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Level structure of ⁹⁹Nb

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The β decay of 97 Sr to 97 Y has been investigated using ion-guide on-line mass separation and a 10 Ge-detector array to record γ - γ coincidences to a detection limit well below that of former studies. Similarities are found in the β -decay patterns of 99 Zr and of its isotone 97 Sr and also in the γ -ray decay rates and branchings of the corresponding levels in their respective daughters ⁹⁹Nb and ⁹⁷Y. This indicates a persisting influence of the $d_{5/2}$ neutron shell closure for 99 Nb. The level structure of 99 Nb and the β -feeding pattern are discussed in the frame of the interacting boson-fermion plus broken pair model and the microscopic quasiparticle phonon model. [S0556-2813(98)06106-8]

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I. INTRODUCTION

Neutron-rich nuclei near $^{96}_{40}$ Zr₅₆ have been investigated in numerous studies motivated by the strong shell closure associated with the neutron $d_{5/2}$ subshell filling when protons occupy the natural parity states below the $g_{9/2}$ orbital [1]. Nuclei with a few valence particles with respect to ⁹⁶Zr are especially interesting owing to the rapidly changing level structure with increasing nucleon number. With the addition of only four neutrons, the ground states of the neighboring N = 60 isotones become strongly deformed [2]. Deformations are as large as $\beta \approx 0.40$ for $^{98}_{38}$ Sr [3] and $^{99}_{39}$ Y [4], while they slightly decrease with increasing Z [5,6,5–8]. Shape coexistence has been reported for the N=59 isotones ${}^{97}Sr$ [9–11] and ⁹⁸Y [12–14], both of which have spherical ground states and deformed states at about 0.5 MeV excitation energy. However, for ⁹⁹Zr a clear indication for an excited deformed band structure is still lacking [15–18]. The lowest-lying levels in ⁹⁷Sr and ⁹⁹Zr present similarities with the levels in the N=57 isotones ⁹⁵Sr [19] and ⁹⁷Zr [20], whereas in their ⁹⁹Mo [21] and ¹⁰¹Mo [22] neighbors one observes an evolution of the level structure versus neutron number. This different behavior versus proton number is related to the vanishing of the $d_{5/2}$ neutron shell closure we have recently discussed for 99 Mo [21]. The N=58 isotones with $Z \le 40$ display clear fingerprints of shell closure effects. The 2_1^+ states in even-even Sr isotopes have almost constant energies at ~ 0.8 MeV from 90 Sr to 96 Sr. Their low collectivity has been established by lifetime measurements [23]. High-spin levels in the odd-proton nucleus ⁹⁷Y have been observed via isomeric decay of an $I^{\pi} = 27/2^{-}$ isomer [24]. The high-spin level structure of ⁹⁷Y has been calculated in the frame of the interacting boson-fermion model (IBFM) [25] and, very recently, by its extended version including a broken neutron pair [26]. The ⁹⁸Zr nucleus has a high-lying 2₁⁺ state at 1223 keV. In contrast, the 2_1^+ energy in 100 Mo is only 536 keV.

Thus, there is a dramatic change of the level structure of $N \le 59$ spherical nuclei associated with the closure of the $g_{7/2}$ - $d_{5/2}$ neutron gap when proton pairs start to occupy the $g_{9/2}$ shell. With its proton number of Z=41, the nucleus ⁹⁹Nb is situated at the transition between the region where the N=56 shell closure is active ($Z \le 40$) and the region of the open shell ($Z \ge 42$). In this context, a detailed study of ⁹⁹Nb levels should provide an insight into the influence of a single $g_{9/2}$ proton on the $d_{5/2}$ neutron-shell closure. Especially appealing is the comparison of levels in the N=58isotones 97 Y and 99 Nb. These are populated by β decay of ⁹⁷Sr and ⁹⁹Zr which have a very similar low-lying level structure [9,15]. Accordingly, levels of the same character might be selected by β decay into 97 Y and 99 Nb and this should provide a means to follow their evolution when the proton number crosses Z=40.

The low-spin levels in 97 Y have been studied by β decay of $I^{\pi} = 1/2^{+97}$ Sr by Pfeiffer *et al.* [27]. Level-lifetime measurements were subsequently performed by the TRISTAN group [10] according to which several levels were interpreted as core + particle configurations. For a better comparison of the levels in the isotones ⁹⁷Y and ⁹⁹Nb we have reinvestigated the 97 Y level scheme using γ - γ coincidence data recorded during former experiments [20,24].

The levels in ⁹⁹Nb have been studied by the decay of $I^{\pi} = 1/2^{+}$ 99Zr, including lifetime measurements [28], and several transfer reactions, compiled in the Nuclear Data Sheets [29]. An extended decay scheme, compared to that published in Ref. [29], was presented by Pfeiffer et al. [30]. The so far reported transitions have intensities of about 1% per decay. It has been shown that experiments at the IGISOL facility, see, e.g., Refs. [20,21], allow the detection of transitions with still lower branchings for most of the neutronrich isotopes in this region. The observation of new weak γ-ray branchings, combined with absolute transition probabilities should provide a basis for firm spin and parity as-

2974

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signments. In addition, the question of whether a $27/2^-$ three quasiparticle state could form an isomer, similar to the one in 97 Y, is still open [21]. If this configuration has a half-life of about 1 ms or longer, high-spin states in 99 Nb could be observed using on-line mass separation with the ion-guide technique.

Finally, shape coexistence has been proposed for 98 Zr, the isotone of 99 Nb, based on the large $\rho^2(0_3^+ \rightarrow 0_2^+)$ value of 0.075 [15]. However, a strongly deformed band built on this 0_3^+ state at 1436 keV has not been observed in a recent prompt-fission study [18]. To our knowledge, there have not been reports of shape coexistence in neutron-rich Nb isotopes. Unfortunately, it is unlikely that a band structure will be observed after the β decay of the $1/2^+$ ground state of 99 Zr.

II. EXPERIMENTS

The main experiment was devoted to the decay of ⁹⁹Zr to ⁹⁹Nb, which is described below. For the purpose of systematics some details of the decay scheme of ⁹⁷Sr to ⁹⁷Y were to be studied in greater detail. The data have been obtained from previous experiments [20,24] which we do not present here.

A. Production of 99Zr and detector setup

The parent nucleus of ⁹⁹Nb, ⁹⁹Zr, was produced by fission of a natural U target induced by 25 MeV protons delivered as a 50 MeV H₂⁺ beam, thus doubling the particle intensity. The beam intensity was typically 10 p μ A. The A = 99 isobars were separated on-line using the IGISOL technique [31–33]. The mass separated beams were collected on a movable tape viewed by various detectors for β particles and γ rays. The ⁹⁹Y ($T_{1/2}$ =1.5 s) and ⁹⁹Zr ($T_{1/2}$ =2.1 s) activities [34] are produced with comparable independent yields of about 5×10^3 ions/s. Hence, the cumulative yield of ⁹⁹Zr, which includes the contribution of the β decay of ⁹⁹Y, is roughly twice that of ⁹⁹Y. As a consequence of the similar half-lives of the ⁹⁹Y and ⁹⁹Zr activities, their discrimination requires acquisition cycles with a long decay period. This reduces the duty cycle and the counting statistics. Therefore, two sets of experiments were performed, following the procedures described in recent papers [20,21]. First, coincidences were recorded with a 10 Ge-detector array (DORIS), a more compact version of the 12 Ge-detector TARDIS array described in [24], providing an improvement of a factor of 1.9 for twofold events. One of the detectors was a planar Ge detector for low-energy radiation. In order to collect the largest number of events, the beam was continuously implanted in the center of the array and the tape was moved at various time intervals in order to enhance one or the other activity. The coincidences were recorded with the EUROGAM data acquisition system [35]. Secondly, a setup consisting of two thin plastic scintillators for β particles, the low-energy Ge detector and a coaxial detector of 23% efficiency covering a range of energies up to 3.6 MeV, was used. The goal of this experiment was to record β - γ -time coincidences and to identify the activities on the basis of growth and decay curves of the γ rays. For this purpose, the separator beam was pulsed and the collecting tape was moved after each beam on/off

sequence. Pure singles, e.g., non- β -gated spectra, of the lowenergy detector were also recorded in order to allow the observation of occasional isomeric transitions. The data were recorded with the VENLA data acquisition system [36] designed at the Accelerator Laboratory in Jyväskylä. A time to digital converter (TDC) marking the time of occurence of events with respect to the beginning of the separator cycle was written as an additional list-mode parameter.

B. Analysis

The coincidence data taken with the DORIS array were first sorted off-line to construct γ - γ and x- γ matrices. Higher-fold events were also investigated, which could have given an indication of a cascade of high multiplicity following the decay of the high-spin isomer postulated to exist in ⁹⁹Nb. The additional experiments yielded further TDC- γ and TAC-γ matrices (where the time-to-amplitude converter recorded the time between β and γ rays for the determination of level lifetimes). For both experiments, energy and efficiency calibrations were performed internally, using data compiled in [29] and our results on the decay of 99Nb to ⁹⁹Mo [21]. Since the ⁹⁹Nb level scheme contains several strong low-energy transitions, we describe in some detail the determination of conversion coefficients which are obtained from coincidence data. Intensity balance considerations for transitions observed in gates set on γ rays placed above the the converted transition yield the total conversion coefficient α . They are reliable since they use medium- or high-energy γ - γ data which can be calibrated accurately. Unfortunately, the measured quantity is $1+\alpha$, which is not very sensitive if α is small. A more direct method, giving α_K , is based on the fluorescence produced by the converted transition. It, however, requires an accurate efficiency calibration of the gated spectra at the low-energies of the K-x rays. This is rather difficult due to the scarcity of internal references. Nevertheless, the K-x rays due to the conversion of some transitions of known multipolarity could be observed under sufficiently clean conditions in various gates set on the Ge detectors. These were the transitions of 98 keV (E2, 99Mo), 130 keV (E2, ⁹⁹Zr), and 36 keV (E1, ⁹⁸Y [12]). The higher-energy part of the on-line efficiency curve of the x-ray detector was determined by comparison with off-line calibrations made with ¹³³Ba, ¹⁵²Eu, and ²⁴¹Am standards [34]. Owing to the distribution of the Ge detectors over the whole solid angle, angular-correlation effects were not likely to play a major role with respect to the rather large statistical uncertainties. These have not been taken into account. In addition, these measurements yielded a value of $\alpha_K = 0.13(3)$ for the 122 keV transition in 99 Zr, confirming its M1 multipolarity.

III. RESULTS

A. Decay scheme of 99Zr to 99Nb

The decay scheme is based mostly on coincidence data using the previously reported transitions [29] as a starting point. As a matter of fact, the detection limit for lines in the singles spectra was poorer than in the coincidence data by about one order of magnitude. Thus, weakly populated I = (1/2,3/2) levels which only decay to the $1/2^-$ isomer remain unidentified if the relative transition intensity is below

1%. This, however, should not modify the conclusions about the presence of very strongly β -fed levels near 1 MeV.

The decay scheme now includes 33 transitions placed between 13 levels. Although transitions weaker than 0.1 relative intensity units have been observed, there are only five new levels with respect to the decay data compiled in Ref. [29]. Some of these have already been reported by Pfeiffer et al. [30] in their decay work or in the transfer-reaction study by Flynn et al. [37]. Transitions and their coincidence relationships are shown in Table I. Several of the listed transitions cannot be placed unambiguously, although there is fair evidence that they belong to the level scheme of ⁹⁹Nb. The coincidence pair 88.8–561.4 keV is assigned to ⁹⁹Nb by the K-x rays originating from the conversion of the 89 keV transition, see Table II. There are no other coincidence relationships involving these lines but their sum of 650.2 keV compares well with the energy difference of the 1015.4 keV level and the isomer at 365.4 keV, so that this cascade probably fits between these levels. The 363.0 and 379.2 keV transitions have very weak coincidences, but which seem to form a consistent set according to which they should be placed below the 387.5 keV level. A transition of 24.3 keV observed randomly in various gates could correspond to the 387.5–363.0 keV energy difference. Nevertheless, we regard the existence of levels at 363 and 379 keV as speculative. Another weak coincidence pair is formed by the lines at 444.5 and 600.4 keV, the sum of which may fit from the 1044.5 keV level to the ground state. However, there are no other data to assign these transitions to 99Nb. A level at 1005.7 keV is uncertain, since based on two sums involving the weak 46.1 keV (a possible Ge-x-ray escape of the strong 56 keV line) and 536.7 keV γ rays (which might be a residual of the strong 536.1 keV line in ⁹⁹Zr). Finally, the weak peaks at 930.8 and 1015.7 keV seen in the single spectra are interpreted as due to coincidence summing of the 461.9 and 546.1 keV transitions, respectively, with the 469.2 keV transition placed below them.

The results confirm the previously adopted level scheme [29], except for the intensity of the 28.4 keV transition to the 930.8 keV level. This fact turns out to be of importance, since it allows a more consistent interpretation of the spin and parity assignments of the ⁹⁹Nb levels. The 28 keV transition was reported with an intensity $I_{\gamma} \approx 3$ relative intensity units. Taking electron conversion into account, the intensity balance of the 930.8 keV level could be achieved only in the absence of direct β feeding. In the present experiment, however, a much lower total intensity *including conversion* of $I_t(28) = 2.4(4)$ is deduced by comparing the peak area of the 56 keV transition in the 461 and 594 keV gates as shown in Fig. 1. The intensity balance now requires allowed β feeding into the 959 keV level and this implies $I^{\pi}(931) = (1/2,3/2)^{+}$ in contrast to the former $5/2^{+}$ assignment.

Conversion coefficients are shown in Table II. The multipolarities formerly assigned to the 28.4, 55.9, and 81.8 keV transitions, mostly on the basis of balancing the level feedings [29], are confirmed. An E2 admixture, 10(8)% using both K and total conversion coefficients, could be present in the 82 keV transition. The conversion coefficient for the new 114.2 keV transition is suggestive of a pure E2 multipolarity. This is, however, not consistent with its lifetime limit

[28], so that it has to be rather a M1 transition with an E2 component.

B. Spin and parity assignments of 99Nb levels

The decay scheme of ⁹⁹Zr to ⁹⁹Nb is shown in Fig. 2 and the properties of the levels are summarized in Table III. The neighbors of ⁹⁹Nb are ⁹⁸Zr and ¹⁰⁰Mo which do not exhibit rotational structure at low energy. For this reason, strongly enhanced *E*2 transitions are not expected to occur between the low-lying levels of ⁹⁹Nb. This assumption is the key for the following spin and parity assignments.

The allowed β branches for the decay of $I^{\pi} = 1/2^{+}$ ⁹⁹Zr to three levels near 1 MeV provide a starting point for the analysis. The strongest cascade is formed by the 546.1 and 469.2 keV transitions. The $\log ft$ value of 4.1 for the level at 1015.4 keV implies $I^{\pi} = (1/2,3/2)^{+}$ and, since the ground state has $I^{\pi} = 9/2^{+}$, at least one of the 546 or 469 keV transitions must be of pure quadrupole character. The lifetime limits of Ref. [28] for both the 1015 and 469 keV levels obviously exclude M2 multipolarities, which results in $I^{\pi} = (5/2,7/2)^{+}$ for the intermediate 469 keV level. The enhancement factors for the 469 and 546 keV transitions, if E2, are 4.8 and >70, respectively. Therefore, we conclude that the 546 keV transition is not an E2 but a E3 and the 469 keV and 1015 keV levels are thus E3 and E3 and E3 and E3 are 4.8 levels are thus E3 and E3 are thus E3 and E3 are the series of the 469 keV and 1015 keV levels are thus E3 and E3 and E3 and E3 and E3 are the series of E3 and E3 and E3 and E3 and E3 are the series of E3 and E3 and E3 are the series of E3 and E3 and E3 are the series of E3 and E3 and E3 are the series of E3 and E3 and E3 are the series of E3 and E3 are the series of E3 and E3 and E3 are the series of E3 and E3 are the series of E3 and E3 are the series of E3 and E3 and E3 are the series of E3 a

The 387.5 keV level is fed from the 469 keV level via the 81.8 keV transition of M1 character (see Table II), which restricts its spin and parity to $I^{\pi} = (5/2,7/2)^{+}$. From the lifetime of the 387 keV level [28] an enhancement of 137 would result for the 387 keV ground-state transition if it were an E2. Therefore, we adopt the M1 alternative and assign $I^{\pi}(387) = 7/2^{+}$. Then, the 628 keV transition from the $3/2^{+}$ level at 1015 keV must be another E2. The transition rate limit of >1.6 deduced from the partial lifetime, is indeed low enough to leave room for a moderately enhanced transition. These assignments were also made by Ohm [28], in contrast to the very first interpretations [39,40].

As a new result, the 931.0 keV level is directly fed in β decay. The logft value of 4.9 implies $I^{\pi}(931) = (1/2,3/2)^{+}$, instead of $I^{\pi} = 5/2^{+}$ [40]. We can use the same kind of argument as above to show that the 462 keV transition to the 469 keV $5/2^{+}$ level would have had an enhancement >89 if an E2. Accordingly, we choose the M1 alternative and assign $I^{\pi}(931) = 3/2^{+}$. Both $3/2^{+}$ levels at 931 and 1015 keV have a similar γ -ray branching pattern. Here also, an E2 transition to the 387 keV level exists. The decay rate for this 543.6 keV transition is >2.4 single-particle units, which is an acceptable limit.

The 959.4 keV level is the last of the levels strongly fed by a direct β -decay branch ($\log ft = 4.4$), which again allows $I^{\pi} = (1/2, 3/2)^{+}$ as the only possible choices. Constrained by the dipole character of the 28 keV transition to the 931 keV level, formerly a $5/2^{+}$ state, Ohm assigned $I^{\pi}(959) = 3/2^{+}$. However, the γ -decay pattern of the 959 keV level is very different from those of the 931 and 1015 keV levels. The largest branching goes to the $1/2^{-}$ isomeric level at 365 keV. The decay to the $5/2^{+}$ level at 469 keV is weak and a transition to the 387 keV $7/2^{+}$ state could not be detected. This

TABLE I. List of γ rays in the decay of 99 Zr to 99 Nb. Intensities are calculated from coincidence data and in most cases also from singles unless specified. For absolute decay intensity multiply I_{γ} by 0.562×10^{-2} .

Energy							
(keV)		Intensity	from	to	Coincidences		
28.4 (1)		0.39 (11)	959	931	56, 82, (387), 462, 469		
46.1 (3)	a	0.12 (6)	(1005	959)	(594)		
55.9 (1)		3.9 (7)	1015	959	<i>K</i> -x _{Nb} , 28, 82, 179, (387),		
					415, 462, 469, (490), 594		
74.3 (4)	b	0.02(1)	544	469	(415), (469)		
81.8 (1)		5.8 (8)	469	387	<i>K</i> -x _{Nb} , (28), (56), (113),		
					(363), (379),		
					387, 462, 490, 546, 575		
84.4 (2)	c	0.16(3)	1015	931	462, 469		
86.7 (3)	b	0.07(2)	631	544	179, (385)		
88.8 (2)	a	0.09(2)			561		
113.4 (4)	b	0.06(2)	1044	931	(462), (469)		
114.2 (2)	c	0.31 (7)	931	817	$(K-x_{Nb}), (348), 387, 429, 817$		
165.6 (3)	b	0.01 (1)	931	765	(221)		
175.2 (5)	a	0.09(3)	(644	469)	(469)		
178.9 (2)		10.2 (16)	544	365	K-x _{Nb} , 56, 87, 221, 250,		
					(385), 386, 415, 471, 500		
198.0 (5)	c	0.06 (2)	1015	817	(429), (817)		
220.9 (2)	b	0.08 (2)	765	544	(166), 179, 250		
250.4 (3)	b	0.04 (1)	1015	765	(179), (221)		
347.5 (3)	b	0.08 (2)	817	469	(114), (469)		
363.0 (5)	a	0.06 (3)			(82), (546), (628)		
379.2 (5)	a	0.10 (3)			(82), (546), (628)		
384.8 (3)	b b	0.06 (2)	1015	631	(87), (179)		
386.5 (3)	d	0.11 (3)	931	544	179		
387.5 (2)	u	14.8 (23)	387	0	<i>K</i> -x _{Nb} , 82, 114, 429, 462, 490, 543, 546, (575), 628		
415.1 (2)		8.3 (7)	959	544	<i>K</i> -x _{Nb} , 56, (74), 179		
429.3 (3)	c	0.38 (8)	817	387	114, 198, 387		
444.5 (4)	a	0.06(2)					
461.9 (2)		20.1 (22)	931	469	<i>K</i> -x _{Nb} , 28, 56, 82, 84, (113), 387, 469, (1043)		
469.2 (2)		100	469	0	<i>K</i> -x _{Nb} , 28, 56, (84), (113), (175), (198), (348), 462, 490, (536), 546, 575, 959, (1043), (1321)		
471.1 (3)	b	0.12 (4)	1015	544	(179)		
490.2 (3)		0.98 (16)	959	469	(K-x _{Nb}), 56, 82, 387, 469		
499.9 (3)	b	0.06(2)	1044	544	(179)		
536.7 (3)	a	1.5 (4)	(1005	469)	(81), (387), (469)		
543.6 (4)	c	1.26 (18)	931	387	$(K-x_{Nb}), (28), (387)$		
546.1 (2)		84.9 (64)	1015	469	<i>K</i> -x _{Nb} , 82, (363), 387, 469, 959, (1321)		
561.4 (3)	a	0.19 (6)			$(K-x_{\rm Nb}), 89$		
575.4 (3)	b	1.63 (42)	1044	469	82, 387, 469		
594.1 (2)		46.9 (54)	959	365	K - x_{Nb} , 56		
600.4 (4)	a	0.06(2)			(444)		
628.0 (2)		4.0 (6)	1015	387	387		
650.0 (2)	d	4.1 (8)	1015	365			
816.7 (3)	c	0.12 (4)	817	0	114, (198)		
930.8 (4)	e	0.9 (3)					
959.4 (3)		0.61 (12)	1975	1015	(56), 469, 546		
1015.7 (4)	e	2.6 (7)			$(K-x_{Nb})$		
1043.4 (4)	c	0.18 (4)	1975	931	(462), (469)		
1321.0 (3)	c	0.24 (6)	2336	1015	(469), (546)		

^aPossible transition of uncertain placement in ⁹⁹Nb.

^bNew transition from this work.

^cReported by Pfeiffer et al. [30].

^dIntensity was determined from singles only.

^ePeak due to coincidence summing.

TABLE II. Conversion coefficients obtained from γ - γ and x- γ coincidences. Theoretical values are obtained from [38].

Energy				Theoretical		
(keV)	Measured		E1	<i>M</i> 1	E2	Adopted
28.4	7.2(26)	a	3.7	7.6	46	<i>M</i> 1
	6.1(20)	b	4.3	8.7	108	
55.9	1.12(25)	c	0.57	1.05	6.7	M1
81.8	0.53(12)	d	0.19	0.35	1.9	M1
	0.50(24)	e	0.22	0.40	2.4	
88.8	1.13(45)	f	0.15	0.28	1.4	E2, (M1)
114.2	0.57(26)	g	0.072	0.14	0.59	M1 + E2
178.9	0.065(27)	h	0.020	0.041	0.119	M1

 $[\]alpha_{K}$ from gate on 462 keV, corrected for contributions of 56 and 82 keV γ rays to $K\alpha$ -x peak.

 $^{{}^{\}rm h}\alpha_K$ from gate on 415 keV, corrected for contribution of the 56 keV to the $K\alpha$ -x peak.

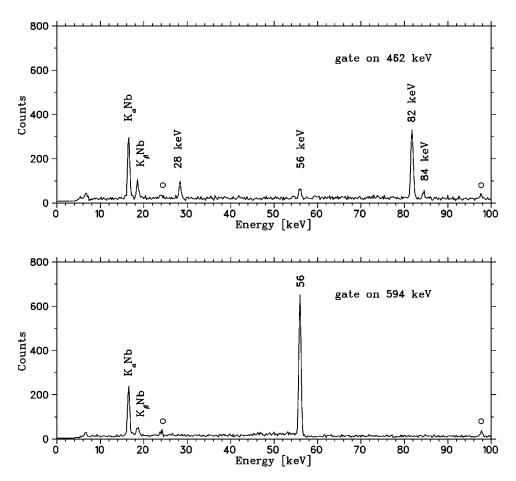


FIG. 1. Low-energy part of projected spectra of the x-ray detector gated by the 462 and 594 keV γ rays. The weak peak at 24 keV (open circle) could be due to a transition in the level scheme of 99 Nb (see text). The other open circle marks the 98 keV transition (99 Mo) due to random coincidences. The area of the 56 keV peak in the 462 keV gate is proportional to the $\gamma + e^-$ intensity of the 28 keV transition. The 594 keV gate is used for normalization of the 56 keV peak area.

 $^{^{}b}\alpha$ from comparison of $I_{\gamma}(594)$ and $I_{\gamma}(462)$ in gate on 56 keV yielding $I_{t}(28) = 2.50(65)$ and 2.33(45) from x- γ and γ - γ data, respectively.

 $^{^{\}rm c}\alpha_{\rm K}$ from gate on 594 keV.

 $^{{}^{\}rm d}\alpha_K$ from gate on 546 keV, corrected for contribution of 469 keV γ ray to $K\alpha$ -x peak.

 $^{^{\}rm e}\alpha$ from comparison of $I_{\gamma}(82)$ and $I_{\gamma}(387)$ in gates 462 and 546 keV.

 $^{{}^{\}rm f}\alpha_{\rm K}$ from gate on 561 keV, this transition is unplaced.

 $^{^{}g}\alpha_{K}$ from gate on 429 keV, corrected for contributions of 28 keV and $K_{\beta}(Y)$ due to 52–428–119 keV cascade in ^{98}Y [12] to the $K\alpha$ -x peak. Pure E2 excluded by the lifetime.

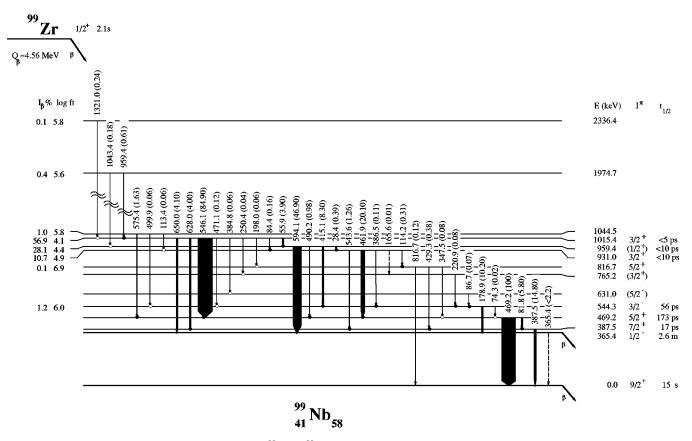


FIG. 2. Decay scheme of ⁹⁹Zr to ⁹⁹Nb from this work and data compiled in Ref. [29].

suggests $I^{\pi}(959) = 1/2^{+}$. This new assignment implies E2 character for the weak 490 keV transition to the 469 keV level, for which the deduced enhancement of >1.2 still can be low enough. Thus, although not unique, this assignment is the most logical one and we will adopt it through the subsequent discussion.

TABLE III. Levels populated in ⁹⁹Nb by β decay of ⁹⁹Zr. The branching of the first-forbidden β feeding to the $1/2^-$ was assumed to be less than 1.0% (logft=5.9). Logft values are calculated using $T_{1/2}(^{99}\text{Zr})=2.1 \text{ s}$ and $Q_\beta=4.54 \text{ MeV}$ [29]. Level lifetimes are from [28,29].

Energy (keV)	β feeding %	$\log ft$	t _{1/2} [ps]	I^{π}
0.0	0			9/2 + [29]
365.4 (2)	0.6 (32)	5.9		1/2 - [29]
387.5 (2)	0.4 (15)	6.6	17 (4)	7/2 + [29]
469.2 (2)	0.4 (39)	6.5	173 (4)	5/2 + [29]
544.3 (3)	1.2 (10)	6.0	56 (10)	3/2 [29]
631.0 (4)	0.01(2)			5/2 [29]
765.2 (4)	0.02(1)	7.8		3/2 + [29]
816.7 (2)	0.12(7)	6.9		5/2 + [29]
931.0 (2)	10.7 (14)	4.9	< 10	3/2 +
959.4 (2)	28.1 (30)	4.4	< 10	$(1/2^{+})$
1015.4 (1)	56.9 (40)	4.1	< 5	3/2 + [29]
1044.5 (2)	0.98 (25)	5.8		(1/2,3/2)
1974.7 (3)	0.44 (8)	5.6		$(1/2,3/2)^+$ [29]
2336.4 (3)	0.13 (4)	5.8		(1/2,3/2)

The 544.3 keV level has been assigned $I^{\pi} = 3/2^{-}$ by various authors [29]. This is consistent with the M1 multipolarity obtained from the conversion coefficient of the 179 keV transition to the $1/2^{-}$ isomer (see Table II). The new 74.3 keV transition, represents an alternative E1 decay to the 469 keV $5/2^{+}$ level.

Other levels are only very weakly fed and no lifetime measurements are available. The 631.0 keV level is fed via the γ ray of 384.8 keV from the $3/2^+$ level at 1015 keV and it decays by the 86.7 keV transition to the $3/2^-$ level at 544 keV. These transitions and the weakness of direct β feeding leave several assignments open, but consistent with $I^{\pi}(631) = 5/2^-$ reported in the (t, α) study of Flynn *et al.* [37]. An E2 decay to the $1/2^-$ isomer cannot be observed, possibly due to the poorer detection limit in the singles spectra.

The 765.2 keV level is placed by the clear cascade of the 250.4 and 220.9 keV transitions, the latter being slightly more intense. This level was also reported in [37] and assigned $I^{\pi} = 3/2^{+}$, which indeed is possible from the transitions. The β feeding is very weak compared to those of the above discussed $3/2^{+}$ states.

The 816.7 keV level was reported in β decay by Pfeiffer *et al.* [30], but not included in the Nuclear Data Sheets evaluation, and in two reaction studies [37,41] which proposed $I^{\pi} = 5/2^{+}$. There is a 114 keV transition from the $3/2^{+}$ state at 931 keV, the lifetime limit [28] and conversion coefficient (see Table II) of which, favor M1 character. Thus, the decay data are consistent with the above stated $I^{\pi}(817) = 5/2^{+}$ assignment.

$$\frac{97}{\text{Sr}} = \frac{1/2^{+}}{1/2^{+}} t_{1/2}^{-0.43 \text{ s}}$$

$$Q = 7.47 \text{ MeV}$$

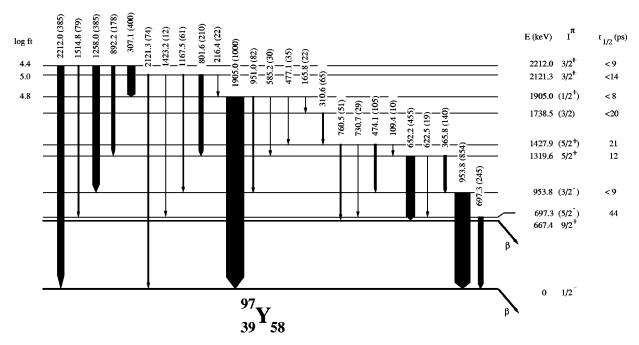


FIG. 3. Partial decay scheme of ⁹⁷Sr based on Refs. [10,27]. Only the levels discussed in the text are shown. The new transitions of 165.8 and 730.7 keV are included and some spin assignments, as discussed in the text, have been modified.

The 1044.5 keV level is a new level. The 575–469 keV cascade occurs, but in reverse order, in the level scheme of 99 Zr [15]. Nevertheless, there is no doubt for the placement of a level in 99 Nb since the 575 keV line is seen also in the 82 and 387 keV gates. The $\log ft$ value is 5.8. Thus, $I^{\pi} = 1/2^{+}$ and $3/2^{\pm}$ are possible, with even parity being more probable.

The two high-lying levels at 1974.6 and 2336.3 keV are rather weakly fed in β decay. It was mentioned that the present experiments cannot rule out the existence of high-energy transitions with intensities of the order of up to 1 relative intensity unit that would depopulate levels directly to the $1/2^-$ isomer. This means that the β feedings to the 1975 and 2336 keV levels shown in Table III are lower limits and some $1/2^+$, $3/2^+$ levels may have remained unobserved. Nevertheless, in spite of a possible shift of the distribution of β strength to higher excitation energies, it remains that the levels at 931, 959, and 1015 keV are very strongly fed directly in β decay.

C. New transitions and revised spin assignments in 97Y

The γ - γ data from our previous experiments [20,24] reveal the existence of two new transitions in the level scheme of 97 Y. A partial level scheme, relevant for the present discussion, is shown in Fig. 3. It is basically the one established by Pfeiffer *et al.* [27]. The level lifetimes later measured by Büscher *et al.* [10] provide a limit to the multipolarities of most of the transitions.

A new transition of 730.7(5) keV and I_{γ} =29(6), using the normalization I_{γ} (1905) = 1000 from Ref. [27], is placed between the 1428 and 697 keV levels, owing to a coinci-

dence with the 697.3 keV γ ray and coincidences between the 310.6 keV (1739 \rightarrow 1428) and 697.3 keV (697 \rightarrow g.s.) lines. According to previously available information the 1428 keV level could be a $5/2^+$ or a $7/2^+$ state and it was discussed as a $7/2^+$ state by Büscher *et al.* [10]. The new decay branch to the 697 keV $3/2^-$ state, however, results in the unique $I^{\pi}(1428) = 5/2^+$ assignment. We subsequently present arguments supporting the exchange of the low-lying $3/2^-$ and $5/2^-$ levels, but this does not modify this conclusion.

The other new 165.8(6) keV transition, with I_{γ} = 22(8), has coincidences with the γ rays of 307.1 keV (2012 \rightarrow 1905) and 310.6 keV (1739 \rightarrow 1428). Hence, it fits between the 1905 and 1739 keV levels. With the new assignments $I^{\pi}(1905) = 1/2^{+}$ (discussed in the following) and $I^{\pi}(1428) = 5/2^{+}$, and the dipole character of the connecting transitions, the 1739 keV level has I = 3/2. Unfortunately, the revised $\log ft$ value of 6.1 does not indicate the parity.

The β decay of $I^{\pi}=1/2^{+}$ 97Sr to 97Y has three strong branches, of clearly allowed character, to the levels at 1905, 2121, and 2212 keV. Pfeiffer *et al.* did not report a value for the first-forbidden β branch to the $1/2^{-}$ ground state [27]. Nevertheless, only a 5% branching is obtained assuming the logft value of 5.9, so that this is of little consequence on the strong branchings. The 2121 and 2212 keV levels, which decay to both $3/2^{-}$ and $5/2^{-}$ states are thus $3/2^{+}$ states. By analogy with the decay of 99 Zr to 99 Nb, it is tempting to assign $I^{\pi}=1/2^{+}$ to the other strongly β -fed level at 1905 keV. We note that $I^{\pi}=1/2^{+}$ was excluded by the transition of 477.1 keV to the 1428 keV level, as long as this latter was assigned $I^{\pi}=7/2^{+}$. The favored decay mode of the 1905 keV

level is to the 1/2 ground state while the transition to the 1320 keV $(I^{\pi} = 5/2^{+})$ level is rather weak. This looks very similar to the decay of the 1/2 + state at 959 keV in ⁹⁹Nb. An inconsistency is, however, the presence of a decay to the 954 keV level (I_{γ} =82) discussed as a 5/2 state by Büscher et al. [10] but not to the 697 keV (presumably 3/2 -) level. In our analysis the latter decay branch could not be found, the intensity being estimated to $I_{\gamma} < 30$. The inconsistency can be removed by exchanging the positions of the $3/2^-$ and 5/2 states, with respect to Ref. [10]. Then, the transition to the 954 keV level becomes an E1, whereas the nonobserved one to the 697 keV level would have to be a M2 (the singleparticle estimate corresponds to five units, which is still below the detection limit of the experiments). As a matter of fact, there is no strong experimental evidence for favoring one or the other assignment for the 697 and 954 keV levels. Pfeiffer et al. [27] favored the placement of the 5/2 level as the highest one and this also yields a reasonable E2 enhancement for the 954 keV 5/2⁻ to 1/2⁻ transition [10]. Nevertheless, a mere quadratic extrapolation of the excitation energies in the isotopes ⁹¹Y, ⁹³Y, and ⁹⁵Y [34] leads to values of 941 keV $(3/2^-)$ and 777 keV $(5/2^-)$, placing $5/2^-$ indeed below the 3/2⁻ level in ⁹⁷Y. A more meaningful argument is that the 954 keV level is more strongly populated by γ -ray cascades (854 intensity units) than the 697 keV level (245 units) [27]. It seems logical that, in the decay of the $I^{\pi} = 1/2^{+}$ level, a 3/2 level receives more feeding than a 5/2 level. To summarize, $I^{\pi}(1905) = 1/2^{+}$ is the most natural assignment for this level and the required exchange of the the $3/2^-$ and 5/2 - levels is not in conflict with the experimental data. This opens the possibility of finding even more analogies between the decays of ⁹⁷Sr and ⁹⁹Zr, which will be dicussed in the next section.

IV. CALCULATIONS OF LEVEL STRUCTURE OF 99Nb

A. Interacting boson-fermion plus broken-pair model calculation

In the systematic investigations of nuclei near the Z=40, N=56 subshell closures the nucleus $^{97}{\rm Y}$ has been recently investigated experimentally and described theoretically in the interacting boson-fermion plus broken-pair model (IBFBPM) [26]. Details about the interacting boson model and its successive extensions up to the inclusion of broken pairs can be found in Refs. [42–55]. In this paper we extend the IBFBPM treatment to the $^{99}{\rm Nb}$ nucleus, with two additional protons in comparison to $^{97}{\rm Y}$. The IBFBPM configuration space of an odd-even nucleus with 2N+1 valence nucleons comprises components of the type [26,55]

 $|N \ bosons \otimes 1 \ fermion \rangle$

$$+|(N-1)|$$
 bosons $\otimes 1$ broken pair $\otimes 1$ fermion \rangle . (4.1)

As in the calculation for ⁹⁷Y, we include broken neutron pairs. The boson-fermion basis states thus contain one-quasiproton-two-quasineutron states in addition to one-quasiproton states. The main difference in the IBFBPM parametrization for ⁹⁹Nb with respect to ⁹⁷Y is given by energies and occupation probabilities of quasiprotons, due to

the two additional protons in the valence shell. For ⁹⁹Nb the BCS occupation probabilities and quasiparticle energies for the $\pi \tilde{g}_{9/2}$, $\pi \tilde{p}_{1/2}$, $\pi \tilde{p}_{3/2}$, $\pi \tilde{f}_{5/2}$, $\pi \tilde{d}_{5/2}$ quasiparticles are $v^2 = 0.13, \ 0.92, \ 0.97, \ 0.98, \ 0.01$ and $E = 1.04, \ 1.37, \ 1.54,$ 1.85, 7.04 MeV, respectively. The main difference between quasiparticle states in ⁹⁹Nb and ⁹⁷Y is in the relative positions of the $\pi \tilde{g}_{9/2}$ and $\pi \tilde{p}_{1/2}$ quasiparticles: in $^{97}{
m Y}$ the $\pi \tilde{p}_{1/2}$ quasiparticle lies below $\pi \tilde{g}_{9/2}$, and in $^{99}{\rm Nb}~\pi \tilde{p}_{1/2}$ lies above $\pi \tilde{g}_{9/2}$. In analogy to the previous IBFBPM calculation for 97 Y [26], the $\pi \tilde{d}_{5/2}$ configuration from the shell above the valence shell is added because of the large non-spin-flip matrix element $\langle \pi d_{5/2} || Y_2 || \pi g_{9/2} \rangle$. All the other IBFBPM parameters are taken to be the same as in the previous calculation for ⁹⁷Y, except for small modifications of the boson parameter h_1 to 0.715 MeV and of the boson-fermion monopole interaction strength A_0^{π} for negative parity states to 0.06 MeV.

The IBFBPM Hamiltonian is diagonalized in the boson-fermion basis (4.1) and we obtain the energy spectra and the wave functions:

$$|J_{k}^{\pi}\rangle = \sum_{jn_{d}vR} \xi_{j,n_{d}vR;J} |\pi\widetilde{j},n_{d}vR;J\rangle$$

$$+ \sum_{jj'j''I_{\nu\nu}I_{\pi\nu\nu}n_{d}vR} \eta_{jj'j''I_{\nu\nu}I_{\pi\nu\nu},n_{d}vR;J}$$

$$\times |[\pi\widetilde{j},(\nu\widetilde{j}',\nu\widetilde{j}'')I_{\nu\nu}]I_{\pi\nu\nu},n_{d}vR;J\rangle. \tag{4.2}$$

Here $\pi \tilde{j}$ stands for proton quasiparticle, $\nu \tilde{j}'$ and $\nu \tilde{j}''$ for neutron quasiparticles. In the boson part of the basis states, the n_d d bosons are coupled to the total boson angular momentum R. The number of s bosons associated with the boson state $|n_d vR\rangle$ is $n_s = N - n_d$ where N is the total number of s and d bosons. In Fig. 4 we present the calculated energy spectrum of ⁹⁹Nb in comparison to the available data and Table IV displays the sizeable components in the wave functions of some states.

Using the IBFBPM wave functions we calculate the E2 and M1 electromagnetic properties of 99 Nb, taking the effective boson charge $e^{\text{vib}}=1$, proton gyromagnetic ratio $g_s^{\pi}=0.4$ $g_s^{\pi,\text{free}}$ and the other values of effective charges and gyromagnetic ratios as used in the previous IBFBPM calculation for 97 Y. The calculated E2 and M1 transitions for low-lying states are given in Table V and the half-lives in Table VI.

B. Microscopic quasiparticle phonon model calculation

The microscopic quasiparticle phonon model (MQPM) was introduced in Refs. [56,57]. The present calculation follows the procedure used for calculation of decay properties of 99 Nb and level structure of 99 Mo [21]. In our MQPM calculation we use essentially the Coulomb-corrected Woods-Saxon single-particle energies with the parametrization of Ref. [58]. The neutron and proton single-particle basis consist of the p-f and s-d-g oscillator major shells complemented by the intruder orbital $0h_{11/2}$ from the next oscillator major shell. The monopole matrix elements of the two-body interaction are scaled by pairing-strength param-

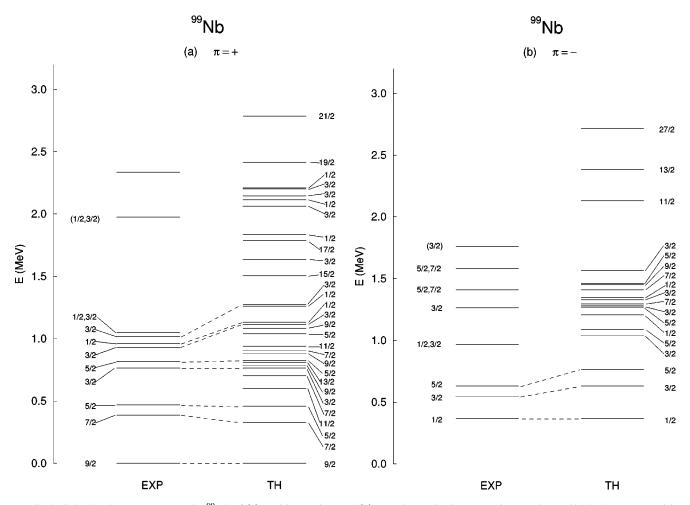


FIG. 4. Calculated IBFBPM states in 99 Nb of (a) positive parity, and (b) negative parity in comparison to the available data. For positive parity the states presented are: all states up to 1.1 MeV; all $1/2^+$, $3/2^+$ states and yrast states for $I^{\pi} \ge 15/2^+$ in the energy interval 1.1 - 2.2 MeV; and yrast states in the energy interval 2.2 - 2.8 MeV. For negative-parity the states presented are all states up to 1.6 MeV; and only yrast states in the energy interval 2.2 - 2.8 MeV.

eters [59], by comparison with semiempirical pairing gaps obtained from proton and neutron separation energies. The low-lying states of odd-A nuclei are composed of various quasiparticles coupled to the few lowest and most important excitations of an even-even core reference nucleus of mass A-1 constructed using the QRPA method of Ref. [60]. The truncation of three-quasiparticle model space is controlled by looking at the convergence of the odd-A spectrum versus the number of included RPA excitations. The odd nuclei ⁹⁹Zr and ⁹⁹Nb were constructed by, respectively, adding a neutron and a proton quasiparticle to the reference nucleus ⁹⁸Zr. This makes it possible to describe the β decay of 99 Zr to levels in ⁹⁹Nb. However, as in the case of the IBFBPM, the experimental excitation spectrum of 100Mo was used as the guide line when determining the phonon energies in the QRPA calculation. The results obtained for the $1/2^+$ and $3/2^+$ states in ⁹⁹Nb and the corresponding calculated logft values for the decay of the 1/2⁺ ground state of ⁹⁹Zr into these states are shown in Table VII.

V. DISCUSSION

A. Nuclear structure of 99Nb in IBFBPM

The region of $A \approx 100$ nuclei has been discussed in the framework of the interacting boson model in Refs. [9,13,17,22,25,26].

As already pointed out, the $\pi \tilde{g}_{9/2}$ and $\pi \tilde{p}_{1/2}$ quasiproton states exchange their positions between $^{97}\mathrm{Y}$ and $^{99}\mathrm{Nb}$. Thus, in $^{97}\mathrm{Y}$ the $1/2^-$ level is the ground state whereas the $9/2^+$ level is the isomeric state, while in $^{99}\mathrm{Nb}$ the situation is the opposite. The nature of these states is the same as in the previous calculation for $^{97}\mathrm{Y}$ [26], i.e., the largest components in the wave functions of the $9/2^+_1$ and $1/2^-_1$ states are the $\pi \tilde{g}_{9/2}$ and $\pi \tilde{p}_{1/2}$ quasiparticle components, respectively.

The next lowest-lying even parity states in 99 Nb are built on the $\pi \tilde{g}_{9/2} \otimes 2_1^+$ configuration, producing a multiplet with $I=5/2^+,\ldots,13/2^+$. In the IBFBPM calculation an effective core is used, lying between 100 Mo with $E(2_1^+)=535$ keV and 98 Zr with $E(2_1^+)=1223$ keV, but much closer to 100 Mo. Thus, the $7/2_1^+$ (387 keV) and the $5/2_1^+$ (469 keV) levels are the members of this multiplet with the lowest spins, in accordance with their strong population by γ -ray cascades after β -decay of the 99 Zr $1/2^+$ ground state. These states are based on the $\pi \tilde{g}_{9/2}$ -one-d-boson multiplets. From the present coincidence data, the existence of two additional levels at 363 and 379 keV has been speculated, having a link to the $7/2^+$ 387 keV level, which could have the I=9/2 and 11/2 assignments and therefore be associated with other members of the $\pi \tilde{g}_{9/2} \otimes 2_1^+$ multiplet. However, the remaining three

TABLE IV. Components in the IBFBPM wave functions of some low-lying states in $^{99}{\rm Nb}.$ Only the components with amplitudes larger than 1% are shown. Boson states with seniority v > $v_{\rm lowest}$ are denoted by n_d R'.

I_k^{π}	$\pi \tilde{j}$ or $[\pi \tilde{j}, (\nu \tilde{j}', \nu \tilde{j}'') I_{\nu \nu}] I_{\pi \nu \nu}$	n_d	R	ξ
9/2 +	$\pi g_{9/2}$	0	0	0.90
	$\pi g_{9/2}$	1	2	-0.39
7/2 +	$\pi g_{9/2}$	1	2	-0.91
	$\pi g_{9/2}$	0 1 1 2 2 0 0 2 1 2 1 2 2 3 1 2 2 3 1 2 2 1 2 2 1 2 0 1 2 1 2 1 2 0 1 2 0 1 2 0 1 2 0 1 2 0 1 2 0 1 2 0 1 2 0 1 2 0 1 2 0 1 2 0 1 2 0 1 2 0 1 2 0 1 2 0 1 2 0 1 2 2 0 1 2 2 0 1 2 2 0 1 2 2 2 3 0 1 2 2 2 2 2 3 0 1 2 2 2 2 2 3 0 1 2 2 2 2 2 3 0 1 2 2 2 2 3 0 1 2 2 2 2 2 3 0 1 2 2 2 2 2 3 0 1 2 2 2 3 0 1 2 2 2 2 2 3 0 1 2 2 2 2 3 0 1 2 2 2 2 2 3 0 1 2 2 2 2 2 2 2 2 2 2 2 2 3 3 0 1 2 2 2 2 2 2 2 3 0 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2	2	-0.14
	$\pi g_{9/2}$	2	4	0.27
	$[\pi g_{9/2}, (\nu s_{1/2}, \nu d_{5/2})2]7/2$	0	0	-0.15
1/2 _	$\pi p_{1/2}$	0	0	0.77
	$oldsymbol{\pi}p_{1/2}$	2	0	0.15
	$\pi p_{3/2}$	1	2	-0.40
	$\pi p_{3/2}$	2	2	-0.16
	$\pi f_{5/2}$	1	2	-0.39
	$m{\pi}f_{5/2}$	2	2	-0.15
5/2 +	$\pi g_{9/2}$	1	2	0.77
	$\pi g_{9/2}$	2	2	-0.56
	$\pi g_{9/2}$		4	0.16
	$\pi g_{9/2}$	3	3	-0.11
3/2 _	$\pi p_{1/2}$	1	2	0.49
	$m{\pi}p_{1/2}$	2	2	0.19
	$\pi p_{3/2}$	0	0	0.67
	$\pi p_{3/2}$	1	2	0.34
	$\pi p_{3/2}$		0	0.10
	$\pi p_{3/2}$	2	2	0.16
	$\pi f_{5/2}$	1	2	-0.16
	$\pi f_{5/2}$	2	4	-0.22
11/2 +	$\pi g_{9/2}$	1	2	0.85
	$\pi g_{9/2}$		2	0.38
	$oldsymbol{\pi} g_{9/2}$	2	4	-0.20
	$[\pi g_{9/2}, (\nu s_{1/2}, \nu d_{5/2})2]11/2$	0	0	0.18
5/2 _1	$\pi p_{1/2}$	1	2	0.58
	$m{\pi}p_{1/2}$	2	2	0.18
	$\pi p_{3/2}$		2	-0.18
	$\pi p_{3/2}$		4	-0.26
	$\pi p_{3/2}$	3	4	-0.12
	$oldsymbol{\pi} f_{5/2}$	0	0	-0.55
	$oldsymbol{\pi} f_{5/2}$		2	-0.27
	$m{\pi}f_{5/2}$		0	-0.10
	$m{\pi}f_{5/2}$		2	-0.19
	$oldsymbol{\pi} f_{5/2}$		4	-0.15
	$[\pi p_{1/2}, (\nu s_{1/2}, \nu d_{5/2})2]5/2$	0	0	0.10
3/2 +	$\pi g_{9/2}$		4	0.31
	$\pi g_{9/2}$		3	0.84
	$\pi g_{9/2}$	3	4	-0.24
	$\pi g_{9/2}$	4	4	0.15
	$\pi g_{9/2}$	4	5	-0.13
	$[\pi g_{9/2}, (\nu s_{1/2}, \nu d_{5/2})2]7/2$	2	2	-0.16

TABLE IV. (Continued).

I_k^{π}	$\pi\tilde{j}$ or $[\pi\tilde{j},(\nu\tilde{j}',\nu\tilde{j}'')I_{\nu\nu}]I_{\pi\nu\nu}$	n_d	R	ξ
9/2 +	$\pi g_{9/2}$	0	0	0.26
	$\pi g_{9/2}$	1	2	0.45
	$\pi g_{9/2}$	2	0	-0.13
	$\pi g_{9/2}$	2	2	0.59
	$\pi g_{9/2}$	3	0	-0.42
	$\pi g_{9/2}$	3	3	0.21
	$[\pi g_{9/2}, (\nu s_{1/2}, \nu d_{5/2})2]9/2$	0	0	0.14
	$[\pi g_{9/2}, (\nu s_{1/2}, \nu d_{5/2})2]11/2$	1	2	0.13
13/2 +	$\pi g_{9/2}$	1	2	-0.91
•	$\pi g_{9/2}$	2	2	0.21
	$\pi g_{9/2}$	2	4	0.26
	$[\pi g_{9/2}, (\nu s_{1/2}, \nu d_{5/2})2]13/2$	0	0	-0.16
5/2 +	$\pi g_{9/2}$	1	2	0.48
-	$\pi g_{9/2}$	2	2	0.47
	$\pi g_{9/2}$	2	4	-0.66
	$[\pi g_{9/2}, (\nu s_{1/2}, \nu d_{5/2})2]5/2$	0	0	0.11
	$[\pi g_{9/2}, (\nu s_{1/2}, \nu d_{5/2})2]7/2$	1	2	-0.15
3/2 +	$\pi g_{9/2}$	2	4	-0.49
-	$\pi g_{9/2}$	3	3	0.11
	$\pi g_{9/2}$	3	4	0.33
	$\pi g_{9/2}$	4	4	0.59
	$\pi g_{9/2}$	4	5	-0.42
	$[\pi g_{9/2}, (\nu s_{1/2}, \nu d_{5/2})2]7/2$	3	3	-0.11
1/2 +	$\pi g_{9/2}$	2	4	0.32
1	$\pi g_{9/2}$	3	4	-0.29
	$\pi g_{9/2}$	4	4	-0.11
	$\pi g_{9/2}$	4	4′	-0.73
	$\pi g_{9/2}$	4	5	0.41
	$[\pi g_{9/2}, (\nu s_{1/2}, \nu d_{5/2})2]7/2$	3	3	0.14
3/2 +	$\pi g_{9/2}$	2	4	0.67
3	$\pi g_{9/2}$	3	3	-0.39
	$\pi g_{9/2}$	4	4	0.40
	$\pi g_{9/2}$	4	5	-0.34
	$[\pi g_{9/2}, (\nu s_{1/2}, \nu d_{5/2})2]5/2$	1	2	0.15

calculated members of this multiplet, $11/2_1^+$, $9/2_2^+$, and $13/2_1^+$ lie at a few hundred keV higher in energy. Therefore, it seems more probable to assume that these two states, if they actually exist, are associated with intruder states. A $9/2^+$ level, although probably at a somewhat higher energy, could be produced by coupling the low-lying intruder 0^+ core state, at 854 keV in 98 Zr and and 695 keV in 100 Mo [34], to the $\tilde{g}_{9/2}$ proton. Therefore, other even-parity levels may exist in this energy range, but β decay from a $1/2^+$ state is not appropriate for their observation.

The weakly populated $3/2^+$ and $5/2^+$ levels at 765 and 817 keV are quite lower than the 2_2^+ and 4_1^+ levels in the core, but they are very close to the calculated $3/2_1^+$ and $5/2_2^+$ states, which are based on the $|\pi \tilde{g}_{9/2}, 33; 3/2\rangle$ and

TABLE V. Calculated E2 and M1 transitions between some low-lying states in comparison to data for 99 Nb.

$\overline{I_i \rightarrow I_f}$	$B(E2) (e^2b^2)$	$B(M1) (\mu_N^2)$		I_{γ}
$(\hbar)(\hbar)$	IBFBPM	IBFBPM	Expt.	IBFBPM
$7/2_1^+ \rightarrow 9/2_1^+$	0.0135	0.0842 a	100	100
$5/2_1^+ \rightarrow 7/2_1^+$	0.0131	0.1308	6	20
\rightarrow 9/2 $_{1}^{+}$	0.0224		100	100
$3/2_1^- \rightarrow 1/2_1^-$	0.0364	0.1036	100	100
$5/2_1^- \rightarrow 3/2_1^-$	0.0087	0.0046	100	100
$\rightarrow 1/2_1^-$	0.0402		b	1216
$3/2_1^+ \rightarrow 5/2_1^+$	0.0107	0.0022		100
\rightarrow 7/2 $_{1}^{+}$	0.0051			37
$5/2_2^+ \rightarrow 3/2_1^+$	0.0025	0.0451		0.02
\rightarrow 5/2 $_{1}^{+}$	0.0028	0.0103	21	2
\rightarrow 7/2 $_{1}^{+}$	0.0100	0.3585	100	100
\rightarrow 9/2 $_{1}^{+}$	0.0049		32	4
$3/2_2^+ \rightarrow 5/2_2^+$	0.0003	0.2802	2	29
\rightarrow 3/2 $_{1}^{+}$	0.0071	0.0006	0.05	0.2
\rightarrow 5/2 $_{1}^{+}$	0.0092	0.0135	100	100
\rightarrow 7/2 $_{1}^{+}$	0.0030		6	7
$1/2_1^+ \rightarrow 3/2_2^+$	0.0015	1.789 ^c	40	34
\rightarrow 5/2 $_2^+$	0.0034			0.1
\rightarrow 3/2 $_{1}^{+}$	0.0062	0.00004		1.2
\rightarrow 5/2 $_{1}^{+}$	0.0062		100 ^d	100
$3/2_3^+ \rightarrow 1/2_1^+$	0.000005	0.0574	5	0.2
\rightarrow 3/2 $_2^+$	0.00002	0.0425	0.2	0.4
\rightarrow 5/2 $_{2}^{+}$	0.0057	0.4363	0.07	58
\rightarrow 3/2 $_1^+$	0.0013	0.0971	0.05	26
\rightarrow 5/2 $_1^+$	0.0073	0.0348	100	100
\rightarrow 7/2 $_{1}^{+}$	0.0043		5	5

 $^{^{}a}$ The measured value for the 387 keV transition is 0.039(11) from Ref. [28].

 $|\pi \tilde{g}_{9/2},24;5/2\rangle$ components, respectively, as the largest components in the corresponding wave functions. The decay of the 765 keV level by a single transition to the $3/2^-$ level at 544 keV is difficult to be interpreted and this could mean that, e.g., the transition to the $1/2^-$ isomer was overlooked. We note that the decay pattern of the $5/2_2^+$ state, associated with the 817 keV level, to the $5/2_1^+$ and $7/2_1^+$ states, resembles the $\Delta n_d = 1$ pattern. The main components in the wave functions of the $5/2_2^+$, $5/2_1^+$, and $7/2_1^+$ states are in accordance with this expectation.

The odd-parity $3/2^-$ (544 keV) and $5/2^-$ (631 keV) levels are seen in proton pickup reactions [37], which indicates important amplitudes of the $p_{3/2}$ and $f_{5/2}$ proton holes instead of the $p_{1/2} \otimes 2^+$ couplings. The calculated low-lying negative-parity triplet states $1/2^-$, $3/2^-$, and $5/2^-$ are based on the $\pi \tilde{p}_{1/2}$, $\pi \tilde{p}_{3/2}$, and $\pi \tilde{f}_{5/2}$ quasiparticles, respectively. The hole-type character of the $3/2_1^-$ and $5/2_1^-$ states is in

TABLE VI. Calculated lifetimes of some states in comparison to data [28] for ⁹⁹Nb.

I_k^{π}	$T_{1/2}$	(ns)
(\hbar)	Expt.	IBFBPM
7/21+	0.017 (4)	0.008
5/21+	0.173 (4)	0.088
$3/2_{1}^{-}$	0.56 (10)	0.063
5/21		0.966 ^a
3/21+		≪ 0.39 ^b
5/2+		0.001
$3/2_{2}^{+}$	< 0.010	0.019
$1/2_{1}^{2}$	< 0.010	0.005 ^c
$3/2_{3}^{+}$	< 0.005	0.004

^aThe half-life is reduced to 0.22 ns by including a tensor term in the M1 operator (see text).

^bThis is an estimate for the upper limit, because only the E1 transition to the $3/2_1^-$ state was observed, although a strong branch to the $1/2_1^-$ level is predicted.

^cObtained by assuming that 94% of the decay intensity goes to odd-parity states.

accordance with the proton pickup data. The same interpretation is made for the $3/2^-$ and $5/2^-$ levels in $^{97}\mathrm{Y}$ after exchanging their positions (see Sec. V C). In the IBFBPM calculation, for realistic parametrizations, we cannot obtain the lowest $3/2^-$ and $5/2^-$ states being dominated by $p_{1/2}\otimes 2^+$ core+particle coupling that was proposed in Ref. [10].

The agreement between the calculated and experimental electromagnetic transitions presented in Table V is reasonable. We note that the decay of the $5/2_1^-$ state requires additional clarification regarding the strongest calculated branch in comparison to experiment. Namely, the calculated $5/2_1$ $\rightarrow 1/2_1^-$ transition is an order of magnitude stronger than the calculated $5/2_1^- \rightarrow 3/2_1^-$ transition, while experimentally there is no information available on the $5/2_1^- \rightarrow 1/2_1^-$ transition. However, it was mentioned previously that the limit of detection for this kind of transition, as a result of the isomeric character of the 1/2 level, is rather high. Thus, we cannot exclude the possibility that the experimental $5/2^-_1 \rightarrow 1/2^-_1$ transition is present and even stronger than the $5/2_1^- \rightarrow 3/2_1^$ transition. Furthermore, we note that the $5/2_1^- \rightarrow 3/2_1^-$ transition is l forbidden in the leading order, leading to a small B(M1) value. Therefore, a sizable contribution may be due to the tensor term in the M1 operator, which was previously included in some IBFM calculations [61,62]. Since the contribution of broken pairs to these two transitions is small, we can estimate a possible influence of the tensor term by performing the standard IBFM calculation with the gyromagnetic ratio $g_T^{\pi} = \frac{1}{20} g_s^{\pi, \text{free}} \langle r^2 \rangle = 5.164$, which gives B(M1) $(5/2_1^- \rightarrow 3/2_1^-) = 0.17 \mu_N^2$. In this case the intensity ratio of the $5/2_1^- \rightarrow 3/2_1^-$ and $5/2_1^- \rightarrow 1/2_1^-$ transitions is 100/33. Thus, by including the tensor term in the M1 operator the two branches depopulating the $5/2_1^-$ state become comparable, which comes closer to the experimental branchings.

We note that the largest calculated B(M1) value is obtained for the $1/2_1^+ \rightarrow 3/2_2^+$ transition. This is in accordance with the strong branch observed for the 28 keV transition in

^bIt is possible that the transition even with such a large branch is not seen experimentally (see text).

^cThe measured value for the 28 keV transition is $> 0.7 \mu_N^2$ from Ref. [28].

^dIt should be noted that the transition of 490 keV represents only 2 and 12% of the strengths of the E1 transitions to the $1/2_1^-$ and $3/2_1^-$ states, respectively.

TABLE VII. Structure of the $1/2^+$ and $3/2^+$ states of 99 Nb and $\log ft$ values for population in decay of the $1/2^+$ ground state of 99 Zr calculated in the MQPM. For comparison with the IBFBPM the $\tilde{g}_{9/2} \otimes 2_1^+$ multiplet is also shown. The first four largest components are listed unless smaller than 1%.

I_k^{π}	E(MeV)	$\log ft$	Structure			
9/21+	0.00		$0.93 \ \widetilde{\widetilde{g}}_{9/2}$	$-0.31 \ \tilde{g}_{9/2} \otimes 2_1^+$	$-0.13 \ \widetilde{g}_{9/2} \otimes 4_1^+$	$+ 0.03 \ \tilde{d}_{5/2} \otimes 2_1^+$
$7/2_{1}^{+}$	0.38		$0.98 \ \widetilde{g}_{9/2} \otimes 2_{1}^{+}$	$+ 0.10 \ \tilde{g}_{9/2} \otimes 4_1^+$	$+ 0.02 \ \widetilde{g}_{9/2} \otimes 4_2^+$	$-0.02 \ \widetilde{p}_{1/2} \otimes 3_{1}^{-}$
5/21+	0.46		$0.99 \ \tilde{g}_{9/2} \otimes 2_1^+$	+ 0.07 $\tilde{p}_{1/2} \otimes 3_1^-$	$-0.06\ \tilde{g}_{9/2} \otimes 4_1^+$	$+ 0.04 \ \widetilde{d}_{5/2}$
$13/2_1^+$	0.49		$1.00 \ \widetilde{g}_{9/2} \otimes 2_{1}^{+}$	$-0.07\ \widetilde{g}_{9/2} \otimes 4_1^+$	$-0.02\ \widetilde{g}_{9/2}\otimes 6_1^+$	$-0.02\ \widetilde{g}_{9/2} \otimes 6_2^+$
$9/2_{2}^{+}$	0.52		$0.95 \ \widetilde{g}_{9/2} \otimes 2_{1}^{+}$	$+ 0.32 \ \tilde{g}_{9/2}$	$+ 0.03 \ \widetilde{g}_{9/2} \otimes 4_1^+$	$+ 0.02 \ \widetilde{g}_{9/2} \otimes 6_1^+$
$11/2_1^+$	0.53		$1.00 \ \widetilde{g}_{9/2} \otimes 2_{1}^{+}$	$-0.11\ \tilde{g}_{9/2}\otimes 4_1^+$	$-0.03\ \widetilde{g}_{9/2} \otimes 4_2^+$	$+ 0.01 \; \tilde{g}_{9/2} \otimes 4_3^+$
$3/2_{1}^{+}$	0.95	5.0	$0.98 \ \widetilde{g}_{9/2} \otimes 4_1^+$	+ $0.06 \ \widetilde{g}_{9/2} \otimes 4_2^+$	$+ 0.03 \tilde{f}_{5/2} \otimes 3_1^-$	$-0.03 \ \widetilde{g}_{9/2} \otimes 4_3^+$
$1/2_{1}^{+}$	1.04	4.1	$1.00 \ \tilde{g}_{9/2} \otimes 4_1^+$	$-0.10\ \tilde{g}_{9/2} \otimes 4_2^+$	$-0.05 \ \tilde{s}_{1/2}$	$-0.04 \ \tilde{f}_{5/2} \otimes 3_{1}^{-}$
$3/2_{2}^{+}$	1.35	4.5	$0.99 \ \widetilde{g}_{9/2} \otimes 4_2^+$	$-0.07\ \widetilde{g}_{9/2} \otimes 4_1^+$	$-0.04\ \widetilde{g}_{9/2}\otimes 6_1^+$	$-0.02\ \widetilde{g}_{9/2} \otimes 6_2^+$
$1/2_{2}^{+}$	1.37	4.4	$1.00 \ \tilde{g}_{9/2} \otimes 4_{2}^{+}$	$+ 0.10 \ \tilde{g}_{9/2} \otimes 4_1^+$	$-0.02\ \tilde{f}_{5/2} \otimes 3_{1}^{-}$	$+ 0.02 \ \widetilde{g}_{9/2} \otimes 4_3^+$
3/23+	1.44	3.9	$1.00 \ \widetilde{g}_{9/2} \otimes 3_1^+$			

spite of its small energy. Experimentally, this B(M1) value is estimated to $>0.7\mu_N^2$.

The levels which collect the bulk of β -decay intensity are the 1/2 $^+$ (959 keV) and the two 3/2 $^+$ states (at 931 and 1015 keV). These levels will be discussed in Sec. V D.

B. Nuclear structure of ⁹⁹Nb in the MQPM

The even-parity levels, which are those relevant for the allowed transitions, are well reproduced. As in the IBFBPM, in the MQPM the leading components of the $9/2_1^+$, $1/2_1^-$, $7/2_1^+$, $5/2_1^+$, $11/2_1^+$, $9/2_2^+$ and $13/2_1^+$ are the $\pi \tilde{g}_{9/2}$ and $\pi \tilde{p}_{1/2}$ quasiparticle states and the $\pi \tilde{g}_{9/2} \otimes 2_1^+$ multiplet, respectively. The lowest spin members of the multiplet, $7/2_1^+$, $5/2_1^+$, are at the same position as in the IBFBPM but the other members of the multiplet are in a different order (see Fig. 4 and Table VII). In the MQPM calculation the $3/2_1^+$ and $5/2_2^+$ levels belong to the $g_{9/2} \otimes 4_1^+$ multiplet. This configuration was also obtained in the IBFBPM calculation for the 817 keV $5/2^+$ level, but it does not seem that the 765 keV level could be identified with this 3/2⁺ state. The states with strong β -decay feedings are some 400 keV too high in the MQPM. They will be discussed in Sec. V D. Finally, the $3/2_1^-$ and $5/2_1^-$ negative-parity states are predicted to be based on the $p_{1/2} \otimes 2_1^+$ doublet in disagreement with the proton pickup data [37]. This follows from the fact that the $p_{1/2}$ one-quasiparticle state is far too low in energy, leading to an unperturbed energy of the $p_{1/2} \otimes 2_1^+$ doublet, that is too low.

C. Comparison of levels in the N = 58 isotones 97 Y and 99 Nb

The singularity of the $d_{5/2}$ subshell closure, which enlarges the level gaps in $^{97}{\rm Nb}$ with 56 neutrons, restricts the possibility for a simple extrapolation of levels to $^{99}{\rm Nb}$. Nevertheless, Fig. 5 shows a smooth evolution of the level sequences, since only the closely lying $5/2^+$ and $3/2^-$ levels change their position between $^{97}{\rm Nb}$ and $^{99}{\rm Nb}$. The systematics of spherical levels ends at $^{99}{\rm Nb}$ since, like their lower-Z neighboring nuclei, Nb isotopes experience the onset of deformation at N=60 with $^{101}{\rm Nb}$ [7].

Alternatively, comparison of levels of N=58 odd-proton nuclei versus a change of proton number by an increment of two is of interest in this particular case. The N=59 parent nuclei of 99 Nb neighbors have the same spin and parity of $I^{\pi}=1/2^{+}$. Consequently, one can expect that their β decay will populate levels of same spins and parities in their respective daughter nuclei 97 Y, 99 Nb, and 101 Tc. A comparison of 99 Zr to 99 Nb and 101 Mo to 101 Tc decays is, however, not very meaningful due to the opening of the neutron shell at Z>40 [21,37] which changes the nature of the levels in 101 Mo [22] and of the onset of deformation in its daughter 101 Tc [63]. In contrast, a close correspondance is found between the levels in 99 Nb and 97 Y shown on the right-hand side of Fig. 5.

The $9/2^+$ and $1/2^-$ single proton quasiparticle levels exchange their positions from 97 Y to 99 Nb, due to the filling of the $p_{1/2}$ subshell at Z=40.

The next 97Y levels at 697 and 954 keV are 3/2 and 5/2 states, where we assume the 697 keV level to be the 5/2 state. This makes ⁹⁷Y the only nucleus among Y and Nb isotopes with N>50 where the $5/2^-$ state is the lowest one. Büscher et al. [10] discussed the 954 keV state as being the $5/2^-$ state from the p_{1/2} $\otimes 2^+$ coupling. They based their arguments on the enhancement > 6.5 if E2 multipolarity for the 954 keV transition is assumed, a value consistent with the rates of the $2^+ \rightarrow 0^+$ transitions in the even-even neighbors. Nevertheless, admixtures of single-particle configurations could lead to a smaller collectivity consistent with the value of 2.9 single-particle units if the 697 keV level is the 5/2 - level. As a matter of fact, transfer reactions of Flynn et al. [37] show the lowest-lying 3/2 (544 keV) and 5/2 (631 keV) levels in ⁹⁹Nb to be mainly of the $p_{3/2}$ and $f_{5/2}$ hole character. There is no obvious reason why these levels would become more collective in ⁹⁷Y.

The state at 1320 keV in 97 Y has I^{π} = 5/2⁺ [29]. The 652 keV transition to the 9/2⁺ isomer has a rate of 11(4) single-particle units [10]. This value compares well with the 4.8 rate of the 469 keV transition in 99 Nb. Therefore, the 1320 keV level in 97 Y and the 469 keV level in 99 Nb are both built on the $g_{9/2} \otimes 2^+$ configuration. This interpretation was

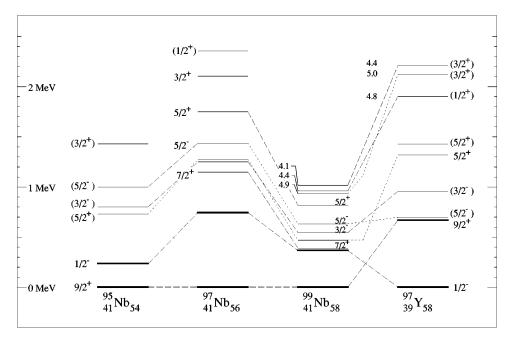


FIG. 5. Systematics of selected low-lying levels in Nb isotopes and in 97 Y, the isotone of 99 Nb. Dashed and dotted lines connect levels of even and odd parity, respectively. The picture displays similarities of the isotones 99 Nb and 97 Y in spite of the shell effects being larger in yttrium than in niobium isotopes and of the exchange of the positions of the $\tilde{p}_{1/2}$ and $\tilde{g}_{9/2}$ quasiparticles.

also proposed in Refs. [10,28].

The level at 1428 keV in 97Y was discussed by Büscher et al. [10] as the $7/2^+$ level belonging to the $g_{9/2} \otimes 2^+$ coupling, corresponding to the 387 keV level in ⁹⁹Nb. However, they noticed the unusually small rate of the E2 transition (even when assuming no M1 component) to the 9/2 + singleparticle state. According to our new data, this level is a 5/2 ⁺ state. It seems impossible in the spherical model space to create another $5/2^+$ level so close to the $5/2^+$ state at 1320 keV. Hence, it could be that the 1428 keV level is the head of a band corresponding to a deformed potential minimum. Firstly, the excitation energy for the $5/2^+$ level in 97 Y is close to the 1436 keV energy of the deformed 0_3^+ state in 98 Zr [15]. Secondly, $I^{\pi} = 5/2^+$ is the probable spin and parity of the lowest band built on a deformed minimum in 97Y since the [422]5/2 Nilsson orbital is the ground state of the deformed ⁹⁹Y [64,65], ¹⁰¹Y [66], and ¹⁰³Y [67] isotopes. Continuing this interpretation, one can find a number of levels, with weak decay branchings to the low-lying levels and energy spacings ressembling those of rotational levels in the heavier Y isotopes. The 3/2 level (of unknown parity) at 1739 keV is indeed a reasonable candidate for the [301]3/2 band head. The 1613 and 1848 keV levels could be very tentatively 5/2 states corresponding to the [303]5/2 band head and the member of the [301]3/2 band, respectively. However, this appealing interpretation of the 1428 keV level seems at variance with the rate >4 (using the half-life upper limit of 8 ps at 3 standard deviations [10]) of the E2 transition from the 1905 keV level to the state at 1428 keV, since a hindrance is generally observed for transitions between different shapes. At least, it would be worthwhile to check the above interpretation by searching for a band structure on top of the 1428 keV level. In ⁹⁹Nb the second 5/2 + state is the 817 keV level for which a two-phonon coupled state seems a more probable configuration (see Sec. V A).

Finally, among the highest-lying levels observed in β decay, both in ⁹⁹Nb and ⁹⁷Y, there are three levels with very strong direct β feedings which should include a large amplitude of the $\pi g_{9/2} \otimes (\nu g_{7/2} \otimes \nu s_{1/2})_{3,4}$ configuration. This can form one 1/2 + and two 3/2 + levels in agreement with our proposed assignments. The $I^{\pi}=3/2^{+}$ levels, at 931 keV (99Nb) and at 2121 keV (97Y), present similar decay properties. Both have a weak branch to the 1/2 - level (even not observed in ⁹⁹Nb), but a strong branch to the lowest lying $(g_{9/2} \otimes 2^+)_{5/2^+}$ level. The other $I^{\pi} = 3/2^+$ levels, at 1015 keV in ⁹⁹Nb and 2212 keV in ⁹⁷Y, have a similar decay pattern, since they decay to almost all levels which can be reached via dipole or electric quadrupole transitions, and also have the lowest logft values (4.4 and 4.1, respectively). On the basis of analogy we have proposed the $I^{\pi} = 1/2^{+}$ assignment for the 1905 keV level in ⁹⁷Y, corresponding to the 959 keV level in ⁹⁹Nb. For both E2 transitions to the $(g_{9/2} \otimes 2^+)_{5/2^+}$ levels a rate larger than 1.2 s.p. units (using the three σ limit for ⁹⁷Y) is deduced. The branching to the possibly deformed 1428 keV level in ⁹⁷Y has no counterpart in ⁹⁹Nb, probably due to the lower energy of the $1/2^+$ level in the latter nucleus. With the new 3/2 - assignment of the 954 keV level in ${}^{97}\text{Y}$ the ratio $B(E1,1/2^+ \rightarrow 3/2^-)/B(E1,1/2^+ \rightarrow 1/2^-)$ is equal to 0.52 for ⁹⁹Nb and 0.66 for ⁹⁷Y. These values are close to each other. As a conclusion of this comparison it turns out that β feedings and γ -decay rates of corresponding levels in ⁹⁷Y and ⁹⁹Nb are very similar.

D. β -decay properties

The N=59 isotones 97 Sr and 99 Zr have the 1/2 $^+$ ground state associated with the $s_{1/2}$ neutron. The two other valence neutrons (outside of the $d_{5/2}$ subshell) mainly occupy the $g_{7/2}$ orbital. Thus, β decays of Sr and Zr parents can be explained by the Gamow-Teller transition of one of the $g_{7/2}$ paired

neutrons, leading to the $\pi \tilde{g}_{9/2} \otimes (\nu \tilde{g}_{7/2} \otimes \nu \tilde{s}_{1/2})_{3,4}$ configuration in daughter nuclei, resulting in three strongly fed states $(I^{\pi}=1/2^+, 3/2^+, 3/2^+)$. The N=57 isotones ⁹⁵Sr [19] and 97 Zr [20] also have a 1/2 $^+$ ground state. In the absence of a strong N = 56 neutron shell closure, the occupation probability of the $g_{7/2}$ neutron orbital should vary smoothly versus neutron number and the β -decay pattern of the N=57 and N=59 isotones should be quite similar, except for some slowing down of the transitions for N = 57. In contrast, experiments show dramatic differences. In the decays of the N=57 isotones ⁹⁵Sr and ⁹⁷Zr, allowed transitions are slower by at least one order of magnitude [34]. In the decay of ⁹⁵Sr to 95 Y, the lowest $\log ft$ values are 6.1 and 5.7 for two levels at 2717 and 2933 keV, respectively, and in the decay of 97Zr to ⁹⁷Nb the lowest logft value is 6.5 for the 2106 keV level. As a matter of fact, the strongest β branchings in terms of intensity are due to first-forbidden decays of the $\nu s_{1/2}$ $\rightarrow \pi p_{1/2}$ type. The reduction of allowed transition rates suggests a dramatic reduction of the occupation of the $g_{7/2}$ neutron pair, which can be regarded as a further evidence for the shell gap at N = 56.

In the IBFM wave functions of the $1/2^+$ ground states of 99 Zr and 97 Sr, the dominant component is the $\nu \tilde{s}_{1/2}$ quasineutron state [9,17,61], while in the IBFBPM wave functions of the 1/2⁺ and 3/2⁺ states around 1 MeV of excitation energy in 99Nb the largest components are the $|\pi \tilde{g}_{9/2}, 24; 1/2\rangle$, $|\pi \tilde{g}_{9/2}, 24; 3/2\rangle$, $|\pi \tilde{g}_{9/2}, 33; 3/2\rangle$, $|\pi \tilde{g}_{9/2}, 44; 1/2\rangle, |\pi \tilde{g}_{9/2}, 44; 3/2\rangle, \text{ i.e., the two-, three-, and}$ four-d-boson components, respectively. As shown in Ref. [9], there are sizeable admixtures of the $|\nu \tilde{g}_{7/2}, 24; 1/2\rangle$ component in the ground states of ⁹⁹Zr and ⁹⁷Sr. This component leads to the population of the main components in the 1/2⁺ and 3/2⁺ states in ⁹⁹Nb and ⁹⁷Y by Gamow-Teller decay of the $\tilde{g}_{7/2}$ neutron to the $\tilde{g}_{9/2}$ proton. The other sizeable admixture in the wave function of parent nuclei is the $|\nu \tilde{d}_{3/2}, 12; 1/2\rangle$ component, which leads in β decay to the population of the $|\pi \tilde{d}_{5/2}, 12; 1/2\rangle$ and $|\pi \tilde{d}_{5/2}, 12; 3/2\rangle$ admixtures in the wave functions of daughter nuclei.

Another contribution arises when one s boson in the main component $|\nu \tilde{s}_{1/2},00;1/2\rangle$ of the $1/2^+$ parent state undergoes a Gamow-Teller decay, in which the $(\nu \tilde{g}_{7/2})_0^2$ pair in the internal structure of the s boson is transformed into the $(\pi \tilde{g}_{9/2} \nu \tilde{g}_{7/2})_1$ configuration. This leads to the population of $\pi \tilde{g}_{9/2} \nu \tilde{g}_{7/2} \nu \tilde{s}_{1/2}$ three-quasiparticle configurations in the intrinsic structure of the $1/2^+$ and $3/2^+$ states of daughter nuclei. In other words, the β decay of 99 Zr populates the $(\nu \tilde{g}_{7/2}, \nu \tilde{s}_{1/2})3$ and $(\nu \tilde{g}_{7/2}, \nu \tilde{s}_{1/2})4$ components of the internal structure of boson states contained in the IBFBPM wave functions of $1/2^+$ and $3/2^+$ states of 99 Nb.

In the MQPM, for the core nucleus all the relevant states, 4_1^+ , 3_1^+ and 4_2^+ , have large $\nu \tilde{g}_{7/2} \nu \tilde{s}_{1/2}$ components in their internal structure (Table VIII). There are five $1/2^+$ and $3/2^+$ states based on these core states coupled to the $\pi \tilde{g}_{9/2}$ quasiproton, namely the two $1/2^+$ ($\pi \tilde{g}_{9/2} \otimes 4_1^+$, $\pi \tilde{g}_{9/2} \otimes 4_2^+$) and three $3/2^+$ states ($\pi \tilde{g}_{9/2} \otimes 4_1^+$, $\pi \tilde{g}_{9/2} \otimes 4_2^+$, $\pi \tilde{g}_{9/2} \otimes 3_1^+$). All these five states have large $\pi \tilde{g}_{9/2} \nu \tilde{g}_{7/2} \nu \tilde{s}_{1/2}$ components and are predicted to be sizably populated in β decay of the $1/2^+$

TABLE VIII. Two-quasineutron internal structure of the 4_1^+ , 3_1^+ , and 4_2^+ states of the reference nucleus $^{98}\mathrm{Zr}$ calculated in the MOPM.

	4 ₁ ⁺ 0.92 MeV	3 ₁ ⁺ 1.32 MeV	4 ₂ ⁺ 1.25 MeV
$\nu \widetilde{g}_{7/2} \nu \widetilde{s}_{1/2}$	-0.63	-0.99	0.74
$ u \widetilde{\widetilde{g}}_{7/2} u \widetilde{d}_{3/2}$	0.24	-0.02	0.01
$ u \widetilde{g}_{7/2} u \widetilde{d}_{5/2}$	0.18	-0.01	0.04
$\nu \widetilde{g}_{7/2} \nu \widetilde{g}_{7/2}$	-0.64	0	-0.66
$ u \widetilde{d}_{5/2} u \widetilde{d}_{3/2}$	0.23	-0.02	0.06
$\nu \tilde{d}_{5/2} \nu \tilde{s}_{1/2}$	0	0.05	0

ground state of 99 Zr. Additional small contributions to the β decay arise due to small admixtures to the dominant $\nu \tilde{s}_{1/2} \otimes 0_1^+$ component in the MQPM wave function of the $1/2^+$ ground state of 99 Zr: $0.177\nu \tilde{d}_{5/2}\otimes 2_1^+$ and $-0.52\nu \tilde{g}_{7/2}\otimes 4_1^+$.

We note that the 4_1^+ state of the MQPM corresponds to the n_d =2 state of the IBFBPM, and 3_1^+ , 4_2^+ states of the MQPM to the n_d =3 states. In this sense we find a qualitative analogy between the MQPM and IBFBPM calculations, only the mixing of the n_d =2 and n_d =3 IBM boson states is stronger for the IBFBPM. However, in both calculations we encounter the same problem: all three calculated low-lying $3/2^+$ states are sizeably populated in β decay, while experimentally one of these states (the lowest one at 765 keV) is weakly populated. A similar problem appears for one of the two $1/2^+$ states.

Concluding, the calculations provide five $1/2^+$, $3/2^+$ states having sizable $\pi \tilde{g}_{9/2} \nu \tilde{g}_{7/2} \nu \tilde{s}_{1/2}$ components in their internal structure, arising from five basis states, leading to sizable population in β decay. Since there is no way to reduce the number of relevant basis states from five to three, we conclude that an additional mixing in the wave functions is needed in order to produce destructive interference among the contributions from the $n_d = 2$ and $n_d = 3$ components to β -decay matrix elements in one $3/2^+$ and one $1/2^+$ state of ⁹⁹Nb. In the present calculation we have not succeeded in simulating such a destructive interference.

VI. SEARCH FOR A HIGH-SPIN ISOMER IN 99Nb

Several experimental [24] and theoretical works [25,26] have been devoted to levels in $^{97}\mathrm{Y}$ populated by the E3 isomeric decay of its $(\pi \tilde{g}_{9/2} \otimes \nu \tilde{g}_{7/2} \otimes \nu \tilde{h}_{11/2})_{27/2^-}$ three-quasiparticle isomer. Especially interesting is the rare opportunity to observe high-spin states generated by a $g_{9/2}$ proton coupled to core excitations or a broken neutron pair. In these states the odd proton has been lifted from the $p_{1/2}$ to the $g_{9/2}$ shell, leaving the $p_{1/2}$ orbital empty. Similar states could be formed in $^{99}\mathrm{Nb}$ by simply filling the $p_{1/2}$ shell with the two extra protons paired to $I^{\pi} = 0^+$. Experimental evidence for a three-quasiparticle isomer in $^{99}\mathrm{Nb}$ was first searched via its β decay to the known yrast levels in $^{99}\mathrm{Mo}$ [21]. This experiment was not successful but did not yet exclude the possibility of the alternative isomeric decay branching to levels in $^{99}\mathrm{Nb}$.

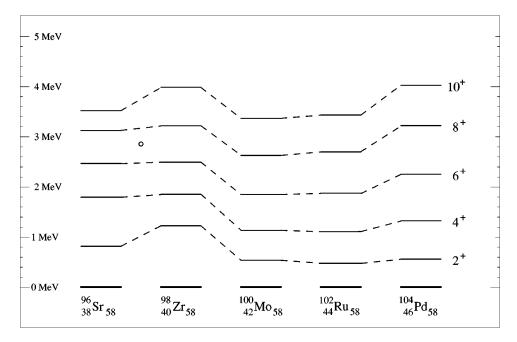


FIG. 6. Yrast levels in the N=58 neighboring isotones of 99 Nb. The distance between the $(\tilde{g}_{7/2} \otimes \tilde{h}_{11/2})_{9^-}$ broken neutron pair (the estimate for 97 Y is shown by an open circle) and the 6^+ yrast level is crucial for the existence of an isomer.

Fingerprints of such a decay are the presence of a strongly converted transition, which can be assigned by the characteristic Nb K-x rays, followed by a cascade of γ rays of comparable intensities and whose energies remind of the spacings between the yrast levels of the even-even core. A sequence of γ rays with these characteristics was searched in the γ - γ and x- γ matrices. In addition, matrices of three- and four-fold events were analyzed, since higher coincidence folds should emphasize the γ rays belonging to cascades of large multiplicity. However, no evidence for the searched patterns was found. Assuming, by analogy with the 97Y data [20], that the independent population of the high-spin isomer is approximately 3% of the total independent Nb population, then about 70 ions must be transported per second to the collection spot. The limit for the detection of a cascade of 500 keV γ rays in this experiment is estimated to 7 ions/sec. Hence, we must conclude that the postulated high-spin configuration in ⁹⁹Nb is not an isomer, the half-life of which is long enough to survive the time (≈ 1 ms) between its production and the measurement.

The isomeric E3 transition corresponds to a neutron transition in which the 9^- broken pair changes to 6^+ . The energies of $(\tilde{g}_{7/2} \otimes \tilde{h}_{11/2})_{9^-}$ broken neutron pairs in the neighboring even-even isotopes are unknown. A crude estimate can be obtained from ^{97}Y by subtracting the experimental $\pi g_{9/2}$ energy of 668 keV from the energy of 3523 keV of the $(\pi \tilde{g}_{9/2} \otimes \nu \tilde{g}_{7/2} \otimes \nu \tilde{h}_{11/2})_{27/2^-}$ configuration. Figure 6 shows the yrast levels in N=58 isotones [11,34]. The estimated 9^- broken pair energy is well below the yrast 8^+ levels in both neighboring core nuclei, and the only possible electromagnetic decay is the E3 transition to the yrast 6^+ level. The experimental $E_{\gamma}(27/2^- \rightarrow 21/2^+)$ energy of 162 keV is even a bit lower than the unperturbed 9^- to 6^+ energy difference of 376 keV (taking the average 6^+ state energies in 96 Sr and 98 Zr), probably due to the coupling of these core states with the $g_{9/2}$ proton. The even-even neighbors for 99 Nb are 98 Zr

and 100 Mo. From the discussion of level structure the latter seems a more appropriate core in which the yrast levels become lower. Thus, with the same assumptions as above, the 9^- to 6^+ energy gap increases and the experimental $E_{\gamma}(27/2^- \rightarrow 21/2^+)$ transition energy should be about 470 keV. For an E3 transition with the same rate characteristics as the one in 97 Y, the estimated lifetime of 0.17 ms is indeed too short to allow observation of the isomer after mass separation. In the present calculation we obtain the $27/2^-$ three-quasiparticle configuration at 2.72 MeV above the $\pi \tilde{g}_{9/2}$ 99 Nb ground state. This is in accordance with the conclusion of the too high energy of the E3 transition to the $[\pi \tilde{g}_{9/2} \otimes 6^+]21/2^+$ level being responsible for the nonobservation of the $27/2^-$ level.

The ultimate condition for isomerism is that the 9^- level should be below the yrast 8^+ level in order to prevent its E1 decay. This gives a maximum energy of 753 keV for the E3 transition, which translates into a half-life of 6 μ s. Therefore, there still should be a fair chance to observe the $27/2^-$ three-quasiparticle configuration in 99 Nb at a recoil separator.

VII. CONCLUSION

The decay schemes of 99 Zr to 99 Nb and of 97 Sr to 97 Y have been reinvestigated. The former now includes several transitions with absolute decay branchings lower than 10^{-3} . A few additional transitions have been found in the latter that allow a new interpretation of some levels. Several level spin and parity assignments have been revised; $I^{\pi}=3/2^+$ for the 931 keV level in 99 Nb and $I^{\pi}=5/2^+$ for the 1428 keV level in 97 Y. The levels at 959 keV in 99 Nb and 1905 keV in 97 Y are proposed to be $1/2^+$ states. This requires the 697 and 954 keV levels in 97 Y to be the $5/2^-$ and $3/2^-$ states, respectively, at variance with previously made assignments. In spite of the compression of level energies in 99 Nb with re-

spect to ${}^{97}\mathrm{Y}$, several levels of the same character are observed. The $5/2^+$ levels at 1320 keV in ${}^{97}\mathrm{Y}$ and 469 keV in ${}^{99}\mathrm{Nb}$ are both built on the $g_{9/2}\otimes 2^+$ configuration. New similarities emerge for three strongly populated levels, two firmly assigned $3/2^+$ states and a probable $1/2^+$ level, which have comparable $\log ft$ values and decay rates in both nuclei. This indicates that the N=56 shell closure is still active for ${}^{99}\mathrm{Nb}$, in contrast to its vanishing with only two extra protons giving a deformed character to ${}^{101}\mathrm{Tc}$.

The observation of high-spin states in ⁹⁹Nb has so far been impossible due to the lack of evidence for a three-particle 27/2 state in decay experiments. The too short level lifetime is interpreted as a consequence of the increase of the softness of the ¹⁰⁰Mo core with respect to the ⁹⁶Sr and ⁹⁸Zr neighbors.

Finally, it is tempting to assign the $I^{\pi} = 5/2^{+}$ 1428 keV

level in 97 Y as the [422]5/2 head of a deformed band. Confirmation of this assumption would put the picture of shape coexistence in the N=58-60 region on a firmer ground. Prompt fission should be the best technique for searching whether any band structure is built on this level.

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