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LETTER TO THE EDITOR

PLANETARY ORBITS IN THE SINGLE STAR SYSTEMS

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Recently, we have shown that the orbital radii of planets and major satellites could be described by the parabolic law $r_n = \text{const.} \times n^2$, where values of n are consecutive integers. The constant depends on a particular system and includes a dimensionless factor f, which is now correlated with some relevant parameters. Using the parabolic law, we discuss a possible existence of the terrestrial– and/or Jovian–type planets in extra–solar single–star systems, assuming variable stellar mass. An extension to the pulsar PSR B1257+12, the only one with three detected planets, is considered.

The old problem of the distribution of planetary orbits in the Solar system will probably be extended, in a near future, to planetary systems around the nearby stars. The prospects for detection of the extra–solar planets are directed to the giant Jupiter–like planets [1] which should be visible using the new generation of telescopes [2]. The sensitivities of astrometric and radial velocity methods in an expected discovery of extra–solar planets will be strongly dependent on planetary masses and their orbital radii. Fortunately, it seems that the so–called ice condensation radius weakly depends on the stellar mass (at least in the interval from 0.1 to 1.0 solar masses) [3–5]. Thus, the first giant Jupiter–like planet may be expected at a distance between 4.5 and 6 AU [4] (1 AU is the distance from the Earth to the Sun). A law for orbital distribution of planets of an extra–solar system is not known. Therefore, it is necessary to explore main properties of our own Solar system, with a hope that the distribution of orbits, and some associated quantities, would be applicable to other planetary systems. Until now only three stars similar to the Sun have been found to have a planet. They are 51 Pegasi B [6], 70 Virginis B and 47 Ursae Majoris B [7,8]. For neutron stars two planets have been detected in PSR B0239+54 [9] and three in PSR B1257+12 [10]. It is reasonable to expect that in the near future a complete planetary system for some stars might be discovered.

Using the astronomical data [11], we have recently demonstrated a general approach to the Solar system, in which semimajor axes of orbits, or planetary orbital radii r_n, are described by the semi–empirical parabolic law [12]

$$r_n = \frac{(fA)^2 M n^2}{G}. \tag{1}$$

Consecutive integer numbers n are associated with the planets. Analogous conclusions were drawn for satellite systems of Jupiter, Saturn and Uranus. Orbital radii depend on the mass M of the Sun (or the mass of the planet, if its satellite system is considered), but not on the mass m of the orbiting planet (or satellite). G is the gravitational constant. f can be defined as a dimensionless factor depending on the particular system. The constant A has then the dimension of angular momentum per square mass, being essentially proportional to $G/\alpha c$ (that follows from the similarity of gravitational and electrostatic force between two identical particles) [12]. α is the fine–structure constant [13] and c is the speed of light. The constant A may arbitrarily be defined as $A = 8\pi^2 G/\alpha c = 2.407 \times 10^{-15}$ J s kg^{-2} in order to obtain $f = 1$ for Jovian planets. The orbit of Jupiter is characterized by $n = 2$, and orbits of Saturn, Uranus, Neptune and Pluto are assigned n values equal to 3,4,5 and 6, respectively, (see Fig. 1). The Jovian planet on the orbit at $n = 1$ failed to be formed, because its orbital radius would fall inside the ice condensation radius, which is estimated to be about 3.4 AU (corresponding to a temperature of about 200 K). Thus, instead of "the first Jovian planet", a subgroup of terrestrial planets appears as the remnants of the mass which has been reduced after the temperature of the Sun had risen. The terrestrial planets, formed by accretion of the remaining rocky planetesimals, follow also the parabolic law of Eq.(1), but with another value of factor f [12]. The interpretation of the terrestrial planets as the subgroup of the Jovian ones presents a radical change from the standard approach settled down long ago by the well–known Titius–Bode law: $r_n = A + BC^n$ (A, B and C are parameters depending on the system, n is an integer) and its various, even recent, modifications [14,15], in which all planets are treated on an equal footing.

A consequence of the parabolic law for orbital radii r_n is that orbital periods T_n scale as n^3, because r_n^3/T_n^2 must be independent on n, due to the third Kepler's law. Therefore, the angular momentum per unit mass of orbiting body, i.e. $2\pi r_n^2/T_n$ depends linearly on integer n [12], being

$$\frac{J_n}{m} = (fA)Mn. \tag{2}$$

Equations (1) and (2) have been obtained semiempirically. A proper theoretical basis will, hopefully, be found. The proposed division of planets (the Jovian and the terrestrial groups) has been obtained also as consequence of a long term behaviour of chaotic systems [16] and by solving the Schrödinger equation using the gravitational potential [17,18], or some symmetries in the planetary system [19].

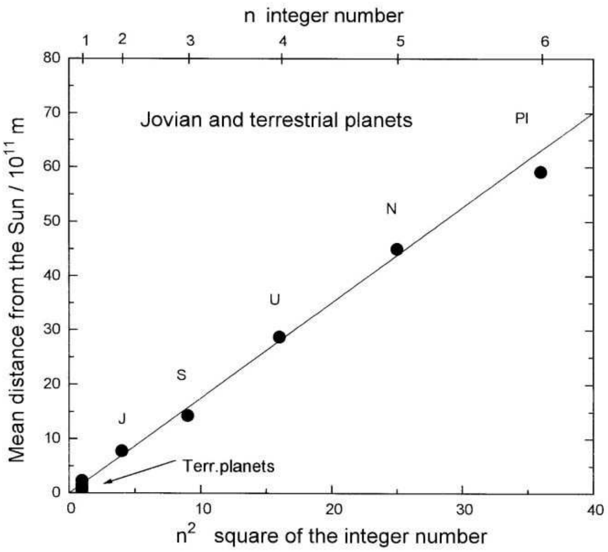

Fig. 1. Correlation of the mean radial distances of the planets from the Sun with an integer number n. Terrestrial planets (Mercury, Venus, Earth and Mars) are shown as a subgroup (at $n = 1$) of the Jovian ones.

To support our approach to the planetary system, we have included also the major satellites of giant planets [12]. Each of the considered quantities, r_n, T_n and J_n/m is proportional to the mass M of the central body (the Sun or a central planet). However, from Eq.(2), it follows that the factor f, chosen as 1.00 for Jovian planets, is equal 0.19 for the terrestrial ones, and 0.53, 0.28 and 1.06 for the satellite systems of Jupiter, Saturn and Uranus, respectively. Mean deviations of f are less than 4% [12]. Factor f is the single adjusting parameter, while the constant A is unique for all systems, making thus the model general and consistent. Some variation of the parameter f has to be expected due to various densities and distributions of mass during the formation of planets and of their satellites. Therefore, we correlate the factor f with mass of all planets or satellites (contributing to the particular system) divided by the mass M of the central body, i.e. with relative mass ($\sum m/M$). A simple correlation of f with $(\sum m/M)^{1/3}$ fits well the terrestrial and Jovian planets, and major satellites of Jupiter, but deviates for satellites of Saturn and Uranus (see Fig. 2). The obliquities of Saturn and Uranus (due to possible disturbances in evolution of the systems) caused the change in orbital distribution of satellites, corresponding to an increase of f, expressed by

$$f = f_0 \left(\frac{\sum m}{M} \right)^{1/3} f_i, \tag{3}$$

where $f_0 = 9.5 \pm 0.6$ is a constant and f_i a correction factor connected with the obliquity.

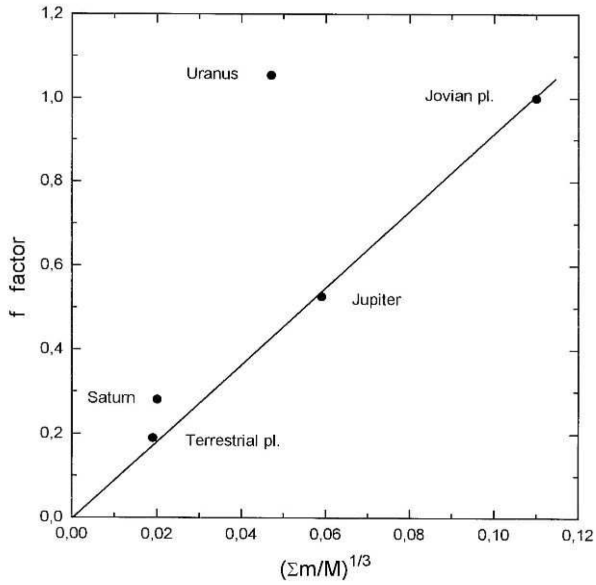

Fig. 2. Correlation of the factor f (Eqs. (1) and (2)) with a ratio of mass of all planets or satellites (belonging to the particular system) to the mass M of the central body for terrestrial and Jovian planets, and major satellites of Jupiter, Saturn and Uranus. Large obliquities of Saturn and particularly of Uranus (27° and 98°, respectively) cause an increase of f by additional factor f_i, according to Eq. (3).

The value of f_i for Saturn is 1.5 ± 0.1 and for Uranus 2.4 ± 0.2. The change of the total angular momentum of the primordial system could have been caused by an impact of a relatively big strange body, resulting also in a considerable obliquity [20–22]. It is plausible that the energy deposited to the gaseous envelope during the impact with the core of the planet caused the extension of the envelope [22]. The satellites are supposed to have been formed later on at greater distances. In such processes, the mass for formation of satellites has not been changed. Another view could be that, after the impact, a certain part of the envelope has been dispersed, diminishing thus the value of the parameter $(\sum m/M)^{1/3}$. A decrease of the relative satellite mass by factor 3.3 for Saturn and 13.2 for Uranus would correspond to factor f_i, in Eq.(3) (see Fig.2). Of course, one can assume that both processes have taken place during the formation of satellites, which would require more elaborated approach.

If we assume that an extra–solar system of Jupiter–like giant planets is similar to the Solar system of Jovian planets, then the factor f would be approximately equal for both systems. The planets would have the discrete orbital radii and their angular momenta per unit mass, according to Eqs. (1) and (2), respectively. With these assumptions, some interesting anticipations may be derived. For stellar mass M of the same order as the mass $M_\odot$ of the Sun, the first giant planet, at $n = 1$, would be missing, and a Jupiter–like planet, at $n = 2$, would exist as the first one at the orbital radius r_2, according to Eq. (1), of course, beyond the ice condensation radius r_i of about 4 AU. For a stellar mass M greater than $M_\odot$, such a planet is expected farther from the star. For some critical mass $M = M_c$, the first giant planet may appear on the closest possible orbit at $n = 1$, in the vicinity of r_i. Then, for M greater than M_c (equal to about 3.4 $M_\odot$), the terrestrial–like planets should not be expected. Value of M_c is approximate, because $(\sum m/M)$ would be larger as one more planet was included, and so would f_c. That would result in a somewhat smaller value of M_c. We have to note that maximum number of Jovian planets seems to be about 6 and, in light of that, one may assume approximately unchanged factor f even for a stellar mass considerably greater than M_c. However, the orbital radius r_1 of the first giant planet at $n = 1$ could then rise with M, even considerably beyond the ice condensation radius.

A completely different situation would appear for the much more common lower–mass stars. Due to Eq. (1), the first giant planet may appear on the orbit at n equal to 3, or higher. Moreover, the orbital radius at $n = 6$ may be inside r_i, resulting in no giant planet at all. That is expected to happen for a stellar mass of about 0.1 $M_\odot$ or less, assuming the unchanged factor f. However, according to Fig. 3, it is reasonable to assume $0.2 < f < 1$. For $f \approx 0.6$, a stellar mass close to 0.3 $M_\odot$ would be accompanied with terrestrial-like planets only. However, the nearest planets to the star could be missing due to "the rotational limit", which is defined by $r_{min} = ((GM/4\pi^2)T_s^2)^{1/3}$, where T_s is the rotational period of the star [12]. A planet initially formed simultaneously with the star may appear only at $r > r_{min}$. Unfortunately, there are no observable data on rotation periods of stars. Exception are pulsars with very short periods of rotation. Consequently, their r_{min} is too small to be effective in preventing formation of nearest planets.

Finally, it must be admitted, that a good prognosis for orbital distribution of extra-solar planets, for stellar masses much greater or much smaller than the mass of the Sun, is very difficult because of an uncertainty of the factor f.

The situation becomes still more complex if the formation of planets occured by a mechanism other than that of the Solar system [23]. For example, recently discovered companion of 51 Pegasi A, called Peg B, cannot be treated by the present model because this single planet does not experience mutual perturbation of other planets, and its distance from the star may be of whatever value governed by the standard laws of classical mechanics. Four possible scenarios for the origin of the Peg B have benn proposed [24]. Our model is applicable only to a system of a single central body with several planets (or satellites), say 4 to 6 [12]. Therefore, an application to the pulsar PSR B0239+54 with only two planets [9] is not considered correct. PSR B1257+12 has three planets [25], what is probably also too low to use Eq. (1). Nevertheless, on may speculate on the factor f. Namely, the most recent data for distances of the three detected planets of this pulsar are $r_A = 0.19$ AU, $r_B = 0.36$ AU and $r_C = 0.47$ AU [10]. For the two largest planets, B and C, it follows from our model $r_C/r_B = ((n_B + 1)/n_B)^2$, and with the assumption that there are no other planets between them, n_B should be equal to 7 and n_C to 8. For the small planet A is assigned $n_A = 5$, according to Eq. (1) (see Fig. 3). The factor f takes the value 0.068 ± 0.001, smaller than that for the terrestrial planets of the Solar system. An absence of planets at n equal 1 to 4 and at n equal 6 has no explanation at present, and the above prediction may be incorrect. However, such a simple analysis may be used for diagnostic purposes, for searching the planets at predicted distances, based on the already detected ones (for example at $n = 6$ for PSR B1257+12, see Fig. 3). The dynamical characteristics of planetary motion around the pulsar are not unlike those of the inner planets of the Solar system [10, 26], although both types of planets may differ in their physical and chemical properties. Thus, the fundamental features of the dynamics of their parent planetary systems should be comparable [10]. Therefore, pulsar planetary systems might obey the parabolic law (see Fig. 3). However, Eq. (3) could, probably, take another form for planets of a pulsar, due to different conditions during formation of planets.

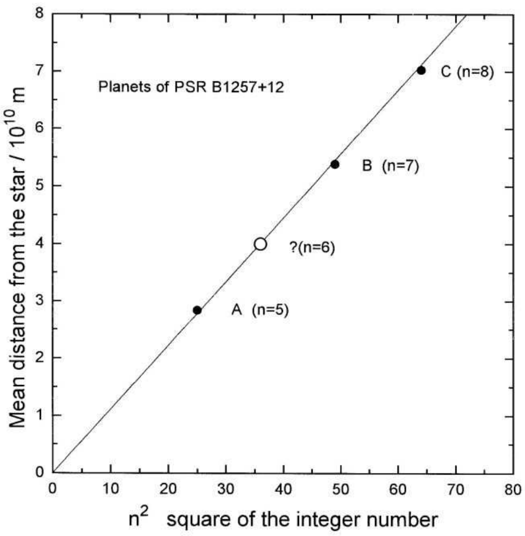

Fig. 3. Correlation of the mean radial distances of the planets from the star PSR B1257+12 with an integer number n. Planets on the orbit $n = 6$ is not detected.

The double- or even triple-star systems [27] cannot be treated by the present model due to unknown values of several parameters.

In conclusion, we expect that orbits of planetary systems with 4 or more planets, initially formed around a single star, follow the parabolic law (Eq. 1), while orbital radii for smaller number of planets are still questionable. The factor f appearing in Eqs. (1) and (2) can be approximated using analogy with the Solar system (Eq. 3). Hopefully, analogous relations will prove valid for pulsar planetary systems.

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PUTANJE PLANETA U SISTEMIMA JEDNE ZVIJEZDE

Nedavno smo pokazali da se velike poluosi putanja planeta i njihovih glavnih satelita mogu opisati kvadratnim zakonom $r_n = konst. \times n^2$, gdje je n suksesivno rastući cijeli broj. Konstanta ovisi o danom sustavu i uključuje bezdimenzijski faktor f, koji je u ovom radu povezan s nekim bitnim parametrima. Na osnovi kvadratičnog zakona, raspravlja se o mogućnosti postojanja planeta terestričkog i/ili jovijalnog tipa u izvan sunčevim sustavima s jednom zvijezdom u ovisnosti o masi te zvijezde. Razmatra se moguća primjena na pulzar PSR B1257+12 s tri planete, što je do sada najveći broj planeta otkrivenih oko neke zvijezde.