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LETTER TO THE EDITOR

SQUARE LAW FOR ORBITS IN EXTRA-SOLAR PLANETARY SYSTEMS

ANTUN RUBČIĆ and JASNA RUBČIĆ

Department of Physics, University of Zagreb, Bijenička 32,
10000 Zagreb, Croatia
E-mail: rubcic@sirius.phy.hr

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The square law $r_n = r_1 n^2$ for orbital sizes r_n (r_1 is a constant dependent on the particular system, and n are consecutive integer numbers) is applied to the recently discovered planets of υ Andromedae and to pulsars PSR B1257+12 and PSR 1828-11. A comparison with the solar planetary system is made. The product nv_n of the orbital velocity v_n with the corresponding orbital number n for planets of υ Andromedae is in good agreement with those for terrestrial planets, demonstrating the generality of the square law in dynamics of diverse planetary systems. "Quantized velocity" of nv_n is very close to 24 kms^{-1}, i.e. to the step found in the quantized redshifts of galaxies. A definite conclusion for planetary systems of pulsars requires additional observations.

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In our previous papers [1,2], the orbital distribution of planets and satellites in the solar system has been described by the simple square law

$$r_n = r_1 n^2 \,. \tag{1}$$

Semimajor axes r_n of planetary and satellite orbits are proportional to the square of consecutive integer numbers n, where r_1 is a constant dependent on the system. We have also applied the square law to the planetary system of the pulsar PSR B1257+12 [3].

Very recently, the planetary system of the nearby star υ Andromedae (from hereafter: υ And) has been discovered using the Doppler radial velocity method [4]. It is the first system of multiple companions with a parent star similar to the Sun. Therefore, it is important to check whether the planets of υ And obey also the square law. Moreover, the planets of the pulsar PSR 1828-11 will be considered, too, although the present findings are not yet confirmed. So far, only three extra-solar planetary systems with more than one observed planet per system have been discovered.

The observational data for υ And and two pulsars are given in Table 1. Note that masses (M) of planets of υ And, and those of the pulsars are of the order of the Jupiter mass (M_J) and Earth mass (M_E), respectively. Question mark added to the planet A of PSR B1257+12 means that original results [5] have been questioned [6] with the suggestion that planet A might be an artefact in the calculations.

TABLE 1. Semimajor axes r_n, masses $(M)\sin(i)$, deduced orbital numbers n, products of n with the corresponding orbital velocity v_n, and the mean values of nv_n for extra-solar planetary systems.

System	$r_n/(10^{11}\mathrm{m})$	$(M)\sin(i)$	n	$nv_n/(\mathrm{km\,s^{-1}})$
υ Andromedae				
υ And b	0.0883	0.71 (M_J)	1	138.52
υ And c	1.242	2.11 ”	4	147.7
υ And d	3.740	4.61 ”	7	<u>148.99</u>
				145.08
PSRB1257 + 12				
A(?)	0.285	0.015 (M_E)	5	410.49
B	0.540	3.4 ”	7	417.50
C	0.705	2.8 ”	8	<u>417.59</u>
				415.20
PSR 1828-11				
A	1.391	3 (M_E)	6	208.57
B	1.975	12 ”	7	204.21
C	3.142	18 ”	9	<u>208.16</u>
				206.98

Data are taken from: Jean Schneider, *Extra-solar Planets Encyclopaedia*, update 15 April 1999. http://www.obspm.fr/planets

In order to determine the orbital numbers n for the particular system, the square roots of orbital semimajor axes have been plotted vs. integer numbers in such a way that all observational points are close to a straight line without an intercept. Deviations of the observational points from the straight line for pulsar planetary systems are found to be less than 2%, while those of υ And less than 6.2% on the average.

The results of the fit to the data in Table 1 are shown in Fig. 1. The square law satisfactorily describes orbital sizes in extra-solar planetary systems, in spite of the fact that only few planets per system have been found. It is evident that some orbits predicted by the square law are not occupied. For the planetary system of υ And, the orbits at n equal to 2, 3, 5, and 6 are vacant. It may be that at these orbits small planets exist, but undetectable by the present methods. Future observations should confirm or disprove these assumptions.

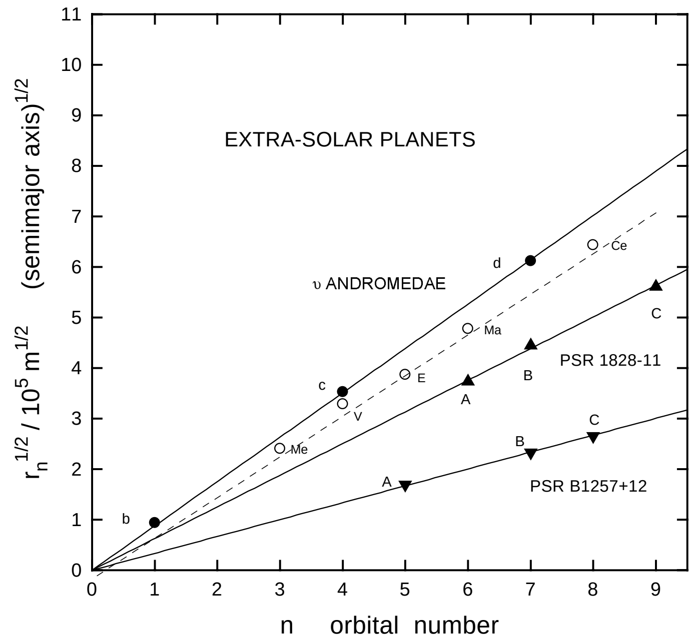

Fig. 1. Correlation of the square root of the semimajor axes r_n with the orbital numbers n for extra-solar planetary systems. Terrestrial planets (open circles, dashed line) are added for comparison.

We have shown [1] that the radius and velocity at the n-th orbit (within the approximation of circular orbits) is proportional to n^2 and $1/n$, respectively. Further investigation [2] has shown that along with the orbital number n, an additional number k may be introduced, resulting in the following relationships

$$r_n = \frac{G}{v_0^2} M \frac{n^2}{k^2} , \tag{2}$$

$$v_n = v_0 \frac{k}{n} , \tag{3}$$

where G is the gravitational constant, M the mass of the central body, and v_0 a fundamental velocity, which may be considered as an important quantity of all considered systems.

The integer number n determines the quadratic increase of orbital radii, while k defines the extension or spacing of orbits. By increasing k, orbits are more closely packed. Thus k may be named the "spacing number" to differ from the main "orbital number" n. Equation (3) states that nv_n is a constant for a given system, and for some other systems it is a multiple of the fundamental velocity v_0. Indeed, this has been demonstrated for the solar system [2], i.e. for its five subsystems: the terrestrial planets and the largest asteroid Ceres ($k = 6$), the Jovian planets ($k = 1$), and satellites of Jupiter ($k = 2$), Saturn ($k = 4$) and Uranus ($k = 1$). For all these subsystems, the value of $nv_n = kv_0$ is given by $(25.0 \pm 0.7)k$ km s^{-1} [2], confirming thus Eq. (3). It has to be pointed out that more accurate value of $nv_n = [(23.5 \pm 0.3)k + (4.0 \pm 1.0)]$ km s^{-1} was obtained (Eq.(12) in Ref. [2]). A similar situation for orbital velocities may be expected in extra-solar systems.

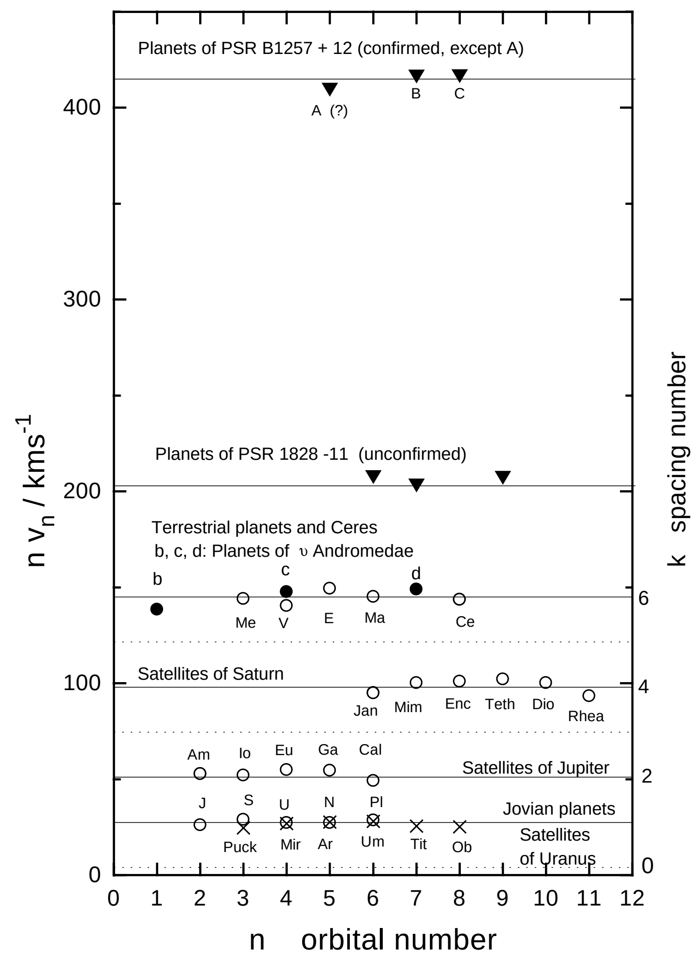

Fig. 2. Correlation of the products of orbital numbers n and orbital velocities v_n with n and the spacing number k, for the solar subsystems and three extra-solar planetary systems.

The correlation of nv_n with n and k is shown in Fig. 2. This figure is based on Fig. 3. of Ref. [2], where only data for the solar system have been taken into account. Here, it is supplemented by the extra-solar system data of υ And and pulsars PSR 1828-11 and PSR B1257+12. Figure 2 demonstrates that new data of the planetary system of υ And, with the mean value of nv_n equal to 145.1 $\mathrm{km\,s^{-1}}$ (see Table 1), are compatible with the data for terrestrial planets of the solar system, for which nv_n has almost the same value of 145.0 $\mathrm{km\,s^{-1}}$ [2]. A similarity among the two planetary systems can be seen also in Fig. 1. Although the number of planets for pulsar planetary systems are small, one may notice the well defined "velocity levels" with the step of nearly 207 $\mathrm{km\,s^{-1}}$. However, one should not take this as a final result because only two nv_n are known. Future discoveries of other pulsar planetary systems will probably change the number of levels defined by k in Fig. 2. Indeed, one may even expect that the step of 207 $\mathrm{km\,s^{-1}}$ might be decreased to $207/8 = 25.9$ $\mathrm{km\,s^{-1}}$, which is nearly equal to that of the solar system. This would lead to the similarity in dynamical properties of diverse systems. However, only future observations should give a definite answer to these expectations.

The velocity about 24 $\mathrm{km\,s^{-1}}$ is deduced from the quantized redshifts of galaxies [7-11] as one of the possible "quantized periods". Some other values like 36, 72 and 144 $\mathrm{km\,s^{-1}}$ are also found. It is a great puzzle why the orbital velocities should be related to the velocities derived from redshifts. However, one suspects that some fundamental link exists among the systems.

Some authors prefer the fundamental velocity of about 144 $\mathrm{km\,s^{-1}}$ [12–14]. This was found for planets in the solar system if one takes all planets as a single system. In the present model, the terrestrial planets are located at the level $k = 6$, and Jovian planets at $k = 1$, because v_0 is addopted to be 24 $\mathrm{km\,s^{-1}}$. In that case, Jovian planets are considered as a subsystem with $n = 2$ for Jupiter, $n = 3$ for Saturn, etc., as can be seen in Fig. 2 (see also Refs. [1–3]). The terrestrial planets could be considered as the remnants of mass of a Jupiter-like planet, which failed to be formed at $n = 1$ [1,14]. However, terrestrial planets may be taken as an independent subsystem, with Mercury at $n = 3$, Venus at $n = 4$, etc., as can be seen in Figs. 1 and 2.

The assumption $v_0 \approx 144$ $\mathrm{km\,s^{-1}}$ will introduce many vacant orbits between Jupiter and Pluto, if the square law for orbital radii is taken into account. Thus, Jupiter will be at $n = 11$, Saturn at $n = 15$, Uranus at $n = 21$, Neptune at $n = 26$ and finally Pluto at $n = 30$. An analysis of the solar-system data suggests that planets of υ And are located at the velocity level $k = 6$, with $v_0 \approx 24$ $\mathrm{km\,s^{-1}}$. If v_0 is taken to be 144 $\mathrm{km\,s^{-1}}$, then k will be equal to one. Consequently, the value of k, e.g., for the Jovian planets would be then 1/6. According to the present model, that does not seem likely, because the "spacing number" k is defined as an integer number and determines the packing of orbits.

There is a hope that the same value v_0 can be attributed to the systems around alike stars. For pulsars, one may suppose that v_0 could be equal to about 26 $\mathrm{km\,s^{-1}}$ and consequently k should be equal to 8 and 16 for PSR 1828-11 and PSR B1257+12, respectively. Although this assumption seems very attractive, it cannot be confirmed without further observations.

In conclusion, one may claim that the square law is adequate for the description of the orbital distribution for diverse systems: solar subsystems, extra-solar planetary systems with stars similar to the Sun and even to planetary systems of pulsars.

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KVADRATNI ZAKON ZA STAZE IZVAN-SUNČEVIH PLANETARNIH SUSTAVA

Kvadratni zakon $r_n = r_1 n^2$ za polumjere staza r_n (r_1 stalnica ovisna o sustavu a n uzastopni cijeli brojevi) primjenjujemo na nedavno otkrivene planete υ Andromede i pulzara PSR B1257+12 i PSR 1828-11. Načinili smo usporedbu sa sunčevim sustavom. Umnožak nv_n za stazne brzine v_n i staznog broja n za planete υ Andromede u dobrom je skladu s vrijednostima za terestrijalne planete. To pokazuje općenitost kvadratnog zakona u dinamici različitih sustava. “Kvantizirana brzina” nv_n vrlo je blizu 24 kms^{-1}, tj. koraku koji se opaža u crvenim pomacima galaksija. Konačan zaključak za planetarne sustave pulzara zahtijeva nove podatke.