A Bethe-Salpeter-equation study with the - enhanced effective QCD coupling

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A BETHE–SALPETER-EQUATION STUDY WITH THE $\langle A^2 \rangle$-ENHANCED EFFECTIVE QCD COUPLING[1]

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Dyson-Schwinger equations provide an effective approach to physics of strong interactions. To reproduce the hadronic phenomenology well, the Dyson-Schwinger approach in the rainbow-ladder approximation must employ an effective interaction between quarks which is fairly strong at intermediate ($Q^2 \sim 0.5$ GeV^2) spacelike transferred momenta. We have recently proposed that such an interaction may originate from the dimension-2 gluon condensate $\langle A^2 \rangle$ which has recently attracted much attention. We showed that the resulting effective running coupling leads to the sufficiently strong dynamical chiral symmetry breaking and successful phenomenology, at least in the light sector of pseudoscalar mesons. In the present paper, we give a more detailed investigation of the parameter dependence of these results.

1. Introduction

In recent years, the dimension-2 gluon condensate $\langle A^a_\mu A^{a\mu} \rangle \equiv \langle A^2 \rangle$ attracted a lot of theoretical attention [1–7], to quote just several of many papers offering evidence that this condensate may be important for the nonperturbative regime of Yang-Mills theories, particularly QCD. Although $\langle A^2 \rangle$ is not gauge invariant, it was even argued that its value in the Landau gauge may have a physical meaning [2, 3, 7]. In our recent paper [8], we argued that $\langle A^2 \rangle$ may be relevant for the Dyson-Schwinger (DS) approach to QCD. Namely, in order that this approach would lead

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[1]A combined paper of two presentations, one by each author, at the 2nd Int. Conference on Nuclear and Particle Physics with CEBAF at Jefferson Lab (NAPP 2003), Dubrovnik 26 – 31 May 2003.

to a successful hadronic phenomenology, an enhancement of the effective quark-gluon interaction seems to be needed at intermediate ($Q^2 \sim 0.5$ GeV^2) momenta[2], and Ref. [8] showed that the gluon condensate $\langle A^2 \rangle$ provides such an enhancement. It also showed that the resulting effective strong running coupling leads to the sufficiently strong dynamical chiral symmetry breaking and successful phenomenology in the light sector of pseudoscalar mesons. However, the issue of the parameter dependence of the results was just commented very briefly. Thus, in the present paper, in Sect. 3, we give a more detailed investigation and presentation of the parameter dependence. A brief recapitulation of the DS approach and the effective interaction is given in the next section.

2. DS approach and its effective interaction

DS approach to hadrons and their quark-gluon substructure [9–11] has strong and clear connections with QCD. Besides being covariant, this approach is chirally well-behaved and nonperturbative. This has been crucial, especially in the light-quark sector of QCD, for successful descriptions of bound states achieved by phenomenological DS studies (e.g., see recent reviews [10, 11] and references therein), where one can treat soundly even the processes influenced by axial anomaly[3], which is really remarkable for a bound-state approach. What happens is that in the process of solving DS equations, one essentially derives a constituent quark model which turns out to be successful over a very wide range of masses. Its main virtue is that it incorporates the correct chiral symmetry behavior through the gap equation for the full, dynamically dressed quark propagator S_q and the Bethe-Salpeter (BS) equation for the bound states of the dynamically dressed quarks (and antiquarks). That is, the constituent quarks arise through the dressing resulting from the dynamical chiral symmetry breaking (DχSB) in the ("gap") DS equation for the full quark propagators, while the *light $q\bar{q}$* pseudoscalar solutions of the BS equation (in a *consistent* approximation) are (*almost* massless) *quasi*-Goldstone bosons of DχSB. Generation of DχSB is well-understood [9, 10, 18–22] in the rainbow-ladder approximation (RLA). Thus, phenomenological DS studies have mostly been relying on RLA and using *Ansätze* of the form

$$[K(k)]^{hg}_{ef} = \mathrm{i}4\pi\alpha_{\mathrm{eff}}(-k^2)\, D^{ab}_{\mu\nu}(k)_0 \left[\frac{\lambda^a}{2}\gamma^\mu\right]_{eg}\left[\frac{\lambda^b}{2}\gamma^\nu\right]_{hf} \tag{1}$$

for interactions between quarks. In this equation, e, f, g, h schematically represent spinor, color and flavor indices and $D^{ab}_{\mu\nu}(k)_0$ is the *free* gluon propagator in the gauge in which the aforementioned DS studies have been carried out almost exclusively, namely the Landau gauge:

$$D^{ab}_{\mu\nu}(k)_0 = \frac{\delta^{ab}}{k^2}\left(-g_{\mu\nu} + \frac{k_\mu k_\nu}{k^2}\right), \tag{2}$$

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[2]We adopt the convention $k^2 = -Q^2 < 0$ for spacelike momenta k.

[3]See, e.g., Refs. [12, 13] for the $\pi^0 \to \gamma\gamma$ transition amplitude $T^{\gamma\gamma}_{\pi^0}$, and Refs. [14–17] for the related transition $\gamma \to \pi^+\pi^0\pi^-$.

while $\alpha_{\text{eff}}(Q^2)$ is an effective running coupling on which we will comment below at length.

The BS equation for the bound-state vertex $\Gamma_{q\bar{q}'}$ of the meson composed of the quark of the flavor q and antiquark of the flavor q', is then

$$[\Gamma_{q\bar{q}'}(k,P)]_{ef} = \int \frac{\mathrm{d}^4\ell}{(2\pi)^4}[S_q(\ell + \frac{P}{2})\Gamma_{q\bar{q}'}(\ell,P)S_{q'}(\ell - \frac{P}{2})]_{gh}[K(k-\ell)]_{ef}^{hg} . \quad (3)$$

The consistent RLA requires that the same interaction kernel (1) be previously used in the DS equation for the full quark propagator S_q. That is, dressed quark propagators $S_q(k)$ for various flavors q,

$$S_q^{-1}(p) = A_q(p^2)\slashed{p} - B_q(p^2) , \qquad (q = \mathrm{u,d,s,}\ldots), \quad (4)$$

are obtained by solving the gap DS equation

$$S_q^{-1}(p) = \slashed{p} - \widetilde{m}_q - \mathrm{i}\,4\pi \int \frac{\mathrm{d}^4\ell}{(2\pi)^4}\,\alpha_{\text{eff}}(-(p-\ell)^2)D_{\mu\nu}^{ab}(p-\ell)_0\,\frac{\lambda^a}{2}\,\gamma^\mu S_q(\ell)\frac{\lambda^b}{2}\,\gamma^\nu. \quad (5)$$

Following the approach of Munczek and Jain [19, 20], the gap equation (5) is un-renormalized, but regularized by an ultra-violet cutoff L. However, this cutoff is huge compared to the QCD scale Λ_{QCD}. (In the present paper, $L = 134$ GeV as in Ref. [20].) In Eq. (5), $\widetilde{m}_q$ is the cutoff-dependent bare mass of the quark flavor q breaking the chiral symmetry explicitly. The case $\widetilde{m}_q = 0$ corresponds to the chiral limit where the current quark mass $m_q = 0$, and where the constituent quark mass $M_q(0) \equiv B_q(0)/A_q(0)$ stems exclusively from the *nonperturbative* phenomenon of DχSB. Of course, calling the "constituent mass" the value of the "momentum-dependent constituent mass function" $M_q(p^2) \equiv B_q(p^2)/A_q(p^2)$ at exactly $p^2 = 0$, and not at some other low $-p^2$, is a matter of a somewhat arbitrary choice. However, it is just a matter of terminology and nothing essential. What is important to get a successful hadronic phenomenology, especially in the light-quark sector $(q = \mathrm{u,d,s})$, is that DχSB is sufficiently strong. This means that the gap equation (5) should yield quark propagator solutions $A_q(p^2)$ and $B_q(p^2)$ giving the dressed-quark mass function $M_q(p^2)$ whose values *at low* $-p^2$ are of the order of typical constituent mass values, namely several hundred MeV, even in the chiral limit. A typical example of such $M_q(p^2)$ is given in Fig. 1, obtained with $\alpha_{\text{eff}}(Q^2)$ (8) proposed originally in our Ref. [8] and further advocated in the present paper.

Indeed, the issue of the origin of the interaction (1), or, equivalently, $\alpha_{\text{eff}}(Q^2)$, which would enable successful phenomenology, is crucial for the DS studies. The form of α_{eff} is only partially known from the fact that at large spacelike momenta it must reduce to $\alpha_{\text{pert}}(Q^2)$, the well-known running coupling of perturbative QCD. However, for momenta $Q^2 \lesssim 1$ GeV^2, where non-perturbative QCD applies, the interactions are still not known; therefore, in phenomenological DS studies, $\alpha_{\text{eff}}(Q^2)$ must be modeled for $Q^2 \lesssim 1$ GeV^2 (e.g., see Refs. [9−11,20−23]). There, one can see that phenomenologically most successful of those modeled interactions have a rather large bump at the intermediate momenta, around $Q^2 \sim 0.5$ GeV^2. For example, in

Fig. 2 compare $\alpha_{\text{eff}}(Q^2)$ used by Jain and Munczek (JM) [20] and by Maris, Roberts and Tandy (MRT) [10, 11, 21, 22]. In any case, successful DS phenomenology re-
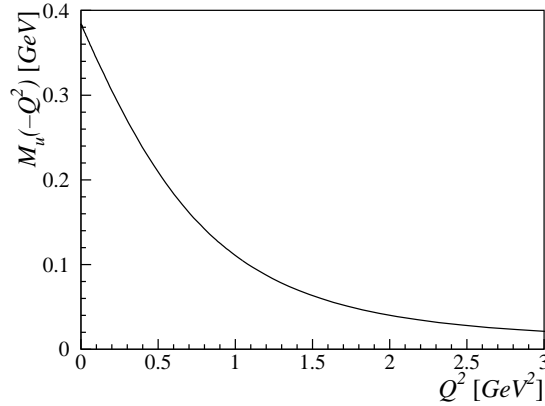

Fig. 1. The effective non-strange ($q = $ u) quark mass function $M_{\text{u}}(-Q^2)$ calculated using the effective coupling (8) proposed in Ref. [8] and the input parameters given by Eqs. (17) and (19).
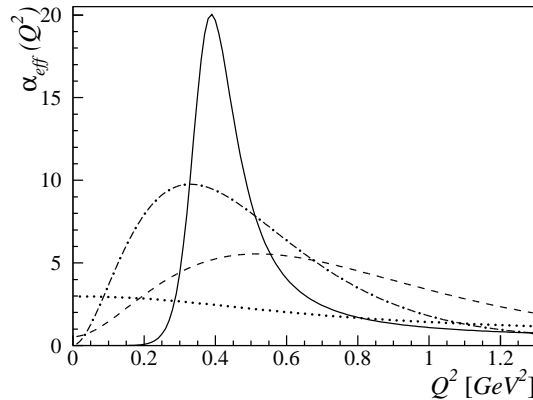

Fig. 2. The momentum dependence of various strong running couplings mentioned in the text. JM [20] and MRT [10, 22] $\alpha_{\text{eff}}(Q^2)$ are depicted by, respectively, dashed and dash-dotted curves. The effective coupling (8) proposed and analyzed in the present paper is depicted by the solid curve, and $\alpha_{\text{s}}(Q^2)$ (8) of Fischer and Alkofer [24] (their fit A) by the dotted curve.

quires that this modeled part of the interaction (1) be fairly strong. That is, regardless of details of the interaction, its *integrated strength* in the infrared must be fairly high to achieve acceptable description of hadrons, notably mass spectra and DχSB [10, 11].

Theoretical explanations on what could be the origin of so strong nonperturbative part of the phenomenologically required interaction are obviously very much

needed, either from the *ab initio* studies of sets of DS equations for Green's functions of QCD (see, e.g., the recent review [9]) or from somewhere outside DS approach. The particularly important result of the *ab initio* DS studies is that, in the Landau gauge, the effects of ghosts are absolutely crucial for the intermediate-momenta enhancement of the effective quark-gluon interaction [9, 24 – 27]. This is obvious in the expression for the strong running coupling $\alpha_s(Q^2)$ in these Landau-gauge studies [9, 24 – 27],

$$\alpha_s(Q^2) = \alpha_s(\mu^2) \, Z(Q^2) \, G(Q^2)^2 \,, \tag{6}$$

where $\alpha_s(\mu^2) = g^2/4\pi$ and $Z(\mu^2)G(\mu^2)^2 = 1$ at the renormalization point $Q^2 = \mu^2$. The gluon renormalization function $Z(-k^2)$ defines the full gluon propagator $D_{\mu\nu}^{ab}(k)$ in the Landau gauge

$$D_{\mu\nu}^{ab}(k) = Z(-k^2)D_{\mu\nu}^{ab}(k)_0 = \frac{Z(-k^2)}{k^2} \, \delta^{ab} \left(-g_{\mu\nu} + \frac{k_\mu k_\nu}{k^2} \right). \tag{7}$$

Similarly, $G(-k^2)$ is the ghost renormalization function which defines the full ghost propagator $D_G^{ab}(k) = \delta^{ab}G(-k^2)/k^2$.

While the *ab initio* DS studies [9, 24 – 27] do find significant enhancement of $\alpha_s(Q^2)$, Eq. (6), until recently this seemed still not enough to yield a sufficiently strong DχSB (e.g., see Sec. 5.3 in Ref. [9]) and a successful phenomenology. However, for carefully constructed dressed quark-gluon vertex *Ansätze*, Fischer and Alkofer [24] have recently managed to obtain good results for dynamically generated constituent quark masses and pion decay constant f_π, although not simultaneously also for the chiral quark-antiquark $\langle \bar{q}q \rangle$ condensate, which then came out somewhat larger than the phenomenological value. Thus, the overall situation is that a progress has been made in this direction [24 – 28], but further investigation and elucidation of the origin of phenomenologically successful effective interaction kernels remains one of primary challenges in contemporary DS studies [10, 11]. This provided the motivation for our paper [8], where we pointed out that such an interaction kernel for DS studies in the Landau gauge resulted from cross-fertilization of the DS ideas on the running coupling of the form (6) [9, 24 – 27] and the ideas on the possible relevance of the dimension-2 gluon condensate $\langle A_\mu^a A^{a\mu} \rangle \equiv \langle A^2 \rangle$ [1 – 7, 29 – 32].

In Ref. [8], we gave arguments that the $\langle A^2 \rangle$-contributions to the OPE-improved gluon (A) and ghost (G) polarization functions (found a long time ago in Refs. [29 – 32], and more recently confirmed by Kondo [4]) lead to an effective coupling $\alpha_{\text{eff}}(Q^2)$ given by

$$\alpha_{\text{eff}}(Q^2) = \alpha_{\text{pert}}(Q^2) \, Z^{\text{Npert}}(Q^2) \, G^{\text{Npert}}(Q^2)^2 \,, \tag{8}$$

where $\alpha_{\text{pert}}(Q^2)$ is the running coupling of perturbative QCD, and

$$Z^{\text{Npert}}(Q^2) \quad = \quad \frac{1}{1 + \dfrac{m_A^2}{Q^2} + \dfrac{C_A}{Q^4}} \,, \tag{9}$$

$$G^{\mathrm{Npert}}(Q^2) \quad = \quad \frac{1}{1 - \dfrac{m_A^2}{Q^2} + \dfrac{C_G}{Q^4}} \; . \tag{10}$$

The functions $Z^{\mathrm{Npert}}(Q^2)$ and $G^{\mathrm{Npert}}(Q^2)$ are the nonperturbative (Npert) parts of the, respectively, gluon and ghost renormalization functions $Z(Q^2)$ and $G(Q^2)$. They crucially depend on the quantity m_A which can be interpreted as a dynamically generated effective gluon mass, and which is proportional to the dimension-2 gluon condensate $\langle A^2 \rangle$. Namely, for the Landau gauge (which we use throughout this paper), the number of QCD colors $N_c = 3$ and the number of space-time dimensions $D = 4$,

$$m_A^2 = \frac{3}{32} \, g^2 \langle A^2 \rangle = -m_G^2 \; , \tag{11}$$

where m_G is a dynamically generated effective ghost mass. (In a subsequent work, Kondo et al. [5] also worked out logarithmic corrections to Eq. (11) thanks to which the dynamical gluon mass (and ghost mass) vanishes as $Q^2 \to \infty$, as it must according to, e.g., Cornwall [33, 34]. However, taking this into account is not necessary at the degree of refinement and precision used in this paper.)

For $g^2 \langle A^2 \rangle$, the Landau-gauge lattice studies of Boucaud et al. [1] yield the value 2.76 GeV^2. This is compatible with the bound resulting from the discussions of Gubarev et al. [2, 3] on the physical meaning of $\langle A^2 \rangle$ (although it is gauge-variant) and its possible importance for confinement. We thus use this value in Eq. (11) and obtain

$$m_A = 0.845 \, \mathrm{GeV}. \tag{12}$$

In our considerations below, this value will turn out to be a remarkably good initial estimate for the dynamical masses m_A and m_G.

The coefficients C_A and C_G, appearing in $Z^{\mathrm{Npert}}(Q^2)$ (9) and $G^{\mathrm{Npert}}(Q^2)$ (10), can, in principle, be related to various other condensates [30 – 32], but some of them are completely unknown at present. Therefore, both C_A and C_G should at this point be treated as free parameters to be fixed by phenomenology. Fortunately, Ref. [8] managed to make the estimate $C_A = (0.640 \, \mathrm{GeV})^4$. This estimate [8] is based on the role of only one condensate [35], the well-known gauge-invariant dimension-4 condensate $\langle F^2 \rangle$ [36], and thus misses some (unknown) three- and four-gluon contributions [31 – 32]. Therefore, and since the true value of $\langle F^2 \rangle$ is still rather uncertain [37], we do not attach too much importance to the above value of C_A but just use it as an initial estimate.

There is no similar estimate for C_G, but one may suppose that it would not differ from C_A by orders of magnitude. We thus try

$$C_G = C_A = (0.640 \, \mathrm{GeV})^4 \tag{13}$$

as an initial guess. It turns out *a posteriori* that this value of C_G leads to a very good fit to phenomenology.

As we discussed in Ref. [8], Eq. (8) can be justified for relatively high Q^2, but not for low Q^2. For example, $\alpha_{\text{pert}}(Q^2)$ must ultimately hit the Landau pole as Q^2 gets lowered. However, this can be handled as in other phenomenological DS studies. Their various choices of $\alpha_{\text{eff}}(Q^2)$ usually also contain $\alpha_{\text{pert}}(Q^2)$, but since handling the Landau pole problem at the fundamental level is out of their scope, they [20 – 22, 38 – 40] just shift[4] the Landau pole to the timelike momenta in all logarithms appearing here: $\ln(Q^2/\Lambda^2_{\text{QCD}}) \rightarrow \ln(x_0 + Q^2/\Lambda^2_{\text{QCD}})$. Presently, we adopt this latter procedure and for $\alpha_{\text{pert}}(Q^2)$ we use throughout the $\overline{\text{MS}}$-scheme two-loop expression used before by JM [20] and in our earlier phenomenological DS studies [38 – 42]. This means, we use throughout the infrared (IR) regulator $x_0 = 10$ (to which all results are almost totally insensitive), the number of quark flavors $N_f = 5$, and $\Lambda_{\text{QCD}} = 0.228$ GeV. These parameters of $\alpha_{\text{pert}}(Q^2)$ are thereby fixed and do not belong among variable parameters such as C_A, C_G, the variation of which is discussed below.

In the present context, the more serious objection to our α_{eff} (8) is that we cannot give an argument in advance that the factor $Z^{\text{Npert}}(Q^2)\,G^{\text{Npert}}(Q^2)^2$ in the proposed $\alpha_{\text{eff}}(Q^2)$ (8) indeed approximates well nonperturbative contributions at low Q^2 (say, $Q^2 < 1$ GeV^2), but can only hope that our results to be calculated will provide an *a posteriori* justification for using it as low as $Q^2 \sim 0.3$ GeV^2 (since Eq. (8) takes appreciable values down to about $Q^2 \sim 0.3$ GeV^2). Of course, $Z^{\text{Npert}}(Q^2)$ and $G^{\text{Npert}}(Q^2)$ must be wrong in the limit $Q^2 \rightarrow 0$, as they are based on the results derived by OPE [4, 29, 32], which certainly fail in that limit. For example, detailed investigations of the $Q^2 \rightarrow 0$ asymptotic behavior in *ab initio* DS studies [9, 24 – 27], settled down to the conclusion that $\alpha_s(Q^2)$ remains finite as $Q^2 \rightarrow 0$, which is also supported by several lattice calculations [43, 44]. On the other hand, if the presently interesting $\langle A^2 \rangle$ condensate is explained by an instanton liquid, the coupling vanishes as $\alpha_s(Q^2) \propto Q^4$ [6], which is closer to the behavior of our $\alpha_{\text{eff}}(Q^2)$ (8). Still, Eqs. (10) enforce, for small Q^2, even much more dramatic suppression of our $\alpha_{\text{eff}}(Q^2)$ (8), which vanishes as Q^{12}. This is an unrealistic artefact of the proposed form (8) when applied down to the $Q^2 \rightarrow 0$ limit. Nevertheless, because of the integration measure in the integral equations in DS calculations, integrands at these small Q^2 (where our $\alpha_{\text{eff}}(Q^2)$ (8) is doubtlessly too suppressed) do not contribute much, at least not to the quantities (such as $\langle \bar{q}q \rangle$ condensate, meson masses, decay constants and amplitudes) calculated in phenomenological DS analyses. Hence, the form of $\alpha_{\text{eff}}(Q^2)$ at Q^2 close to zero is not very important[5] for the outcome of these phenomenological DS calculations. This is because the most important for the success of phenomenological DS calculations seems to be the enhancement at somewhat higher values of Q^2 – e.g., see the humps at $Q^2 \sim 0.4$ to 0.6 GeV^2 in the JM [20] or MRT [21, 22] $\alpha_{\text{eff}}(Q^2)$, dashed curves and dash-dotted curves in Fig. 2. Our $\alpha_{\text{eff}}(Q^2)$ (8) exhibits such an enhancement centered around

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[4]As pointed out already by, e.g., Cornwall [33], dynamically generated gluon mass can provide the physical reason for such a change in the arguments of logarithms. That is, $x_0 \propto m_A^2/\Lambda^2_{\text{QCD}} \sim 10$.

[5]Of course, the $Q^2 \rightarrow 0$ domain would give an important contribution in a case with a sufficiently strong (but still integrable) divergence in $\alpha_{\text{eff}}(Q^2)$, such as the delta function in Ref. [23].

$Q^2 \approx m_A^2/2$, as shown by the solid curve representing it in Fig. 2. This enhancement is readily understood when one notices that Eq. (8) has four poles in the complex Q^2 plane, given by

$$(Q^2)_{1,2} = \frac{1}{2}\left(m_A^2 \mp \mathrm{i}\sqrt{4C_G - m_A^4}\right), \qquad \text{poles of } G^{\text{Npert}}(Q^2), \qquad (14)$$

$$(Q^2)_{3,4} = \frac{1}{2}\left(-m_A^2 \mp \mathrm{i}\sqrt{4C_A - m_A^4}\right), \qquad \text{poles of } Z^{\text{Npert}}(Q^2). \qquad (15)$$

For $\min\{C_G, C_A\} > m_A^4/4$ there is no pole on the real axis, but a saddle point in the middle of two complex conjugated poles. For the DS studies, which are almost exclusively carried out in Euclidean space, spacelike k^2 (i.e., $Q^2 > 0$ in our convention) is the relevant domain and is thus pictured in Fig. 2. There, the maximum of $\alpha_{\text{eff}}(Q^2)$ (8) at the real axis is at $Q^2 \approx m_A^2/2$, i.e., the real part of its *double* poles $(Q^2)_{1,2}$. The height and the width of the peak is influenced by both C_G and m_A. The enhancement of $\alpha_{\text{eff}}(Q^2)$ (8) is thus crucially determined by the $\langle A^2 \rangle$ condensate through Eq. (11), and by the manner this condensate contributes to the ghost renormalization function, which enters squared into the effective coupling (8).

3. Phenomenology with the condensate-enhanced coupling

We solved the DS equations for quark propagators and BS equations for pseudoscalar $q\bar{q}$ ($q = \mathrm{u}, \mathrm{d}, \mathrm{s}$) bound states in the same way as in our previous phenomenological DS studies [38–41]. This essentially means as in the JM approach [20], except that instead of JM's $\alpha_{\text{eff}}(Q^2)$, Eq. (8) is employed in the RLA interaction (1). We can thus immediately present the results because we can refer to Refs. [38–41] for all calculational details, such as procedures for solving DS and BS equations, all model details, as well as expressions for inputs such as the aforementioned IR-regularized $\alpha_{\text{pert}}(Q^2)$ and explicit expressions for calculated quantities, e.g., for f_π.

3.1. In the chiral limit

In the chiral limit, where the bare (and current) quark masses vanish, the only parameters are those defining our $\alpha_{\text{eff}}(Q^2)$ (8), namely m_A, C_A and C_G. It turns out that the initial estimates (12) and (13), motivated above, need only a slight modification to provide a very good description of the light pseudoscalar sector: it is enough to increase the estimate $m_A = 0.845$ GeV by just 5%. That is, the parameter set

$$C_A = (0.640 \text{ GeV})^4 = C_G, \quad m_A = 0.884 \text{ GeV} \qquad (16)$$

leads to (to begin with) an excellent description of DχSB, which gives rise to Goldstone bosons which are also massless pseudoscalar $q\bar{q}$ bound states. This is seen in

the first line of Table 1: our good chiral limit values of the pion decay constant ($f_\pi \approx 88$ MeV) and the $\bar{q}q$ condensate ($\langle \bar{q}q \rangle \approx (-214 \text{ GeV})^3$) satisfy the Gell-Mann-Oakes-Renner (GMOR) relation (two last columns in Table 1 very well, at the level of a couple of percent. These chiral-limit results are similar to, e.g., the corresponding results with JM $\alpha_{\text{eff}}(Q^2)$, which are also given in Table 1 (in the last line) for comparison.

TABLE 1. *The chiral-limit results for f_π and $\langle \bar{q}q \rangle$ and the test of the GMOR relation for our α_{eff} (8) and the JM one [20]. The quark condensate and the* current *quark mass m are calculated at the renormalization scale $\mu = 1$ GeV. In DS approach, good values of f_π automatically lead to good description of $\pi^0 \to \gamma\gamma$, since the empirically successful amplitude $T_{\pi^0}^{\gamma\gamma} = 1/4\pi^2 f_\pi$ is always obtained analytically in this approach in the chiral limit [12, 13].*

$\alpha_{\text{eff}}, C_G,$ C_A, m_A	$\langle \bar{q}q \rangle$ (GeV^3)	f_π (GeV)	$-\dfrac{\langle \bar{q}q \rangle}{f_\pi^2}$ (GeV)	$\lim_{m \to 0} \dfrac{M_\pi^2}{2m}$ (GeV)
Eqs. (8) and (16)	$(-0.214)^3$	0.0882	1.261	1.293
Eqs. (8) and (17)	$(-0.217)^3$	0.0905	1.241	1.289
JM α_{eff} [20]	$(-0.227)^3$	0.0898	1.368	1.401

The behavior of the momentum-dependent constituent mass function $M_q(p^2) \equiv B_q(p^2)/A_q(p^2)$ is also *qualitatively* similar both to $M_q(p^2)$ found earlier by JM [20] and ourselves [38–42] with JM α_{eff} and to $M_q(p^2)$ obtained now with our α_{eff} (8) but with different parameters (this is exemplified by $M_u(-Q^2)$ in Fig. 1). Quantitatively, for the parameters (16) and the chiral limit ($\widetilde{m}_q = 0$), the constituent quark mass $M_q(0) = 0.306$ GeV. This is almost 25% below both our old results for $M_u(0)$ [39] obtained with JM α_{eff} *and* our present $M_u(0)$ in Fig. 1, pertaining to the refitted parameters (17) and (much less importantly) to $\widetilde{m}_u \neq 0$ (19). However, the quantitative differences of such a size are not a problem, since calculations in practice show that a successful reproduction of the hadronic phenomenology requires just that values of this (anyway unobservable) quantity *at low Q^2* are of the order of several hundred MeV, i.e., of the order of typical constituent quark mass, $M_q(0) \sim M_{\text{nucleon}}/3 \sim M_\rho/2$.

The constituent quark mass in the chiral limit, directly related to the $\langle \bar{q}q \rangle$ condensate, is also very convenient for illustrating the dependence of the key DχSB phenomenon on the model parameters. If we vary C_G (for fixed values of m_A and C_A) away from its phenomenologically favorable value in Eq. (16), which gives sufficient enhancement of α_{eff}, the dynamically generated constituent quark mass $M_q(0)$ quickly drops. Beyond some critical value of C_G, it is always exactly zero, meaning that the DχSB is then completely absent (see Fig. 3). The sensitivity of our results to C_G is understandable, since from Eqs. (14) it is clear that C_G,
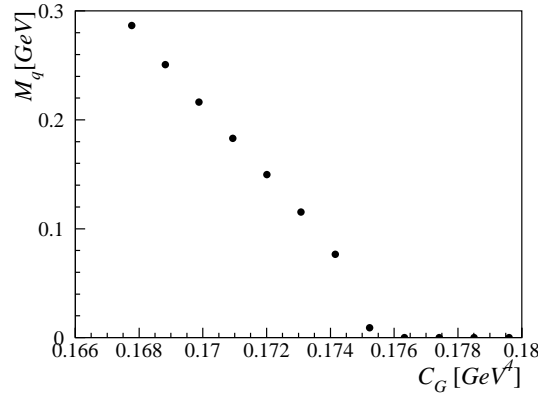
Fig. 3. The dependence of the dynamically generated constituent quark mass $M_q(0)$ on the parameter C_G illustrates the disappearance of $D\chi SB$ for unfavorable values of C_G: when for given values of m_A and $\widetilde{m}_q$ (here $m_A = 0.884$ GeV and $\widetilde{m}_q = 0$) C_G deviates from the value that gives sufficient enhancement of α_{eff}, the dynamically generated mass $M_q(0)$ quickly falls. Moreover, beyond some critical value of C_G, it is always exactly zero since the $D\chi SB$ phenomenon then completely disappears.

in combination with m_A, influences the height and width of the peak of $\alpha_{\mathrm{eff}}(Q^2)$ (8) for spacelike momenta. In spite of this sensitivity, we were able to find other combinations of parameter values which lead to good results. For example, the values

$$C_A = (0.6060\,\mathrm{GeV})^4 = C_G, \quad m_A = 0.8402\,\mathrm{GeV} \tag{17}$$

yield the second line of Table 1. This indicates that there may be an interesting interplay between m_A and C_G, and it motivates us to find how the phenomenologically favorable values of m_A and C_G are related. However, we will do it below in the more realistic, massive case, away from the chiral limit. There, the quark bare masses (and the related current masses) deviate from zero so that empirical masses of pseudoscalar mesons can be obtained.

3.2. Away from the chiral limit

We start by noting that both of the two sets of (m_A, C_A, C_G) values quoted above as successful in the chiral limit, Eqs. (16) and (17), give also a good fit away from the chiral limit. As the first shot, we adopt without any change the explicit breaking of chiral symmetry from JM, that is, the bare mass parameters $(\widetilde{m}_q)$ of light quarks $(q = \mathrm{u, d, s})$ leading to the broad phenomenological fit with *their* α_{eff} [20], namely

$$\widetilde{m}_{\mathrm{u}} = \widetilde{m}_{\mathrm{d}} = 3.1 \cdot 10^{-3}\ \mathrm{GeV} \quad \text{and} \quad \widetilde{m}_{\mathrm{s}} = 73 \cdot 10^{-3}\ \mathrm{GeV}. \tag{18}$$

These values of $\widetilde{m}_{\mathrm{u,d}}$ lead to an excellent description of the pion as a quasi-Goldstone boson of $D\chi SB$ also in conjunction with our α_{eff} (8) and Eq. (16), as witnessed by the first line in Table 2, where we predict pion mass, weak decay constant, and

$\pi^0 \rightarrow \gamma\gamma$ amplitude very close to their empirical values (in the seventh line of Table 2). For the same reason as in the chiral limit, the results are again quite sensitive to changes of C_G but not of C_A. Table 3 illustrates this relatively weak sensitivity to the changes of C_A for the case of the parameter set (17) and (19). Table 3 shows, one can increase (or decrease) C_A by a factor of 2, and the results change little.

TABLE 2. *The masses and decay constants of pions and kaons, and the $\pi^0 \rightarrow \gamma\gamma$ decay amplitude $T_{\pi^0}^{\gamma\gamma}$, obtained in DS approach with our $\alpha_{\text{eff}}(Q^2)$ (8). The first two lines result from the initial parameters $m_A, C_{A,G}$ (16) and the quark bare mass parameters (18) fixed already by the broad JM phenomenological fit [20]. These masses (18) with another $(m_A, C_{A,G})$ parameter set (17) give the third and the fourth line. Similarly, the fifth and the sixth line result from $\alpha_{\text{eff}}(Q^2)$ with $m_A, C_{A,G}$ given by Eq. (17), and the slightly altered bare masses (19). The last two lines are the corresponding experimental values.*

$\alpha_{\text{eff}}, C_G, C_A,$ $m_A, \widetilde{m}_{\text{u}}, \widetilde{m}_{\text{s}}$	H	M_H (MeV)	f_H (MeV)	$T_{\pi^0}^{\gamma\gamma}$ (MeV^{-1})
Eqs. (8), (16)	π	136.70	91.2	0.272×10^{-3}
and (18)	K^+	520.72	112.1	
Eqs. (8), (17)	π	136.17	93.0	0.256×10^{-3}
and (18)	K^+	516.28	112.5	
Eqs. (8), (17)	π	134.96	92.9	0.256×10^{-3}
and (19)	K^+	494.92	111.5	
experimental	π^0	134.9766 ± 0.0006	91.9 ± 3.5	$(0.274 \pm 0.010) \times 10^{-3}$
values	K^+	493.677 ± 0.016	112.8 ± 1.0	

TABLE 3. *This table illustrates rather weak sensitivity to the changes of C_A. The "set A" of input parameters is given by Eqs. (19) and (17) with the change $C_A \rightarrow 2C_A$. The "set B" of input parameters is given by the same equations with the change $C_A \rightarrow C_A/2$. Meson masses and meson decay constants are in units of GeV, while s$\bar{\text{s}}$ stands for the non-physical pseudoscalar s$\bar{\text{s}}$ bound state.*

	set A		set B	
H	M_H	f_H	M_H	f_H
π	0.132609	0.0877711	0.136671	0.0955129
K^+	0.490013	0.107034	0.499146	0.113735
s$\bar{\text{s}}$	0.716287	0.131859	0.727431	0.133728

The second line of Table 2 reveals that the parameter set (16) and (18) works somewhat less well in the strange sector, as the kaon mass is 5% too high. However, a deviation of this size is not worrisome in the present circumstances where we know that the model interaction anyway misses some aspects (such as the $Q^2 \to 0$ behavior and non-ladder contributions), and where we just want to point out that the $\langle A^2 \rangle$ condensate is a possible source of the needed enhancement of $\alpha_{\text{eff}}(Q^2)$. In fact, the empirical success in the strange sector is quite reasonable considering that we used the standard JM mass parameters [20], (as we did also in [38−41]) and no refitting was performed there (although $\alpha_{\text{eff}}(Q^2)$ was different).

Nevertheless, it is interesting to see what changes are brought by refitting. If one, for example, tries the values of m_A, C_G and C_A given by Eq. (17) instead of Eq. (16), one gets the third and the fourth line in Table 2 instead of, respectively, the first and second line. Thus, the improvement achieved thereby is not significant, indicating that we should try changes of the bare quark masses $\widetilde{m}_q$. It turns out that slight changes of the values (18) are sufficient to achieve agreement with experiment in both the non-strange and strange sectors. For example, the parameter set which gives the fifth and sixth lines of Table 2, thus reproducing the empirical mass of both π^0 and K^+ together with good results for their decay constants and $\pi^0 \to \gamma\gamma$ amplitude $T_{\pi^0}^{\gamma\gamma}$, is given by m_A, C_G and C_A from Eq. (17) and by the bare quark masses

$$\widetilde{m}_{\text{u}} = \widetilde{m}_{\text{d}} = 3.046 \cdot 10^{-3} \text{ GeV}, \quad \widetilde{m}_{\text{s}} = 67.70 \cdot 10^{-3} \text{ GeV}. \tag{19}$$

This parameter set, Eqs. (17) and (19), is also the one giving the gap equation solutions resulting in the momentum-dependent constituent mass function $M_q(-Q^2)$ displayed in Fig. 1.

The parameter set (17) and (19) also gives us a good description of the η–η' complex, along the lines of our Refs. [39, 42]. Although it means employing just a minimal extension of the DS approach, we must relegate this to another paper [45].

The preferred parameter set (17) and (19) is a result of a systematic examination of refitting possibilities performed by studying the dependence on the input parameters $x = (\widetilde{m}_{\text{u}}, \widetilde{m}_{\text{s}}, m_A, C_G, C_A)$ of the function

$$F[x] = \sum_y \left(\frac{y_{\text{exp}} - y_{\text{th}}}{y_{\text{exp}}} \right)^2 \times 100\% \,, \tag{20}$$

namely the sum of squared differences of the four experimentally measured (y_{exp}) and presently theoretically calculated (y_{th}) quantities $y \in \{M_{\pi^0}, f_{\pi^\pm}, M_{K^0}, f_{K^\pm}\}$. We kept choosing $C_A = C_G$ for simplicity, since we find that moderate variations of C_A do not affect our results much anyway, as already illustrated in Table 3.

Minimization of Eq. (20) shows different respective characters of the α_{eff} parameters (m_A, C_G, C_A) and the mass parameters ($\widetilde{m}_{\text{u}}, \widetilde{m}_{\text{s}}$). The point (19) in the parameter subspace ($\widetilde{m}_{\text{u}}, \widetilde{m}_{\text{s}}$) is the location of a non-degenerate minimum of the function (20). Thus, the possible values of the bare quark masses ($\widetilde{m}_{\text{u}}, \widetilde{m}_{\text{s}}$) can be precisely restricted by demanding that the function (20) is below a certain value.

Figure 4 shows $F = 5.0\%$ and $F = 2.5\%$ curves in the $(\widetilde{m}_{\mathrm{u}}, \widetilde{m}_{\mathrm{s}})$ plane, with m_A and $C_G(= C_A)$ fixed at Eq. (17). At the minimum, for $(\widetilde{m}_{\mathrm{u}}, \widetilde{m}_{\mathrm{s}})$ values (19), we obtain $F \approx 1.5\%$.
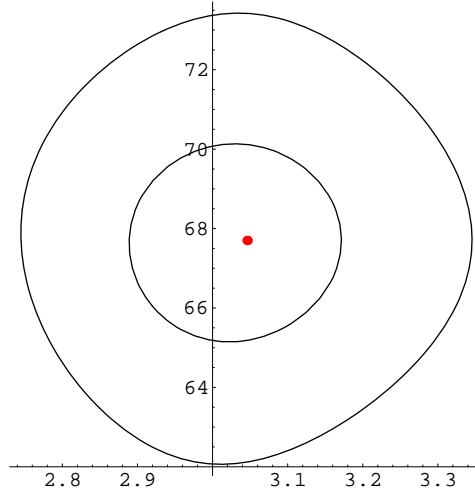
Fig. 4. The curves are the solutions of the equations $F = 2.5\%$ and $F = 5.0\%$ in $(\widetilde{m}_{\mathrm{u}}, \widetilde{m}_{\mathrm{s}})$ plane. The point is the position of the simple, non-degenerate minimum at the bare quark mass values (19).
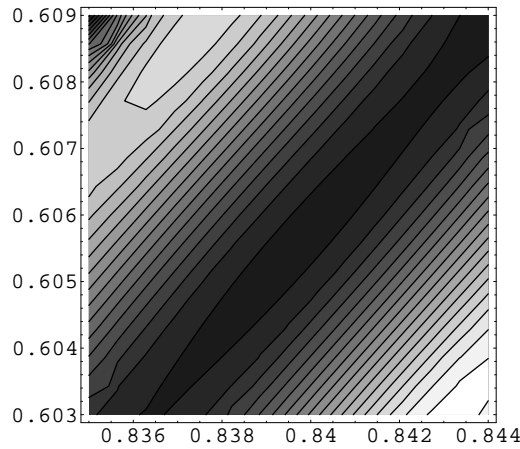

Fig. 5. F vs. $(m_A, C_G^{1/4})$ contour plot. The darkest color corresponds to $F \sim 1.5\%$, defining the valley of the minimal F. Conversely, the lighter the shade of gray, the larger the value of F, i.e., the overall difference between the calculated and experimental quantities.

In contrast to the bare quark masses $(\widetilde{m}_{\mathrm{u}}, \widetilde{m}_{\mathrm{s}})$, the parameters defining α_{eff} cannot be determined so unambiguously. By this we do not mean just the afore-

mentioned weak sensitivity to C_A. They also cannot be fixed by minimization of F (20) in the same sense as the bare quark masses, even though the results are very sensitive to m_A and C_G. The point is that F has no simple minimum in the $(m_A, C_G^{1/4})$–plane as it has in $(\widetilde{m}_u, \widetilde{m}_s)$ plane: Figure 5 reveals a minimum in the form of a narrow, straight "valley" described very well by a linear relation between m_A and $C_G^{1/4}$. Thus, in spite of high sensitivity to m_A and C_G, there are many pairs of these quantities which give a fit comparable (within few percent) to that resulting from the values (17), as long as they approximately satisfy the linear relation

$$(C_G)^{1/4} = 0.7742 \, m_A - 0.0444 \text{ GeV}. \tag{21}$$

That is, the function (20) measuring the difference between the calculated and experimental values of $M_{\pi^0}, f_{\pi^\pm}, M_{K^0}, f_{K^\pm}$ has a degenerate minimum in the shape of a narrow valley. It is bounded by the values $(C_G)_{\min} \approx (0.6 \text{ GeV})^4$ and $(C_G)_{\max} \approx (0.9 \text{ GeV})^4$ in the sense that between these values we managed to find solutions providing excellent fits (F of the order 1.5%) to the empirical values.

4. Conclusion

The dimension-2 gluon condensate $\langle A^2 \rangle$ allowed the derivation [8] of a suitably enhanced $\alpha_{\text{eff}}(Q^2)$. This effective interaction leads to the sufficiently strong DχSB and successful phenomenology at least in the light sector of pseudoscalar mesons. This opens the possibility that instead of modeling $\alpha_{\text{eff}}(Q^2)$, its enhancement at intermediate Q^2 may be understood in terms of gluon condensates, which seem to provide an important mechanism proposed and studied for the first time in our recent Ref. [8]. The systematic examination of the parameter space, i.e., various fitting possibilities set forth in the present paper, leads to the conclusion that this scenario is compatible with reasonable values of both $\langle A^2 \rangle$-condensate and the gauge-invariant dimension-4 gluon condensate $\langle F^2 \rangle$ [36]. In the relevant momentum region, $\alpha_{\text{eff}}(Q^2)$ (and thus also the solutions of DS and BS equations and results for calculated measurable quantities) depend only very weakly on C_A, which parametrizes contributions of dimension-4 condensates to the gluon propagator. The essential parameters C_G and m_A, on which the dependence is very strong, are not independent. Thus, due to the relation (21), Eq. (8) is an essentially one-parameter model for α_{eff}, albeit on a relatively small interval of C_G. This can be interpreted as another instance that what counts is the integrated strength of the interaction. Over the possible range, we have a continuous set of parameter pairs (m_A, C_G); their values are such that they give higher peaks at smaller squared momenta, resulting in similar integrated strengths. We find that the phenomenologically allowed range of values of the dynamically generated gluon mass m_A is in agreement with the lattice results [1] on $\langle A^2 \rangle$ in the Landau gauge. Also, phenomenologically allowed values of C_G, which parametrizes contributions of dimension-4 condensates to the ghost propagator, are such that they might be a sign that C_G is indeed mostly determined by the dimension-4 gluon condensate $\langle F^2 \rangle$ [36].

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BETHE–SALPETEROVA JEDNADŽBA S EFEKTIVNIM QCD VEZANJEM I $\langle A^2 \rangle$-POJAČANJEM

Dyson-Schwingerove jednadžbe predstavljaju važan pristup fizici jakih međudjelovanja. Da bi se dobro opisala hadronska fenomenologija, Dyson-Schwingerov pristup u ljestvičastom približenju mora rabiti efektivno međudjelovanje među kvarkovima koje je prilično jako na srednjim ($Q^2 \sim 0.5$ GeV^2) prostornim prijenosima impulsa. Nedavno smo predložili da to međudjelovanje može nastati od gluonskog kondenzata $\langle A^2 \rangle$ dimenzije 2, koji je u posljednje vrijeme privukao dosta pažnje. Pokazali smo da dobiveno efektivno vezanje vodi na dovoljno jako lomljenje dinamičke kiralne simetrije i uspješnu fenomenologiju, barem u sektoru lakih pseudoskalarnih mezona. U ovom radu opisujemo detaljno istraživanje parametarskih ovisnosti.