIT bag model [2] and do not consider bag models refined by, e.g., meson solitons.)

Now, although the effective instanton-induced quark interactions are usually (at least in model calculations) considered in the local approximation, they stem from instantons which are nonperturbative structures of an average spatial scale of about $\frac{1}{600}$ MeV $\approx \frac{1}{3}$ fm. They are, therefore, of just the right scale to help capture the intermediate-range QCD effects. (It is, by now, quite certain that they are not responsible for confinement [5], as thought previously.) If they contribute to the mass shifts in the same direction as one-gluon exchange, the latter is freed from a part of its task in producing the mass shifts and can be reduced in magnitude by reducing the value of the strong-coupling constant. Such smaller coupling constants would be more appropriate for the short-distance physics and would hopefully be truly perturbative.

Having so listed the motivation for checking what happens if one tries to introduce instantons in the MIT bag model, let us now explain how we go about doing this.

## II. INCORPORATION OF THE INSTANTON-INDUCED INTERACTION IN THE MIT BAG MODEL

We shall consider the vacuum-averaged version of the instanton-induced effective interaction between quarks derived for the case of instanton liquid by Nowak *et al.* [6], but transformed to $x$ space. It is essentially the same as the Shifman-Vainshtein-Zakharov (SVZ) interaction of Ref. [7]. It is convenient to separate it in zero-body, one-body, two-body, and three-body pieces:

$$\mathcal{L}_I = \mathcal{L}_0^I + \mathcal{L}_1^I + \mathcal{L}_2^I + \mathcal{L}_3^I \ . \tag{1}$$

What we termed the "zero-body" part $\mathcal{L}_0^I$ is simply a con-

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stant, a $c$-number combination of current quark masses $m_u, m_d, m_s$, instanton density $n$, and instanton average size $\rho$; i.e., it does not contain any quark field operators. It corresponds to a constant energy density, which is the same for any state (just like the bag constant $B$), so that its matrix element for a given hadron $H$ is simply a part of the volume energy; i.e., it is of the same form as, and should contribute to, the usual volume energy $B(4\pi/3)R_H^3$. That is, the $\mathcal{L}_0^I$ contribution amounts to the renormalization of $B$. However, the bag constant is, in practice, not obtained by calculating various contributions to $B$ and then summing them all up, but it is determined as a whole by a phenomenological fit to the hadron spectrum. Thus $B$ contains all contributions, including the one due to $\mathcal{L}_0^I$, so that the explicit inclusion of $\mathcal{L}_0^I$ in the instanton-induced mass shifts would be double counting. Of course, the fitted value of $B$ will change after the relevant parts of $\mathcal{L}_I$ are included in the calculation of hadron masses, namely, those parts containing the quark field operators:

$$\mathcal{L}_1^I=-n\left[\frac{4\pi^2}{3}\rho^3\right]\{\mathcal{F}_u\bar{u}_Ru_L+(u\leftrightarrow d)+(u\leftrightarrow s)\}+(R\leftrightarrow L)\ , \tag{2}$$

$$\begin{aligned}\mathcal{L}_2^I=-n\left[\frac{4\pi^2}{3}\rho^3\right]^2&\{\mathcal{F}_u\mathcal{F}_d[(\bar{u}_Ru_L)(\bar{d}_Rd_L)+\tfrac{3}{32}(\bar{u}_R\lambda^au_L\bar{d}_R\lambda^ad_L-\tfrac{3}{4}\bar{u}_R\sigma_{\mu\nu}\lambda^au_L\bar{d}_L\sigma^{\mu\nu}\lambda^ad_L)]\\&+(u\leftrightarrow s)+(d\leftrightarrow s)\}+(R\leftrightarrow L)\ ,\end{aligned} \tag{3}$$

$$\begin{aligned}\mathcal{L}_3^I=-n\left[\frac{4\pi^2}{3}\rho^3\right]^3&\mathcal{F}_u\mathcal{F}_d\mathcal{F}_s\frac{1}{3!}\frac{1}{N_c(N_c^2-1)}\epsilon_{f_1f_2f_3}\epsilon_{g_1g_2g_3}\\&\times\left\{\left[1-\frac{3}{2(N_c+2)}\right](\bar{q}_R^{f_1}q_L^{g_1})(\bar{q}_R^{f_2}q_L^{g_2})(\bar{q}_R^{f_3}q_L^{g_3})+\frac{8}{3(N_c+3)}(\bar{q}_R^{f_1}q_L^{g_1})(\bar{q}_R^{f_2}\sigma_{\mu\nu}q_L^{g_2})(\bar{q}_R^{f_3}\sigma^{\mu\nu}q_L^{g_3})\right\}+(R\leftrightarrow L)\ .\end{aligned} \tag{4}$$

The left (and right) projected components are standard: e.g.,

$$u_{L,R}=\gamma_\pm u\equiv\tfrac{1}{2}(1\pm\gamma_5)u\ , \tag{5}$$

etc. $\mathcal{F}_f$'s are the characteristic factors (corresponding to inverse effective masses) composed of current quark masses $m_f$ ($f=u,d,s$), average instanton size $\rho$, and quark condensate $\langle 0|\bar{q}q|0\rangle=(-240\text{ MeV})^3$, e.g.,

$$\mathcal{F}_u\equiv\left[m_u\rho-\frac{2\pi^2}{3}\rho^3\langle 0|\bar{q}q|0\rangle\right]^{-1}\ , \tag{6}$$

and analogously for $d$ and $s$ flavors. In the three-body interaction $\mathcal{L}_3^I$, the indices $f_i, g_i$ ($i=1,2,3$) run over flavors $u$, $d$, and $s$. (For example, $g_2=u$ means $q_L^{g_2}\equiv u_L$.) Summation over repeated indices is understood, so that the first term of $\mathcal{L}_3^I$ (which leads in $1/N_c$ and does not contain $\sigma_{\mu\nu}$) is simply the quark determinant. The three-body interaction looks surprisingly simpler than one would expect from, e.g., Shifman-Vainshtein-Zakharov (SVZ) version [7]. This remarkable simplification has been detailed by Nowak [8], who Fierzed away otherwise very complex color structures in the three-body piece SVZ interaction [7] and lumped it in simple prefactors containing $N_c$.

However, as each piece of $\mathcal{L}_3^I$ always contains all three flavors, it would, in our calculation, contribute only for the $\Lambda$ baryon, which we therefore skip and thus avoid the need to calculate the $\mathcal{L}_3^I$ contribution to baryon mass shifts. ($\Sigma^0$ also contains all three flavors and may have a nonvanishing $\mathcal{L}_3^I$ contribution. The latter, however, must vanish in the isospin limit to yield the mass shift equal to the ones of $\Sigma^\pm$, the isospin partners of $\Sigma^0$.)

The total instanton-induced mass shift of any baryon $|B\rangle$ (except $\Lambda$ where also $\mathcal{L}_3^I$ can contribute) is, thus, just

$$E_I^B=\Delta M_B^{(1)}+\Delta M_B^{(2)}=\langle B|{:}-\mathcal{L}_1^I-\mathcal{L}_2^I{:}|B\rangle\ . \tag{7}$$

It is appropriate to give $\Delta M_B^I$ as the normal ordered interaction sandwiched between ordinary bag states $|B\rangle$ composed of valence quarks only, so that there are no vacuum contributions, because we use the vacuum-averaged instanton-induced interaction, which already includes all relevant vacuum contributions.

The instanton-induced mass shifts have already been studied by Kochelev [9] for nonstrange quark bags, but there are crucial differences between his calculation and ours. First, Kochelev did not include the one-body term, but only the two-body term $\mathcal{L}_2^I$. Dropping of the one-body term would be justified in the constituent quark model, where one uses quark masses already "dressed" by QCD, so that the self-mass part of the instanton effects is already included in the constituent quark mass parameters. However, we use the current quark masses, as appropriate in the bag model, so that the effect of $\mathcal{L}_1^I$ should be included. We find this contribution absolutely crucial, being not only larger than the $\mathcal{L}_2^I$ contribution, but also of the opposite sign.

The second difference is that we employ the MIT bag model, whereas Kochelev used his own variant of the bag, called the chiral bag model and developed in Ref. [10], so that it be in agreement with the sum rules. (However, it differs substantially from the more standard versions of the chiral bag, e.g., [3,4,11]).

Moreover, Kochelev assumed the instanton density $n$

to be equal to the density in the nonperturbative QCD vacuum between the bag radius $R$ and some $R_{\mathrm{ch}} \simeq \frac{2}{3}R$, and then falling to zero. (For example, in this respect it is like some "bag with skin.")

On the other hand, in this work we want to stick firmly with the original MIT bag model. The only modification is the inclusion of the instanton-induced interaction. It is hoped it would describe intermediate-range ($\approx \frac{1}{3}$ fm) physics anywhere quarks can go within the bag, just as one-gluon exchange should take care of the short-distance quark-quark interactions anywhere inside the impenetrable cavity. It is thus appropriate to assume a constant (though as yet undetermined) instanton density $n = n_0$ throughout the bag, just as $\alpha_c$ has the same value everywhere. Clearly, $n$ inside should be significantly smaller than the instanton density $n_c$ in the true, nonperturbative QCD vacuum—otherwise, the vacuum inside the bag would depart very much from the trivial perturbative vacuum and would start looking more and more like the nonperturbative one. Already in Ref. [12], Shuryak argued, on general grounds, that the instanton density should be substantially depleted inside a quark bag. In the present framework, this value of $n$ appropriate for the interior of the MIT bag should come out as a result of our model calculation. Thus our $n$ also looks like a step function, but it falls from the true vacuum value ($n = n_c \approx 8\times 10^{-4}$ GeV$^4$ [12]) to a smaller (but in principle nonzero) value to be determined below.

## III. INSTANTONS AND THE $p$-$n$ MASS DIFFERENCE

If we go beyond isospin symmetry and take $u$ and $d$ quark masses to be different, $m_u \neq m_d$, Eq. (7) yields a nonvanishing instanton contribution to the proton-neutron ($p$-$n$) mass difference:

$$\Delta E_I^{pn} = \tilde{n}\frac{4\pi^2}{3}\rho^{-1}[\mathcal{J}_u I_u - \mathcal{J}_d I_d] \;, \tag{8}$$

$$I_f = \int \overline{\Psi}_f \Psi_f d^3 r \;, \quad f = u,d,s \;, \tag{9}$$

where $\Psi_f$ is the usual ground-state wave function (given, e.g., in Ref. [13]) of a bagged quark of mass $m_f$. $\tilde{n}$ is the "dimensionless instanton density" obtained by expressing $n$ in units of the inverse average instanton size $\rho$, $n \equiv \tilde{n}\rho^{-4}$. We shall consistently use the commonly accepted value $\rho = \frac{1}{600}$ MeV $\simeq \frac{1}{3}$ fm [14].

The proton-neutron mass difference in the MIT bag model was studied by Chodos and Thorn [15], then Desphande *et al.* [16], and later by, e.g., Bickerstaff and Thomas [17]. In order to make a consistent comparison with, and usage of, their results [17], we give $\Delta E_I^{pn}$ for the quark masses they used, $m_u = 7.86$ MeV and $m_d = 12.14$ MeV:

$$\Delta E_I^{pn} = \tilde{n}\times 83.137 \text{ MeV} \;. \tag{10}$$

(The value that $\tilde{n}$ may take will be determined below.)

It turns out, however (see, e.g., Ref. [18]) that the contributions to the $p-n$ difference from the quark kinetic energy $E_K$, color-magnetic energy due to one-gluon exchange $E_c$, and the electromagnetic energy $E_{\mathrm{EM}}$, are essentially sufficient to fit the whole $p-n$ mass difference. For example, denoting these contributions by $\Delta E_X$ ($X = K, c, EM, I$),

$$\begin{aligned}\Delta E_{pn} &= \Delta E_K^{pn} + \Delta E_c^{pn} + \Delta E_{\mathrm{EM}}^{pn} \\ &= (-2.09 + 0.30 + 0.50) \text{ MeV} \\ &= -1.29 \text{ MeV} \;,\end{aligned} \tag{11}$$

and this is equal to the experimental value of $\Delta E_{pn}$. How, then, can the instanton contribution $\Delta E_I^{pn}$ fit in? One possibility is that phenomenology tells us, via $\Delta E_{pn}$, that the instanton density inside the bag must be very small indeed, $\tilde{n} \leq 10^{-3}$. The other, more attractive, possibility is that instanton effects may allow the reduction of the model parameter playing the role of the strong-coupling constant $\alpha_c$. Since $\Delta E_c^{pn}$ (as well as $E_c$) is proportional to $\alpha_c$, and $\Delta E_c^{pn}$ has the same sign as the instanton contribution $\Delta E_I^{pn}$, it is tempting to assume that $\Delta E_c^{pn}$ can, in fact, be reduced by reducing $\alpha_c$ to a value acceptable for perturbation theory, while the decrease in $\Delta E_c^{pn}$ would be compensated by $\Delta E_I^{pn}$. Nevertheless, the reduction of $\alpha_c$ reduces not only $\Delta E_c^{pn}$, but also the absolute magnitude of $E_c$, and this in turn may jeopardize the fit to the experimental baryon masses. We address this problem in the following sections.

## IV. INSTANTON-INDUCED MASS SHIFTS OF BARYONS

Unlike for the $p$-$n$ mass difference, the isospin breaking does not play a significant role for the mass shifts (7). We thus take $m_d = m_u$ so that $I_d = I_u$. The isosymmetric version of the mass shifts due to $\mathcal{L}_1^I$ is then

$$\Delta M_N^{(1)} = \tilde{n}\rho^{-1}4\pi^2\mathcal{J}_u I_u = \Delta M_{\Delta_{3/2}}^{(1)} \;, \tag{12}$$

$$\Delta M_\Sigma^{(1)} = \Delta M_\Lambda^{(1)} = \tilde{n}\rho^{-1}\frac{4\pi^2}{3}[2\mathcal{J}_u I_u + \mathcal{J}_s I_s] = \Delta M_{\Sigma_{3/2}^*}^{(1)} \;, \tag{13}$$

$$\Delta M_\Xi^{(1)} = \tilde{n}\rho^{-1}\frac{4\pi^2}{3}[\mathcal{J}_u I_u + 2\mathcal{J}_s I_s] = \Delta M_{\Xi_{3/2}^*}^{(1)} \;, \tag{14}$$

$$\tilde{n}\rho^{-1}4\pi^2\mathcal{J}_s I_s = \Delta M_{\Omega_{3/2}^-}^{(1)} \;. \tag{15}$$

In order to make some later comparisons with the bag model fit of DeGrand *et al.* [13], we will quote the numerical results for quark masses they used, namely, $m_u = m_d = 0$, $m_s = 280$ MeV:

$$\Delta M_N^{(1)} = \tilde{n}\times 26\,971 \text{ MeV} = \Delta M_{\Delta_{3/2}}^{(1)} \;, \tag{16}$$

$$\Delta M_\Sigma^{(1)} = \Delta M_\Lambda^{(1)} = \tilde{n}\times 23\,949 \text{ MeV} = \Delta M_{\Sigma_{3/2}^*}^{(1)} \;, \tag{17}$$

$$\Delta M_\Xi^{(1)} = \tilde{n}\times 20\,927 \text{ MeV} = \Delta M_{\Xi_{3/2}^*}^{(1)} \;, \tag{18}$$

$$\tilde{n}\times 17\,905 \text{ MeV} = \Delta M_{\Omega_{3/2}^-}^{(1)} \;. \tag{19}$$

Results for $m_u = m_d = 8$ MeV and the more standard strange mass $m_3 = 200$ MeV differ from (16)–(19) only slightly, by 2% to 5%.

The mass shifts due to the two-body term are given basically by a simple integral over the square of the sum of squared Bessel functions only for the (isosymmetric) nucleons, with $m_u = m_d$. For the $\Sigma$, $\Xi$, and $\Lambda$, the corresponding expressions are more involved because the significantly different mass of the strange quark complicates them slightly:

$$\Delta M_N^{(2)} = -9\mathcal{K}_{ud}\left[\frac{\mathcal{N}^2}{8\pi}\right]^2 \int_{V_{\rm bag}} (A_+^2 j_0^2 + A_-^2 j_1^2)^2 d^3r \ , \tag{20}$$

$$\Delta M_\Sigma^{(2)} = \Delta M_\Xi^{(2)} = -\mathcal{K}_{\rm us}\frac{\mathcal{N}^2\mathcal{N}_s^2}{(8\pi)^2}\int_{V_{\rm bag}} \{9(A_+^2 j_0^2 + A_-^2 j_1^2)(S_+^2\tilde{j}_0^2 + S_-^2\tilde{j}_1^2) - 10(A_+ j_0 S_- \tilde{j}_1 - A_- j_1 S_+ \tilde{j}_0)^2\} d^3r \ , \tag{21}$$

$$\Delta M_\Lambda^{(2)} = -6\mathcal{K}_{\rm ud}\left[\frac{\mathcal{N}^2}{8\pi}\right]^2 \int_{V_{\rm bag}} (A_+^2 j_0^2 + A_-^2 j_1^2)^2 d^3r - 3\mathcal{K}_{\rm us}\frac{\mathcal{N}^2\mathcal{N}_s^2}{(8\pi)^2}$$
$$\times \int_{V_{\rm bag}} [(A_+ j_0 S_+ \tilde{j}_0 + A_- j_1 S_- \tilde{j}_1)^2 - (A_- j_1 S_+ \tilde{j}_0 - A_+ j_0 S_- \tilde{j}_1)^2] d^3r \ . \tag{22}$$

$j_i$ and $\tilde{j}_i$ $(i=0,1)$ are the $i$th spherical Bessel functions of the arguments $x_u r/R_{\rm bag}$ and $x_s r/R_{\rm bag}$, respectively:

$$j_i \equiv j_i(x_u r/R) \ , \quad \tilde{j}_i \equiv j_i(x_s r/R) \quad (i=0,1) \ , \tag{23}$$

where $x_f$ is the bag eigenvalue of the lowest mode of the quark of the mass $m_f$ $(f=u,d,s)$. The energy of the quark of the flavor $f$ is correspondingly

$$\omega_f = \sqrt{x_f^2/R_{\rm bag}^2 + m_f^2} \ .$$

We have used the abbreviations

$$A_\pm^2 \equiv \frac{\omega_u \pm m_u}{\omega_u} \ , \quad S_\pm^2 \equiv \frac{\omega_s \pm m_s}{\omega_s} \ , \tag{24}$$

$$\mathcal{K}_{f_1 f_2} = n\left[\frac{4\pi^2}{3}\rho^3\right]^2 \mathcal{F}_{f_1}\mathcal{F}_{f_2} \ , \quad f=u,d,s \ . \tag{25}$$

Now we are in the isosymmetric limit, so that $x_u = x_d$, $\omega_u = \omega_d$, and $\mathcal{F}_u = \mathcal{F}_d$. $\mathcal{N}$ is the usual normalization for $u$ and $d$, and $\mathcal{N}_s$ for $s$ quark wave functions.

Here we quote the numerical values for $m_u = m_d = 0$, $m_s = 280$ MeV:

$$\Delta M_N^{(2)} = -\bar{n} \times 12\,468 \text{ MeV} \ , \tag{26}$$

$$\Delta M_\Sigma^{(2)} = \Delta M_\Xi^{(2)} = -\bar{n} \times 6341 \text{ MeV} \ , \tag{27}$$

$$\Delta M_\Lambda^{(2)} = -\bar{n} \times 10\,413 \text{ MeV} \ . \tag{28}$$

(For moderately different masses, e.g., $m_u = m_d = 8$ MeV and $m_s = 200$ MeV, results again differ only slightly.)

For the baryons from the decuplet, the $\mathcal{L}_2^I$ contribution vanishes, $\Delta M_{\rm decuplet}^{(2)} = 0$. The total instanton contribution $E_I^B \equiv \Delta M_B^{(1)} + \Delta M_B^{(2)}$ is thus (for $m_u = m_d = 0$, $m_s = 280$ MeV and $R = 5$ GeV$^{-1}$)

$$E_I^N = \bar{n} \times 14\,503 \text{ MeV} \ , \tag{29a}$$

$$E_I^\Sigma = \bar{n} \times 17\,608 \text{ MeV} \ , \tag{29b}$$

$$E_I^\Xi = \bar{n} \times 14\,631 \text{ MeV} \ , \tag{29c}$$

$$E_I^{\Delta_{3/2}} = \bar{n} \times 26\,971 \text{ MeV} \ , \tag{29d}$$

$$E_I^{\Sigma^*_{3/2}} = \bar{n} \times 23\,949 \text{ MeV} \ , \tag{29e}$$

$$E_I^{\Xi^*_{3/2}} = \bar{n} \times 20\,927 \text{ MeV} \ , \tag{29f}$$

$$E_I^{\Omega^-_{3/2}} = \bar{n} \times 17\,905 \text{ MeV} \ . \tag{29g}$$

($\Lambda$ is left out here, since we would also need the three-body piece to form $E_I^\Lambda$, the total instanton contribution for this baryon. The sum of one and two-body instanton contributions for $\Lambda$ is $\Delta M_\Lambda^{(1)} + \Delta M_\Lambda^{(2)} = \bar{n} \times 13536$ MeV.)

## V. ESTIMATING THE INSTANTON DENSITY $\bar{n}$ INSIDE THE BAG

Determining the value of $\bar{n}$ consistent with the MIT bag interior also means exploring the possibility of reducing the value of $\alpha_c$, which would improve the consistency of the perturbative approach inside the bag. This is how we can accommodate $\Delta E_I^{pn}$, i.e., our result for instanton contribution to the proton-neutron mass difference. The decrease of $\alpha_c$ would reduce $\Delta E_c^{pn}$ and this reduction would be compensated by $\Delta E_I^{pn}$. However, the reduction of $\alpha_c$ decreases not only $\Delta E_I^{pn}$, but also the absolute magnitude of the chromomagnetic energy $E_c$. Since $E_c$ is negative for nucleons and other octet baryons, it will not be possible to compensate, by the total instanton contribution $E_I$, the decrease in the absolute value of $E_c$ resulting from the decrease of $\alpha_c$. This is because $E_I$ is *positive* since the one-body contribution exceeds the two-body one. The nucleon-$\Delta$ splitting would also be spoiled for the same reasons.

Still, the decrease in $\alpha_c$ along the good fit to the hadron masses may be achieved anyway if, along the inclusion of $E_I$, other contributions to the bag energy are also changed. (For example, the zero-ppoint energy $-Z_0/R$ is negative, and the increase in the parameter $Z_0$ may compensate for the decrease of $\alpha_c$ and the inclusion of $E_I$.) If we start from, e.g., the bag model fit of DeGrand *et al.* [13], where the bag model parameters were chosen so that they reproduce the experimental nucleon mass $M_N$, then we demand that after the inclusion of the instanton contribution $E_I$ (that is, allowing $\tilde{n}$ inside the bag to deviate from $\tilde{n}=0$), the bag model parameters change so that the nucleon mass remains at the empirical value:

$$\delta M_N=\frac{\delta M_N}{\delta R_N}\delta R_N+\frac{\delta M_N}{\delta \alpha_c}\delta\alpha_c+\frac{\delta M_N}{\delta Z_0}\delta Z_0 +\frac{\delta M_N}{\delta B}\delta B+\frac{\delta M_N}{\delta \tilde{n}}\delta\tilde{n}=0 . \tag{30}$$

(We do not consider the possibility of varying the quark masses, because we have to adopt the values used by Ref. [13] if we want to use the results of Refs. [13] and [17], and to compare our results with theirs. Bickerstaff and Thomas [17] themselves studied the effects of small and different $u$ and $d$ masses on top of the isosymmetric fit of DeGrand *et al.* [13].)

Fortunately, it is not necessary to perform a full refitting of the bag-model parameters varied in Eq. (30), as it is a good approximation to "freeze" the bag radii because of the pressure-balance condition

$$\frac{dM_N}{dR_N}=0 . \tag{31}$$

Namely, we have seen that since the $p$-$n$ mass difference is small, there is room only for relatively small instanton effects. More precisely, even if almost the whole of $\Delta E_c^{pn}$ is supplanted by $\Delta E_I^{pn}$, the maximal $\tilde{n}$ that can be accommodated by $\Delta E_{pn}$ would be $\tilde{n}\approx 0.3\times 10^{-2}$, so that $E_I^N$ cannot be excessively large. Also, we commented that the conflicting behaviors of $\Delta E_I^{pn}$, $E_I$, and $E_c$ with $\tilde{n}$ and $\alpha_c$ cannot allow large instanton effects. Therefore, the fit of Ref. [13] cannot be altered very much, so that the pressure balance condition (31) still holds in a good approximation after the inclusion of $E_I$. Thus the variation of the radius $R_N$ cannot contribute to the energy variation equation (30) comparably to the variations of other parameters; i.e., we can neglect the term

$$\frac{\delta M}{\delta R}\delta R .$$

Only very large variations of $R$ would change the energy balance substantially. (Indeed, a change of $R$ by as much as 10% changes the mass only by order of 1%.) This is a significant simplification of Eq. (30) since the remaining parameters ($Z_0$, $B$, $\alpha_c$, $n$) enter linearly.

We therefore consider only the change of $Z_0^{\text{old}}$, $\alpha_c^{\text{old}}$, and $B_{\text{old}}$ [where "old" indicates the values of Ref. [13], i.e., 1.84, 0.55, and $(0.145\ \text{GeV})^4$, respectively] to new values $Z_0$, $\alpha_c$, and $B$ when the instanton-induced interaction is turned on, i.e., when $\tilde{n}$ is allowed to deviate from zero. Equation (30) then leads (in the case of nucleon $N$) to

$$M_N^{\text{bag}}-E_Q^N=\frac{Z_0}{Z_0^{\text{old}}}E_0^N+\frac{\alpha_c}{\alpha_c^{\text{old}}}E_M^N+\frac{B}{B_{\text{old}}}E_V+E_I , \tag{32}$$

where $E_Q^N$, $E_M^N$, $E_V^N$, and $E_0^N$ are, respectively, the kinetic, chromomagnetic, volume, and zero-point energies for the nucleon ($N$) as given in Table III of DeGrand *et al.* [13]. Reference [13] also fits the mass of the $\Omega_{3/2}^-$ bag to the experimental value, just as for the nucleon and delta bags. Since we are interested in the interplay of instanton effects and one-gluon exchange, we are especially concerned with not spoiling the nucleon-delta splitting, which is usually attributed to one-gluon exchange. We thus demand that equations analogous to Eq. (30) for the nucleon, or, equivalently, (32), also hold for $\Delta_{3/2}$ and $\Omega_{3/2}^-$. For the fourth equation necessary for determining the four unknowns (the new $Z_0$, $\alpha_c$, and $B$, but also $\tilde{n}$ sitting in $E_I$ and $\Delta E_I^{np}$) we have chosen the equation for $p-n$ mass difference in the presence of instanton-induced interaction $\mathcal{L}_I$:

$$\Delta E_{pn}=\Delta E_K^{pn}+\Delta E_{EM}^{pn}+\frac{\alpha_c}{\alpha_c^{\text{old}}}\Delta E_c+\Delta E_I^{pn} . \tag{33}$$

Why this choice? As solving Eq. (33) for the instanton density $\tilde{n}$ shows, this equation links the positivity of $\tilde{n}$ and the expected decrease of $\alpha_c$, the strength of one-gluon exchange. This decrease was anticipated on physical grounds in Sec. I, and it indeed occurs, as we will see. In addition to the discussion in Sec. III, which indicates that $p$-$n$ mass difference might hold an important message for instantons in the MIT bag, the issue of positivity of the instanton density (proportional to the number of instantons plus the number of anti-instantons) was the other reason for choosing (33) for the fourth determining equation. Namely, we could have, of course, also taken an equation analogous to the aforementioned equations for $N$, $\Delta$, or $\Omega$, but for a baryon other than those already used. In that case, however, there is nothing to act automatically against the solutions with negative instanton density, and this physical requirement would have to be imposed as an additional constraint. (Indeed, if we use $\Xi$ or $\Sigma$, it turns out that, although the results for $\alpha_c$ and the absolute magnitude of instanton effects are qualitatively rather similar as when $p$-$n$ mass difference is used, the instanton density comes out positive in the case of the $\Xi$ equation, but negative in the case of the fourth equation stemming from $\Sigma$.)

In Eq. (33), we have used the values of Ref. [17] for $\Delta E_K$, $\Delta E_{\text{EM}}$, $\Delta E_c$, and $\Delta E_{pn}$. For the quark masses used by Ref. [17] ($m_u=7.86$ MeV, $m_d=12.14$ MeV),

$$\Delta E_I^{pn}=\tilde{n}\times 83.14\ \text{MeV} . \tag{34}$$

We make the same approximation as that used by Bickerstaff and Thomas [17]; i.e., we use the energy differences $\Delta E_X^{pn}$ ($X=K,\text{EM},c,I$) computed for the small and different quark masses $m_u, m_d$ on top of the isosymmetric fit to hadron masses done in Ref. [13] with $m_u=m_d=0$, $m_s=280$ MeV.

Solving the system of four linear equations consisting of (33), (32), and analogies of (32) for $\Omega_{3/2}^-$ and for $\Delta_{3/2}$, yields $\alpha_c=0.52$, $Z_0=2.11$, $B=(0.151\ \text{GeV})^4$ and $\tilde{n}=0.205\times 10^{-3}$. This $\tilde{n}$ corresponds to $n=(71.79\ \text{MeV})^4$ for $\rho=\frac{1}{600}$ MeV and is even more depleted with respect to the $n$ in the nonperturbative QCD vacuum than we expected. (For example, it is roughly $\frac{1}{30}$ of Shuryak's estimate [12] for nonperturbative vacuum.)

For $m_u=m_d=0$ and $m_s=280$ MeV, i.e., the values used in Table II of Ref. [13], the instanton contribution to the energies for the octet baryons are

$$E_I^N = 3.0 \text{ MeV} ,$$

$$E_I^\Sigma = 3.6 \text{ MeV} ,$$

$$E_I^\Xi = 3.0 \text{ MeV}$$

(as for $\Lambda$, $\Delta M_\Lambda^{(1)} + \Delta M_\Lambda^{(2)} = 2.8$ MeV), and, for the decuplet baryons,

$$E_I^{\Delta_{3/2}} = 5.5 \text{ MeV} ,$$

$$E_I^{\Sigma^*_{3/2}} = 4.9 \text{ MeV} ,$$

$$E_I^{\Xi^*_{3/2}} = 4.3 \text{ MeV} ,$$

$$E_I^{\Omega^-_{3/2}} = 3.7 \text{ MeV} .$$

The results for $m_u = m_d = 8$ MeV and $m_s = 200$ MeV are very similar.

With these instanton-induced additions, and with the changed $E_0$, $E_V$, and $E_M$ contributions (while $E_Q$ and $E_E$ stay as they are in Table III of Ref. [13]), the bag masses are, for the octet,

$$M_{\text{bag}}^N = M_{\text{expt}}^N = 938 \text{ MeV} ,$$

$$M_{\text{bag}}^\Sigma = 1142 \text{ MeV} ,$$

$$M_{\text{bag}}^\Xi = 1285 \text{ MeV}$$

(for $\Lambda$, modulo $\mathcal{L}_3^I$ contribution, $M_{\text{bag}}^\Lambda = 1103$ MeV), and, for the decuplet,

$$M_{\text{bag}}^{\Delta_{3/2}} = M_{\text{expt}}^{\Delta_{3/2}} = 1233 \text{ MeV} ,$$

$$M_{\text{bag}}^{\Sigma^*_{3/2}} = 1385 \text{ MeV} ,$$

$$M_{\text{bag}}^{\Xi^*_{3/2}} = 1529 \text{ MeV} ,$$

$$M_{\text{bag}}^{\Omega^-_{3/2}} = M_{\text{expt}}^{\Omega^-_{3/2}} = 1672 \text{ MeV} .$$

The fit to the baryon masses therefore remains good.

## VI. CONCLUSION

We have found that, at least in the simplest MIT bag model, the instanton-induced baryon mass shifts are not similar to the one-gluon exchange effects (unlike the situation found in the nonrelativistic constituent quark model [1]). The instanton-induced mass shifts in the MIT bag model are small, of the order of a few MeV, just as the reduction of the strong-coupling constant is about 6%. That is, the change in $\alpha_c$ is in the desired direction, but it is quantitatively so marginal that we cannot claim that the inclusion of the instanton effects has successfully supplanted or supplemented the one-gluon exchange and has brought about the desired improvement in consistency of the perturbative description of the bag interior. It has turned out, however, that the "perturbative" bag interior cannot support sufficiently high instanton density for ensuring an important role of instantons in the MIT bag model. The value of $\bar{n}$ we have estimated as appropriate for the MIT bag interior is 1–2 orders of magnitude lower than $\bar{n}$ estimated, e.g., by Nowak *et al.* [6] or Shuryak [12], but for the true, nonperturbative QCD vacuum. Such instanton densities would induce gigantic mass shifts of hundreds of MeV. At a conceptual level, this actually shows us another reason why the MIT bag model cannot tolerate high instanton densities. If $\bar{n}$ is too high, the "dressing" of quarks would also be large, and a too large quark self-mass would lead to *de facto* constituent, nonrelativistic quarks, while the MIT bag is a relativistic model. Therefore instanton densities above a certain value would lead to a contradiction with the relativistic way the model was formulated. The relativistic nature of the MIT bag model, which requires the inclusion of the one-body of the instanton-induced interaction, is the reason why the inclusion of instantons gives different results than in the nonrelativistic constituent model [1].

Finally, let us remark that there are physical situations where even the weak instanton interaction, which produces unimportant effects on MIT bag model spectroscopy, can have interesting effects. Production of strangeness in nucleons, and some of its consequences, is one such example. The work on these issues is in progress.

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