β decay of odd-A As to Ge isotopes in the interacting boson-fermion model

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β decay of odd-*A* As to Ge isotopes in the interacting boson-fermion model

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The structure of odd-mass isotopes of As and Ge is described in the framework of the proton-neutron interacting boson-fermion model. The energy levels and the electromagnetic properties of ^{69,71,73}As and ^{69,71,73}Ge are calculated and compared with the experiment. The β-decay rates from the As isotopes to the Ge isotopes are calculated. The calculated decays tend to be stronger than the observed ones. This may indicate a mixture of components outside the model space in the wave functions of actual nuclei. The effect of the higher-order terms in the decay operators seems small.

I. INTRODUCTION

Nuclei in the $A=70$ mass region display a rich variety of complex phenomena. The properties of these nuclei have been investigated in a number of experimental and theoretical works. Both the shape transitions and shape coexistence occur in these nuclei. These features have been explained with the help of various theoretical models. In this work we apply the interacting boson [1] and the interacting boson-fermion model [2] in a consistent description of a series of even-even Ge and odd-*A* As and Ge isotopes. The main application we consider here is focused on the description of β-decay rates in odd mass nuclei. β-decay rates are very sensitive to the details of wave functions. Each decay rate involves matrix elements with wave functions of three nuclei and therefore can provide a fine test of the nuclear model. The proton-neutron interacting boson-fermion model (IBFM2) [2] was proposed [3] and applied in the description of β decay [4–7]. Recently this approach was very successful in the analysis of β transitions from spherical Rh to Pd odd-mass nuclei [8] and from Cs to Xe O(6) γ-soft isotopes of mass number $A=125, 127, 129$ [9].

II. EVEN Ge NUCLEI

The strong variation in energy of the first excited 0^+ state in the even-even Ge isotopes, being lowest for ^{72}Ge where it is the lowest excited state overall, suggests the presence of an intruder state. This feature can be described in an IBM type of calculation by introducing an s' boson, with an appropriate energy, into the standard model that involves s and

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d bosons only. Although the intruder state is the dominant component in one excited state, the admixture of the intruder component even in the ground state is not insignificant. The introduction of s' and d' bosons in IBM2 has successfully explained low-lying 0^+ states in this region [10]. Because our main interest is focused on β-decay calculations (where there have not been attempts to introduce such complex model), we ignore those 0^+ states and describe other states in the usual IBM2 model [1]. The even-even Ge nuclei (used as cores for the odd-mass As and Ge neighbors) are therefore described as systems of proton and neutron bosons with the Hamiltonian

$$H^{\mathrm{B}} = \epsilon_d(n_{d_\nu} + n_{d_\pi}) + \kappa(Q_\nu^{\mathrm{B}} \cdot Q_\pi^{\mathrm{B}}) + \frac{1}{2}\xi_2[(d_\nu^\dagger s_\pi^\dagger - d_\pi^\dagger s_\nu^\dagger) \cdot (\tilde{d}_\nu s_\pi$$
$$- \tilde{d}_\pi s_\nu)] + \sum_{K=1,3} \xi_K([d_\nu^\dagger d_\pi^\dagger]^{(K)} \cdot [\tilde{d}_\pi \tilde{d}_\nu]^{(K)}), \qquad (1)$$

where

$$Q_\nu^{\mathrm{B}} = d_\nu^\dagger s_\nu + s_\nu^\dagger \tilde{d}_\nu + \chi_\nu[d_\nu^\dagger \tilde{d}_\nu]^{(2)}, \qquad (2)$$

$$Q_\pi^{\mathrm{B}} = d_\pi^\dagger s_\pi + s_\pi^\dagger \tilde{d}_\pi + \chi_\pi[d_\pi^\dagger \tilde{d}_\pi]^{(2)} \qquad (3)$$

are the boson quadrupole operators.

In Table I we present the IBM2 parameters for a series of nuclei considered in this paper. The parameters (except for ^{74}Ge) are taken from Ref. [11] where the same approach was applied to high-spin states. These parameters present a slight readjustment of parameters in Ref. [10]. The number of proton bosons for all core nuclei is 2 (number of proton pairs above $Z=28$) while the number of neutron bosons is 4 and 5 for ^{68}Ge and ^{70}Ge, respectively (number of neutron pairs above $N=28$), and 5 and 4 for ^{72}Ge and ^{74}Ge, respectively (number of neutron hole pairs below $N=50$).

TABLE I. IBM2 parameters. The unit is Mev except for the dimensionless χ_ν and χ_π. The parameter $\chi_\pi = -1.20$ is fixed for all nuclei.

Odd nuclei	Core nucleus	ϵ_d	κ	χ_ν	ξ_2	$\xi_1 = \xi_3$
^{69}As, ^{69}Ge	^{68}Ge	1.42	−0.20	1.3	0.06	−0.10
^{71}As, ^{71}Ge	^{70}Ge	1.22	−0.11	1.4	0.08	−0.04
^{73}As	^{72}Ge	1.10	−0.11	1.7	0.10	−0.13
^{73}Ge	^{74}Ge	1.10	−0.11	1.6	0.10	−0.13

The energy levels are shown in Fig. 1. The experimental data are taken from Refs. [12–15]. Except for 0_2^+ states, the agreement between the experiment and the calculation is very good. The observed first excited 0^+ states are considered to be the intruder states arising from the particle-hole excitation over the closed core. As we have already noticed, they are outside of the model space of the simple IBM and can be described quite well by introducing s' and d' bosons [10]. The electromagnetic transition operators are

$$T_B^{(E2)} = e_\pi^B Q_\pi^B + e_\nu^B Q_\nu^B \tag{4}$$

and

$$T_B^{(M1)} = \sqrt{\frac{3}{4\pi}} (g_\pi^B L_\pi^B + g_\nu^B L_\nu^B). \tag{5}$$

The boson effective charge $e_\pi^B = e_\nu^B = 0.0631$ e b, which was determined from $B(E2; 0_1^+ \to 2_1^+)$ in ^{70}Ge [13], was used for all isotopes including the odd ones. The boson g factors for all isotopes are $g_\nu^B = 0$, $g_\pi^B = 1.0$ μ_N [11].

Figure 2 shows the electric quadrupole moments and the magnetic dipole moments. The observed electric moments are negative in ^{72,74}Ge while the calculated values are positive and small in magnitude. The calculated magnetic moments are systematically smaller than the experimental data.

The branching ratios are shown in Table II together with $B(E2)$ and $B(M1)$ values. The calculated strongest branches generally agree with the observed strongest ones. The re-
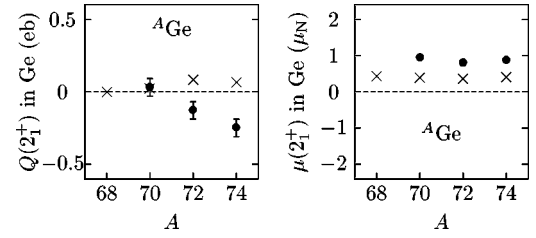

FIG. 2. The calculated electric quadrupole and magnetic dipole moments in even-even Ge isotopes are compared with the experimental data. The symbol • with the error bar shows the experimental values while × shows the results of the calculations.

duced transition probabilities show reasonable agreement as long as the magnitudes are large enough. In ^{68}Ge, the experimental $B(E2; 2_2^+ \to 2_1^+)$ value is very small. This may mean that 2_2^+ has an appreciable mixture of the component outside of the model. The two main problems appearing in the present calculation of electromagnetic properties—the sign of the electric quadrupole moments of 2_1^+ states in ^{72,74}Ge and too small calculated $B(E2; 2_2^+ \to 0_1^+)$ values in ^{70,72,74}Ge—do not appear in the calculation with a very similar parametrization in the model that includes s' and d' bosons [10]. This indicates that the admixture of intruder components in the ground state and in the 2_1^+ state is not insignificant and will influence the results of our calculations of electromagnetic and β decay properties of odd-mass isotopes.

III. ODD-MASS As AND Ge NUCLEI

Odd-A As and Ge nuclei are described in the IBFM2 [2] by coupling an odd nucleon to the even-even Ge boson core. The Hamiltonian is written as

$$H = H^B + H^F + V^{BF}, \tag{6}$$

where H^B is the IBM2 core Hamiltonian (1). H^F is the Hamiltonian of the odd fermion:
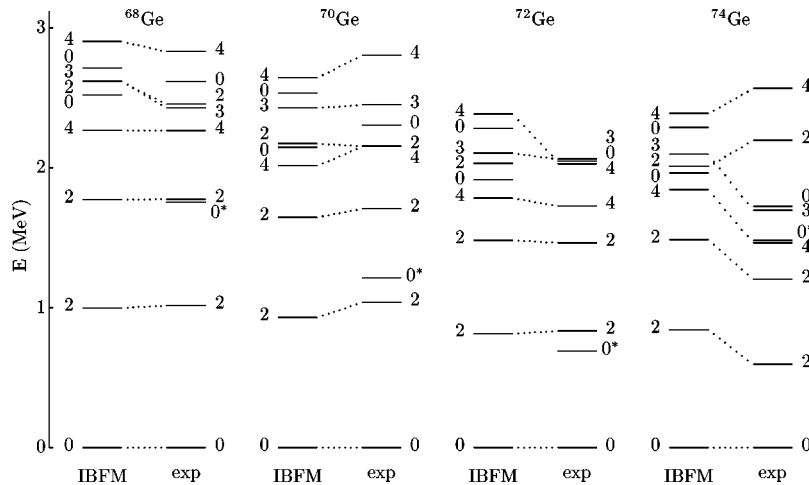

FIG. 1. The energy levels of the positive-parity states in the even-even Ge isotopes. The experimental levels labeled by 0^* are considered to be intruder states, which are ignored in the present study.

TABLE II. Branching ratios in the even-even Ge isotopes.

Nucleus	Level (Mev)	Transition	$B(E2)$(W.u.) (IBM2)	$B(E2)$(W.u.) (Expt.)	$B(M1)$(W.u.) (IBM2)	$B(M1)$(W.u.) (Expt.)	I_γ (IBM2)	I_γ (Expt.)
^{68}Ge	1.777	$2_2^+ \to 2_1^+$	28.3	0.49 (21)	0.10	0.0080 (14)	100	100 (4)
		$2_2^+ \to 0_1^+$	0.11	0.141 (25)			2.6	44.2 (17)
	2.268	$4_1^+ \to 2_2^+$	0.48				0.0	
		$4_1^+ \to 2_1^+$	29.8	13.9 (18)			100	100
^{70}Ge	1.708	$2_2^+ \to 2_1^+$	26.7	111 (60)	0.12	0.0025^{+31}_{-17}	100	100 (1)
		$2_2^+ \to 0_1^+$	0.14	1.0 (5)			3.6	85 (1)
	2.154	$4_1^+ \to 2_2^+$	0.53				0.0	0.8 (2)
		$4_1^+ \to 2_1^+$	33.2	24 (6)			100	100 (1)
^{72}Ge	1.464	$2_2^+ \to 2_1^+$	31.7	62 (11)	0.09	0.00016 (5)	100	100.0 (12)
		$2_2^+ \to 0_1^+$	0.04	0.130 (24)			0.7	14.16 (17)
	1.728	$4_1^+ \to 2_2^+$	0.95				0.0	
		$4_1^+ \to 2_1^+$	32.7	37 (5)			100	100
^{74}Ge	1.204	$2_2^+ \to 2_1^+$	23.3	45 (5)	0.11	0.00105 (25)	100	100 (1)
		$2_2^+ \to 0_1^+$	0.02	0.75 (9)			0.1	46 (3)
	1.464	$4_1^+ \to 2_2^+$	0.51				0.0	
		$4_1^+ \to 2_1^+$	25.6	37 (5)			100	100

$$H^{\mathrm{F}} = \sum_i \varepsilon_i n_i, \tag{7}$$

where ϵ_i is the quasiparticle energy of the ith orbital while n_i is its number operator. V^{BF} is the interaction between the bosons and the odd particle:

$$V^{\mathrm{BF}} = \sum_{i,j} \Gamma_{ij}([a_i^\dagger \tilde{a}_j]^{(2)} \cdot Q_{\rho'}^{\mathrm{B}}) + \sum_{i,j} \Gamma'_{ij}([a_i^\dagger \tilde{a}_j]^{(2)} \cdot Q_\rho^{\mathrm{B}})$$
$$+ A \sum_i n_i n_{d_{\rho'}} + \sum_{i,j} \Lambda_{ki}^j \{:[[d_\rho^\dagger \tilde{a}_j]^{(k)} a_i^\dagger s_\rho]^{(2)} : \cdot [s_{\rho'}^\dagger \tilde{d}_{\rho'}]^{(2)}$$
$$+ \text{Hermitian conjugate}\}. \tag{8}$$

The modified annihilation operator $\tilde{d}$ is defined by $\tilde{d}_m = (-1)^m d_{-m}$. The symbols ρ and ρ' denote π (ν) and ν (π) if the odd fermion is a proton (a neutron). The creation operator of the odd particle is written as $a_{jm}^\dagger$, while the modified annihilation operator is defined as $\tilde{a}_{jm} = (-1)^{j-m} a_{j-m}$. The orbital dependences of the quadrupole and the exchange interactions are [16]

$$\Gamma_{i,j} = (u_i u_j - v_i v_j) Q_{i,j} \Gamma, \tag{9}$$

$$\Lambda_{k,i}^j = -\beta_{k,i} \beta_{j,k} \left(\frac{10}{N_\rho(2j_k+1)} \right)^{1/2} \Lambda, \tag{10}$$

where

$$\beta_{i,j} = (u_i v_j + v_i u_j) Q_{i,j}, \tag{11}$$

$$Q_{i,j} = \langle l_i, \tfrac{1}{2}, j_i \| Y^{(2)} \| l_j, \tfrac{1}{2}, j_j \rangle. \tag{12}$$

The electromagnetic transition operators are

$$T^{(\mathrm{E2})} = e_\pi^{\mathrm{B}} Q_\pi^{\mathrm{B}} + e_\nu^{\mathrm{B}} Q_\nu^{\mathrm{B}} + \sum_{i,j} e'_{i,j} [a_i^\dagger \tilde{a}_j]^{(2)}, \tag{13}$$

where

$$e'_{i,j} = -\frac{e_\rho^{\mathrm{F}}}{\sqrt{5}} (u_i u_j - v_i v_j) \langle i \| r^2 Y^{(2)} \| j \rangle \tag{14}$$

and

$$T^{(\mathrm{M1})} = \sqrt{\frac{3}{4\pi}} \left(g_\pi^{\mathrm{B}} L_\pi^{\mathrm{B}} + g_\nu^{\mathrm{B}} L_\nu^{\mathrm{B}} + \sum_{i,j} e_{i,j}^{(1)} [a_i^\dagger \tilde{a}_j]^{(1)} \right), \tag{15}$$

where

$$e_{i,j}^{(1)} = -\frac{1}{\sqrt{3}} (u_i u_j + v_i v_j) \langle i \| g_l \mathbf{l} + g_s \mathbf{s} \| j \rangle. \tag{16}$$

For the odd proton in As, $e_\pi^{\mathrm{F}} = 1.5\ e$, while for the odd neutron in Ge, $e_\nu^{\mathrm{F}} = 0.5\ e$ has been used [11]. For the odd particle, the spin g factor is reduced by a factor of 0.7.

As the ground states of parent odd As nuclei have negative parity and our main topic is β transitions, in the present work we shall limit our attention to negative-parity states in odd As and Ge isotopes.

TABLE III. Single-particle energies (MeV) of the proton orbitals in As.

Nucleus	$f_{7/2}$	$p_{3/2}$	$f_{5/2}$	$p_{1/2}$	$g_{9/2}$	$d_{5/2}$
^{69}As	−7.0	−1.5	0.5	1.6	3.9	7.9
^{71}As	−7.0	−1.5	0.5	1.6	3.6	7.6
^{73}As	−7.0	−1.5	0.5	1.6	3.0	7.0

TABLE IV. Parameters in the boson-fermion interaction (MeV) for As isotopes.

Nucleus	Γ	Γ'	A	Λ
[69]As	0.3	0.70	−0.3	0.25
[71]As	0.3	0.66	−0.3	0.25
[73]As	0.3	0.58	−0.3	0.25

A. As isotopes

The odd-A As isotopes are described by coupling an odd proton to the even-even Ge cores (see Table I). In order to obtain proton quasiparticle energies and occupation probabilities, the BCS equations are solved with $\Delta = 12/\sqrt{A}$ MeV and single-particle energies shown in Table III. The set of single-particle energies is very similar to the one used in the shell model Monte Carlo calculations of Ref. [17]. In the IBFM2 calculation for negative-parity states, proton orbitals of odd orbital angular momentum are included. The mass dependence of boson-fermion interactions is presented in Table IV. In addition to the quadrupole and the monopole interactions between the odd proton and the neutron bosons and the exchange interaction, the quadrupole interaction between the odd proton and the proton bosons is used.

The level structure of odd-A As and Ge nuclei has been already discussed within the frame of the IBFM1 [2]. In this model there is no distinction between neutron and proton bosons. Therefore, instead of two different parameters χ (χ_ν and χ_π), only one, somehow average, value is used. In the IBFM1 calculations for [67]As [18], [71]Ge [19–21], [71]As [20], [73]Ge [22], and [73]As [22,23], χ was negative. In IBFM2 language this is obvious for Ge isotopes. Neutrons interact with proton bosons for which χ_π (Table I) is negative. For As isotopes this means that an important (or even dominant) part of the quadrupole interaction comes from the interaction between protons and proton bosons. Our calculations (Table IV) are in agreement with these results. In fact, we have found that with the quadrupole interaction between protons and neutron bosons alone, a systematic description of As isotopes was not possible.

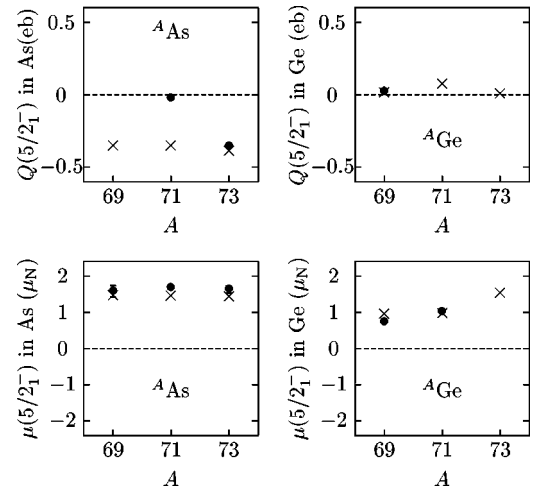In Fig. 3 we present the comparison of calculated and

FIG. 4. The calculated electric quadrupole and magnetic dipole moments are compared with the experimental data. The symbol • shows the experimental values while × shows the results of the calculations.

experimental levels of negative parity in [69]As, [71]As, and [73]As, in Fig. 4 the electric quadrupole and magnetic dipole moments for the $5/2_1^-$ respective levels, and in Tables V–VII the branching ratios in these isotopes. The experimental data are taken from Refs. [24–26].

The main features of the low-energy spectra of odd-A As isotopes have been reproduced reasonably well. A reasonable agreement between theory and experiment is obtained for most transitions, with leading branches being reproduced. However, for some weaker transitions there are discrepancies between the calculated and experimental intensities by about an order of magnitude. The main discrepancy appears for the transition $1/2_3^- \rightarrow 3/2_1^-$ in [73]As, which is much too large in the calculation. The calculated branching ratios in [71]As are very close to those calculated in IBFM1 [20]. The [70]Ge (^{3}H,d) [71]As reaction data [27] show that the $5/2_1^-$ ground state of [71]As has strong $f_{5/2}$ component while the $1/2_1^-$ state has a sizable $p_{1/2}$ component. These experimental results are in accordance with the calculated wave functions, where the percentage of $f_{5/2}$ and $p_{1/2}$ components is 93 and 7 in $5/2_1^-$ and 57 and 35 in $1/2_1^-$. In [73]As the percentage of $p_{3/2}$, $f_{5/2}$, and $p_{1/2}$ components is (93, 1, 3) in $3/2_1^-$, (1, 94, 6) in $5/2_1^-$,
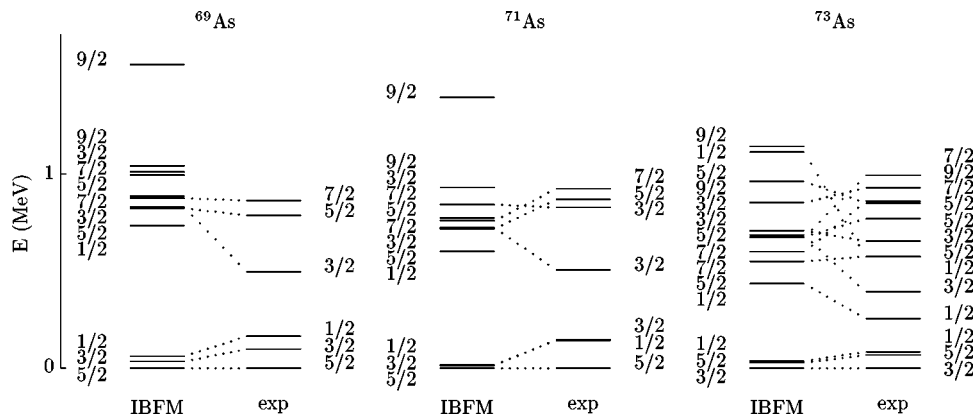

FIG. 3. The energy levels of the negative-parity states in As.

TABLE V. Branching ratios in ^{69}As.

Level (MeV)	Transition	I_γ(IBFM2)	I_γ(Expt.)
0.098	$3/2_1^- \to 5/2_1^-$	100	100
0.164	$1/2_1^- \to 3/2_1^-$	100	100
	$1/2_1^- \to 5/2_1^-$	7.5	
0.497	$3/2_2^- \to 1/2_1^-$	3.6	28 (4)
	$3/2_2^- \to 3/2_1^-$	100	100 (6)
	$3/2_2^- \to 5/2_1^-$	9.4	28 (4)
0.789	$5/2_2^- \to 3/2_2^-$	38.2	10.7 (12)
	$5/2_2^- \to 1/2_1^-$	17.4	15.9 (16)
	$5/2_2^- \to 3/2_1^-$	59.4	100 (10)
	$5/2_2^- \to 5/2_1^-$	100	28 (4)
0.864	$7/2_1^- \to 5/2_2^-$	0.2	9.0 (5)
	$7/2_1^- \to 3/2_2^-$	0.0	
	$7/2_1^- \to 3/2_1^-$	1.3	5.5 (6)
	$7/2_1^- \to 5/2_1^-$	100	100

(16, 56, 27) in $1/2_1^-$, (73, 5, 18) in $1/2_2^-$, and (11, 79, 10) in $3/2_2^-$ states. This composition of wave functions is consistent with (^{3}H,d) and (d,^{3}He) reaction data. Both reactions indicate that the dominant components in the wave functions are $f_{5/2}$ in $5/2_1^-$ and $p_{3/2}$ in $3/2_1^-$ while the $3/2_2^-$ is weakly populated. The data also indicate that the $p_{1/2}$ strength is shared between $1/2_1^-$ and $1/2_2^-$ levels. We notice that in Fig. 2 the sign of the experimental electric quadrupole moment for the $5/2_1^-$ level in ^{73}As was assumed negative (only the absolute value is known). This is done on the basis of the occupation

TABLE VI. Branching ratios in ^{71}As.

Level (MeV)	Transition	I_γ(IBFM2)	I_γ(Expt.)
0.143	$1/2_1^- \to 5/2_1^-$	100	100
0.147	$3/2_1^- \to 1/2_1^-$	0.0	
	$3/2_1^- \to 5/2_1^-$	100	100
0.506	$3/2_2^- \to 3/2_1^-$	100	100 (5)
	$3/2_2^- \to 1/2_1^-$	7.1	27 (14)
	$3/2_2^- \to 5/2_1^-$	8.2	
0.829	$3/2_3^- \to 3/2_2^-$	9.3	
	$3/2_3^- \to 3/2_1^-$	100	100 (14)
	$3/2_3^- \to 1/2_1^-$	30.3	9.3 (7)
	$3/2_3^- \to 5/2_1^-$	29.4	
0.870	$5/2_2^- \to 3/2_3^-$	0.0	
	$5/2_2^- \to 3/2_2^-$	28.8	
	$5/2_2^- \to 3/2_1^-$	36.4	40 (1)
	$5/2_2^- \to 1/2_1^-$	27.0	1.8 (7)
	$5/2_2^- \to 5/2_1^-$	100	100.0(7)
0.925	$7/2_1^- \to 5/2_2^-$	0.0	
	$7/2_1^- \to 3/2_3^-$	0.0	
	$7/2_1^- \to 3/2_2^-$	0.0	
	$7/2_1^- \to 3/2_1^-$	1.1	5.8 (16)
	$7/2_1^- \to 5/2_1^-$	100	100 (3)

TABLE VII. Branching ratios in ^{73}As.

Level (MeV)	Transition	I_γ(IBFM2)	I_γ(Expt.)
0.067	$5/2_1^- \to 3/2_1^-$	100	100
0.084	$1/2_1^- \to 5/2_1^-$	0.0	
	$1/2_1^- \to 3/2_1^-$	100	100
0.254	$1/2_2^- \to 1/2_1^-$	71.2	2.85 (24)
	$1/2_2^- \to 5/2_1^-$	0.2	
	$1/2_2^- \to 3/2_1^-$	100	100 (1)
0.393	$3/2_2^- \to 1/2_2^-$	1.2	6.99 (20)
	$3/2_2^- \to 1/2_1^-$	2.0	10.14(19)
	$3/2_2^- \to 5/2_1^-$	0.6	<0.6
	$3/2_2^- \to 3/2_1^-$	100	100 (10)
0.575	$1/2_3^- \to 3/2_2^-$	0.1	56.1 (12)
	$1/2_3^- \to 1/2_2^-$	5.8	100.0(12)
	$1/2_3^- \to 1/2_1^-$	4.5	16.2 (4)
	$1/2_3^- \to 5/2_1^-$	0.3	
	$1/2_3^- \to 3/2_1^-$	100	
0.578	$5/2_2^- \to 1/2_3^-$	0.0	
	$5/2_2^- \to 3/2_2^-$	24.3	1.0 (5)
	$5/2_2^- \to 1/2_2^-$	0.8	1.4 (7)
	$5/2_2^- \to 1/2_1^-$	50.7	<0.6
	$5/2_2^- \to 5/2_1^-$	64.9	71 (20)
	$5/2_2^- \to 3/2_1^-$	100	100 (12)
0.655	$3/2_3^- \to 5/2_2^-$	0.2	<7
	$3/2_3^- \to 1/2_3^-$	0.0	
	$3/2_3^- \to 3/2_2^-$	5.7	13.4 (3)
	$3/2_3^- \to 1/2_2^-$	100	100 (9)
	$3/2_3^- \to 1/2_1^-$	16.3	19.4 (6)
	$3/2_3^- \to 5/2_1^-$	12.7	7.0 (4)
	$3/2_3^- \to 3/2_1^-$	52.7	9.51 (13)
0.770	$5/2_3^- \to 3/2_3^-$	0.0	
	$5/2_3^- \to 5/2_2^-$	0.4	2 (1)
	$5/2_3^- \to 1/2_3^-$	0.0	
	$5/2_3^- \to 3/2_2^-$	4.0	9 (2)
	$5/2_3^- \to 1/2_2^-$	0.5	15 (2)
	$5/2_3^- \to 1/2_1^-$	0.4	<1
	$5/2_3^- \to 5/2_1^-$	8.3	60.9 (9)
	$5/2_3^- \to 3/2_1^-$	100	100.0(22)
0.861	$7/2_1^- \to 5/2_3^-$	0.2	
	$7/2_1^- \to 3/2_3^-$	0.0	
	$7/2_1^- \to 5/2_2^-$	5.5	0.70 (14)
	$7/2_1^- \to 3/2_2^-$	0.1	4.6 (5)
	$7/2_1^- \to 5/2_1^-$	100	62 (3)
	$7/2_1^- \to 3/2_1^-$	5.0	100 (5)
0.929	$9/2_1^- \to 7/2_1^-$	0.2	
	$9/2_1^- \to 5/2_3^-$	0.0	
	$9/2_1^- \to 5/2_2^-$	0.2	
	$9/2_1^- \to 5/2_1^-$	100	100 (3)

probabilities of proton configurations. The proton $f_{5/2}$ quasiparticle configuration in As isotopes is particle like, which results in a negative quadrupole moment in the state where it is the dominant component. Consistently, the state that is dominated by a holelike $p_{3/2}$ quasiparticle must have a posi-

TABLE VIII. Single-particle energies (MeV) of the neutron orbitals in Ge.

Nucleus	$f_{7/2}$	$p_{3/2}$	$f_{5/2}$	$p_{1/2}$	$g_{9/2}$	$d_{5/2}$
^{69}Ge	−4.0	−0.4	0.9	1.4	2.4	6.4
^{71}Ge	−4.0	−0.2	0.6	1.4	2.4	6.4
^{73}Ge	−4.0	0.0	0.3	1.4	2.4	6.4

TABLE IX. Parameters in the boson-fermion interaction (MeV) for Ge isotopes.

Nucleus	Γ	Γ'	A	Λ
^{69}Ge	0.3	0	−0.2	0.6
^{71}Ge	0.3	0	−0.2	0.6
^{73}Ge	0.3	0	−0.2	1.35

tive quadrupole moment (as is the case for the $3/2_1^-$ level in ^{75}As).

B. Ge isotopes

In the odd-A Ge isotopes we couple an odd neutron to the even-even Ge cores (see Table I). With the single-particle neutron energies shown in Table VIII (again, very similar to the set used in Ref. [17]) and $\Delta = 12/\sqrt{A}$ MeV, neutron quasiparticle energies and occupation probabilities are obtained in the BCS calculation. For negative-parity states, neutron orbitals of odd orbital angular momentum are included in the IBFM2 calculation. The mass dependence of boson-fermion interactions is presented in Table IX. Here we use the quadrupole and the monopole interactions between the odd neutron and the proton bosons and the exchange interaction.

In Fig. 5 we present a comparison of the calculated and experimental levels of negative parity in ^{69}Ge, ^{71}Ge, and ^{73}Ge, in Fig. 2 the electric quadrupole and magnetic dipole moments for the $5/2_1^-$ respective levels, and in Tables X, XI, and XII the branching ratios in these isotopes.

The calculated wave functions of $5/2_1^-$ and $1/2_1^-$ levels in ^{69}Ge are dominated by $f_{5/2}$ and $p_{1/2}$ neutron components, respectively, in accordance with the transfer reaction data. With the exception of the $5/2_3^-$ level, the whole decay pattern for negative-parity levels up to 1.5 MeV excitation energy is reasonably well described. The established correspondence between calculated and experimental levels enables a detailed calculation of β transitions into these levels.

The structure of ^{71}Ge is well described, too. The decay pattern (Table XI) is very similar to the one obtained in the IBFM1 calculation [21]. We have adopted the same assignment of calculated to experimental levels as in Ref. [21].

Following the arguments therein, we have used the $1/2_3^-$ assignment for the level at 886.9 keV excitation energy. On the basis of calculated decay properties (where only in few cases the strongest branch was not correctly predicted, but the main experimental branches were predicted as the strong ones), we are positive that at least up to 1.2 MeV excitation energy the wave functions are good enough for calculations of β transitions. There was an argument that $3/2_3^-$ at 0.831 MeV and $5/2_4^-$ at 1.212 MeV are coupled to the intruder state of the core and therefore outside of the model space [19]. This may explain the disagreement seen in higher $3/2^-$ and $5/2^-$ states. However, contributions of intruder components can be expected in many states of odd-mass Ge and As isotopes that we have investigated. On the basis of electromagnetic decays alone, it is not possible to positively determine which of them are predominantly based on the intruder configuration.

For ^{73}Ge a unique correspondence between the IBFM2 and experimental negative-parity levels can be achieved only up to excitation energy of 0.5 MeV. At higher excitation energies the spin assignments are unreliable but the number of low-spin levels up to approximately 1 MeV corresponds to the theoretical prediction.

IV. β DECAY FROM As TO Ge NUCLEI

In the framework of IBFM2 the Fermi $\Sigma_k t^\pm(k)$ and the Gamow-Teller $\Sigma_k t^\pm(k)\sigma(k)$ transition operators [28] can be constructed by the transfer operators [2,3,16,28]

$$A_m^{\dagger(j)} = \zeta_j a_{jm}^\dagger + \sum_{j'} \zeta_{jj'} s^\dagger [\tilde{d} a_{j'}^\dagger]_m^{(j)} \quad (\Delta n_j = 1, \Delta N = 0), \quad (17)$$
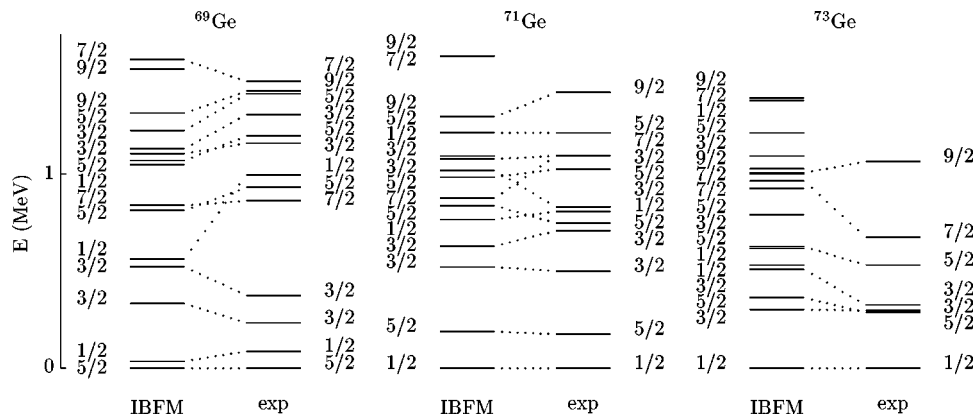

FIG. 5. The energy levels of the negative-parity states in Ge.

TABLE X. Branching ratios in ^{69}Ge.

Level (MeV)	Transition	I_γ(IBFM2)	I_γ(Expt.)
0.087	$1/2_1^- \to 5/2_1^-$	100	100
0.233	$3/2_1^- \to 1/2_1^-$	43.2	48.3 (13)
	$3/2_1^- \to 5/2_1^-$	100	100 (3)
0.374	$3/2_2^- \to 3/2_1^-$	0.7	4.6 (8)
	$3/2_2^- \to 1/2_1^-$	100	100.0 (15)
	$3/2_2^- \to 5/2_1^-$	0.1	31.5 (8)
0.862	$7/2_1^- \to 3/2_2^-$	0.4	0.76 (13)
	$7/2_1^- \to 3/2_1^-$	0.1	8.4 (21)
	$7/2_1^- \to 5/2_1^-$	100	100 (3)
0.933	$5/2_2^- \to 7/2_1^-$	0.0	
	$5/2_2^- \to 3/2_2^-$	0.5	32 (7)
	$5/2_2^- \to 3/2_1^-$	16.7	8
	$5/2_2^- \to 1/2_1^-$	35.5	24 (7)
	$5/2_2^- \to 5/2_1^-$	100	100 (5)
0.995	$1/2_2^- \to 5/2_2^-$	0.0	
	$1/2_2^- \to 3/2_2^-$	7.9	9 (6)
	$1/2_2^- \to 3/2_1^-$	26.8	41 (9)
	$1/2_2^- \to 1/2_1^-$	0.7	
	$1/2_2^- \to 5/2_1^-$	100	100 (21)
1.160	$3/2_3^- \to 1/2_2^-$	0.0	
	$3/2_3^- \to 5/2_2^-$	0.2	
	$3/2_3^- \to 7/2_1^-$	0.0	
	$3/2_3^- \to 3/2_2^-$	24.0	
	$3/2_3^- \to 3/2_1^-$	7.3	23 (3)
	$3/2_3^- \to 1/2_1^-$	100	100 (3)
	$3/2_3^- \to 5/2_1^-$	19.5	20 (3)
1.196	$5/2_3^- \to 3/2_3^-$	0.0	
	$5/2_3^- \to 1/2_2^-$	0.0	
	$5/2_3^- \to 5/2_2^-$	0.5	
	$5/2_3^- \to 7/2_1^-$	6.8	
	$5/2_3^- \to 3/2_2^-$	0.1	100 (9)
	$5/2_3^- \to 3/2_1^-$	100	16 (3)
	$5/2_3^- \to 1/2_1^-$	0.5	
	$5/2_3^- \to 5/2_1^-$	0.7	56 (3)
1.307	$3/2_4^- \to 5/2_3^-$	0.0	
	$3/2_4^- \to 3/2_3^-$	0.6	
	$3/2_4^- \to 1/2_2^-$	4.3	
	$3/2_4^- \to 5/2_2^-$	1.1	
	$3/2_4^- \to 7/2_1^-$	0.1	
	$3/2_4^- \to 3/2_2^-$	91.4	44 (22)
	$3/2_4^- \to 3/2_1^-$	97.9	13 (7)
	$3/2_4^- \to 1/2_1^-$	100	100 (4)
	$3/2_4^- \to 5/2_1^-$	8.5	92 (4)
1.415	$5/2_4^- \to 3/2_4^-$	0.7	
	$5/2_4^- \to 5/2_3^-$	0.0	
	$5/2_4^- \to 3/2_3^-$	16.3	
	$5/2_4^- \to 1/2_2^-$	0.0	
	$5/2_4^- \to 5/2_2^-$	2.1	

TABLE X. (*Continued.*)

Level (MeV)	Transition	I_γ(IBFM2)	I_γ(Expt.)
	$5/2_4^- \to 7/2_1^-$	3.1	
	$5/2_4^- \to 3/2_2^-$	100	39 (6)
	$5/2_4^- \to 3/2_1^-$	7.2	22 (6)
	$5/2_4^- \to 1/2_1^-$	35.1	
	$5/2_4^- \to 5/2_1^-$	12.4	100 (11)

$$B_m^{\dagger(j)} = \theta_j s^\dagger \tilde{a}_{jm} + \sum_{j'} \theta_{jj'} [d^\dagger \tilde{a}_{j'}]_m^{(j)} \quad (\Delta n_j = -1, \Delta N = 1).$$

(18)

The former creates a fermion, while the latter annihilates a fermion, simultaneously creating a boson. Either operator increases the quantity $n_j + 2N$ by one unit. The conjugate operators are

$$\tilde{A}_m^{(j)} = (-1)^{j-m} \{A_{-m}^{\dagger(j)}\}^\dagger = \zeta_j^* \tilde{a}_{jm} + \sum_{j'} \zeta_{jj'}^* s [d^\dagger \tilde{a}_{j'}]_m^{(j)}$$

$$\times (\Delta n_j = -1, \Delta N = 0),$$

(19)

$$\tilde{B}_m^{(j)} = (-1)^{j-m} \{B_{-m}^{\dagger(j)}\}^\dagger = -\theta_j^* s a_{jm}^\dagger - \sum_{j'} \theta_{jj'}^* [\tilde{d} a_{j'}^\dagger]_m^{(j)}$$

$$\times (\Delta n_j = 1, \Delta N = -1),$$

(20)

where the asterisks mean complex conjugate. These decrease the quantity $n_j + 2N$ by one unit.

The IBFM image of the Fermi $\Sigma_k \mathbf{t}^\pm(k)$ and the Gamow-Teller transition operator $\Sigma_k t^\pm(k) \sigma(k)$ are

$$O^F = \sum_j -\sqrt{2j+1} [P_\nu^{(j)} P_\pi^{(j)}]^{(0)},$$

(21)

$$O^{GT} = \sum_{j'j} \eta_{j'j} [P_\nu^{(j')} P_\pi^{(j)}]^{(1)},$$

(22)

where

$$\eta_{j'j} = -\frac{1}{\sqrt{3}} \langle l' \tfrac{1}{2}; j' \| \sigma \| l \tfrac{1}{2}; j \rangle$$

$$= -\delta_{l'l} \sqrt{2(2j'+1)(2j+1)} W(lj' \tfrac{1}{2} 1; \tfrac{1}{2} j).$$

(23)

The transfer operators $P_\rho^{(j)}$ are chosen from Eqs. (17)–(20) depending on the nuclei.

The ft value is

$$ft = \frac{6163}{\langle M_F \rangle^2 + (G_A/G_V)^2 \langle M_{GT} \rangle^2}$$

(24)

(in units of second), where $(G_A/G_V)^2 = 1.59$ and

$$\langle M_F \rangle^2 = \frac{1}{2I_i + 1} |\langle I_f \| O^F \| I_i \rangle|^2,$$

(25)

TABLE XI. Branching ratios in ^{71}Ge.

Level (MeV)	Transition	I_γ(IBFM2)	I_γ(Expt.)
0.175	$5/2_1^- \to 1/2_1^-$	100	100
0.500	$3/2_1^- \to 5/2_1^-$	4.2	0.5 (1)
	$3/2_1^- \to 1/2_1^-$	100	100.0 (4)
0.708	$3/2_2^- \to 3/2_1^-$	0.7	
	$3/2_2^- \to 5/2_1^-$	0.0	7.0 (5)
	$3/2_2^- \to 1/2_1^-$	100	100 (2)
0.747	$5/2_2^- \to 3/2_2^-$	0.0	
	$5/2_2^- \to 3/2_1^-$	2.4	74 (1)
	$5/2_2^- \to 5/2_1^-$	30.3	100 (2)
	$5/2_2^- \to 1/2_1^-$	100	62 (2)
0.808	$1/2_2^- \to 5/2_2^-$	0.0	
	$1/2_2^- \to 3/2_2^-$	2.2	
	$1/2_2^- \to 3/2_1^-$	40.9	24 (5)
	$1/2_2^- \to 5/2_1^-$	100	100 (12)
	$1/2_2^- \to 1/2_1^-$	66.6	62 (5)
0.831	$3/2_3^- \to 1/2_2^-$	0.0	
	$3/2_3^- \to 5/2_2^-$	0.0	
	$3/2_3^- \to 3/2_2^-$	0.8	
	$3/2_3^- \to 3/2_1^-$	1.6	1.9 (10)
	$3/2_3^- \to 5/2_1^-$	2.0	
	$3/2_3^- \to 1/2_1^-$	100	100.0 (19)
1.027	$5/2_3^- \to 3/2_3^-$	0.4	1.0 (5)
	$5/2_3^- \to 1/2_2^-$	0.0	
	$5/2_3^- \to 5/2_2^-$	27.0	21.6 (7)
	$5/2_3^- \to 3/2_2^-$	0.5	
	$5/2_3^- \to 3/2_1^-$	100	100 (10)
	$5/2_3^- \to 5/2_1^-$	52.8	21.0 (19)
	$5/2_3^- \to 1/2_1^-$	9.1	36.2 (10)
1.095	$3/2_4^- \to 5/2_3^-$	0.0	
	$3/2_4^- \to 3/2_3^-$	18.2	0.2 (1)
	$3/2_4^- \to 1/2_2^-$	0.1	0.52 (6)
	$3/2_4^- \to 5/2_2^-$	1.1	1.2 (2)
	$3/2_4^- \to 3/2_2^-$	0.6	0.30 (4)
	$3/2_4^- \to 3/2_1^-$	0.8	2.0 (2)
	$3/2_4^- \to 5/2_1^-$	8.2	7.4 (2)
	$3/2_4^- \to 1/2_1^-$	100	100.0 (14)
1.096	$7/2_1^- \to 3/2_4^-$	0.0	
	$7/2_1^- \to 5/2_3^-$	0.1	
	$7/2_1^- \to 3/2_3^-$	0.0	
	$7/2_1^- \to 5/2_2^-$	0.5	
	$7/2_1^- \to 3/2_2^-$	0.1	
	$7/2_1^- \to 3/2_1^-$	0.0	
	$7/2_1^- \to 5/2_1^-$	100	100
1.139	$3/2_5^- \to 7/2_1^-$	0.0	
	$3/2_5^- \to 3/2_4^-$	0.0	
	$3/2_5^- \to 5/2_3^-$	0.0	
	$3/2_5^- \to 3/2_3^-$	5.1	1.0 (3)
	$3/2_5^- \to 1/2_2^-$	9.2	2.6 (4)

TABLE XI. (*Continued.*)

Level (MeV)	Transition	I_γ(IBFM2)	I_γ(Expt.)
	$3/2_5^- \to 5/2_2^-$	10.4	4.3 (3)
	$3/2_5^- \to 3/2_2^-$	0.4	2.6 (1)
	$3/2_5^- \to 3/2_1^-$	100	6.1 (3)
	$3/2_5^- \to 5/2_1^-$	3.3	8.5 (2)
	$3/2_5^- \to 1/2_1^-$	27.8	100.0 (5)
1.212	$5/2_4^- \to 3/2_5^-$	0.0	
	$5/2_4^- \to 7/2_1^-$	0.0	
	$5/2_4^- \to 3/2_4^-$	9.6	
	$5/2_4^- \to 5/2_3^-$	0.6	
	$5/2_4^- \to 3/2_3^-$	0.8	
	$5/2_4^- \to 1/2_2^-$	0.0	
	$5/2_4^- \to 5/2_2^-$	25.5	28.8 (5)
	$5/2_4^- \to 3/2_2^-$	25.7	53 (15)
	$5/2_4^- \to 3/2_1^-$	14.8	100.0 (18)
	$5/2_4^- \to 5/2_1^-$	33.7	62.1 (8)
	$5/2_4^- \to 1/2_1^-$	100	85.6 (15)

$$\langle M_{\mathrm{GT}} \rangle^2 = \frac{1}{2I_i+1} |\langle I_f \| O^{\mathrm{GT}} \| I_i \rangle|^2. \tag{26}$$

The coefficients η_j, $\eta_{jj'}$, θ_j, and $\theta_{jj'}$ appearing in Eqs. (17)–(20) are [2]

$$\zeta_j = u_j \frac{1}{K'_j}, \tag{27}$$

$$\zeta_{jj'} = -v_j \beta_{j'j} \left(\frac{10}{N(2j+1)} \right)^{1/2} \frac{1}{KK'_j}, \tag{28}$$

$$\theta_j = \frac{v_j}{\sqrt{N}} \frac{1}{K''_j}, \tag{29}$$

$$\theta_{jj'} = u_j \beta_{j'j} \left(\frac{10}{2j+1} \right)^{1/2} \frac{1}{KK''_j}. \tag{30}$$

N is N_π or N_ν, depending on the transfer operator, and K, K'_j, and K''_j are determined by

TABLE XII. Branching ratios in ^{73}Ge.

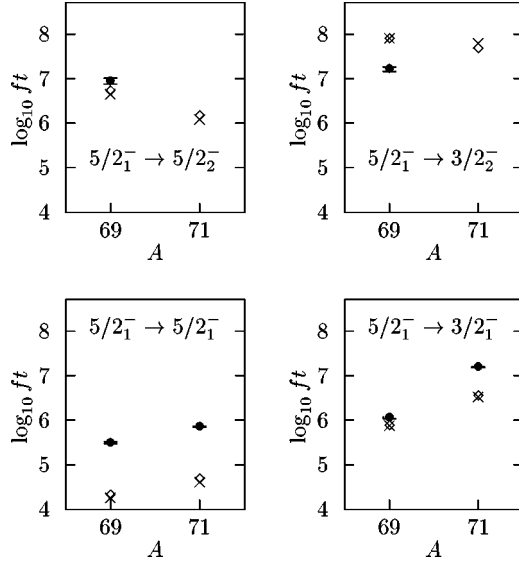| Level (MeV)	Transition	I_γ(IBFM2)	I_γ(Expt.)
0.364	$3/2_1^- \to 5/2_1^-$	0.0	
	$3/2_1^- \to 1/2_1^-$	100	100.0 (18)
0.392	$3/2_2^- \to 3/2_1^-$	12.9	
	$3/2_2^- \to 5/2_1^-$	41.1	
	$3/2_2^- \to 1/2_1^-$	100	100.0 (21)
0.741	$7/2_1^- \to 3/2_2^-$	0.1	
	$7/2_1^- \to 3/2_1^-$	1.2	25.7 (17)
	$7/2_1^- \to 5/2_1^-$	100	100 (3)

FIG. 6. The $\log_{10}ft$ values of the β decay from the As to the Ge isotopes. The symbol • shows the experimental values with their errors, while the symbol × shows the results of calculations with the conventional operators. The symbol ◊ shows the results of calculations with the additional d-boson number conserving terms.

$$K = \left(\sum_{jj'} \beta_{jj'}^2 \right)^{1/2} \tag{31}$$

and the conditions

$$\sum_{\alpha J} \langle \text{odd}; \alpha J \| A^{\dagger j} \| \text{even}; 0_1^+ \rangle^2 = (2j+1)u_j^2, \tag{32}$$

$$\sum_{\alpha J} \langle \text{even}; 0_1^+ \| B^{\dagger j} \| \text{odd}; \alpha J \rangle^2 = (2j+1)v_j^2. \tag{33}$$

When the odd fermion is a hole with respect to the boson core, u_j and v_j are interchanged in Eqs. (27)–(33).

The systematics of $\log_{10}ft$ values is presented in Fig. 6. In Tables XIII and XIV the calculated $\log_{10}ft$ values are compared with the experimental values in ^{69}Ge and ^{71}Ge, respec-

TABLE XIII. $\log_{10}ft$ values for levels in ^{69}Ge.

Level	$\log_{10}ft$(IBFM2)	$\log_{10}ft$(Expt.)
$3/2_1^-$	5.88	6.05 (2)
$3/2_2^-$	7.90	7.21 (5)
$3/2_3^-$	5.07	6.79 (4)
$3/2_4^-$	6.46	6.71 (6)
$3/2_5^-$	6.73	7.02 (6)
$5/2_1^-$	4.26	5.49 (2)
$5/2_2^-$	6.65	6.94 (7)
$5/2_3^-$	5.33	6.65 (5)
$5/2_4^-$	5.49	6.80 (6)
$7/2_1^-$	7.54	6.98 (5)
$7/2_2^-$	6.54	6.81 (5)
$7/2_3^-$	5.96	6.20 (5)

TABLE XIV. $\log_{10}ft$ values for levels in ^{71}Ge.

Level	$\log_{10}ft$(IBFM2)	$\log_{10}ft$(Expt.)
$3/2_1^-$	6.52	7.19 (1)
$3/2_2^-$	7.79	
$3/2_3^-$	5.73	
$3/2_4^-$	5.21	6.33 (1)
$3/2_5^-$	7.34	6.94 (1)
$5/2_1^-$	4.60	5.85 (1)
$5/2_2^-$	6.08	
$5/2_3^-$	5.63	6.87 (2)
$5/2_4^-$	5.55	6.84 (2)
$7/2_1^-$	7.60	8.79 (25)

tively (the ground states of parent ^{69}As and ^{71}As nuclei are $5/2_1^-$ levels). The hierarchy of values for transitions into different states of each angular momentum is reproduced for ^{69}Ge (except for the transition to the $3/2_3^-$ level, which is predicted to have a rather small $\log_{10}ft$ value). The same is true for ^{71}Ge. We notice that the theory predicts that the smallest $\log_{10}ft$ value among all $3/2^-$ levels in ^{71}Ge has the $3/2_4^-$ level. This result is in agreement with the experimental data. The only available experimental $\log_{10}ft$ value in ^{73}Ge is for the $1/2_1^-$ level ($\log_{10}ft = 5.4$). The corresponding theoretical value (4.27) is the smallest calculated.

Although the calculated distributions of the β feeding are in agreement with the experimental data, a systematic effect can be seen. For most decays the calculated $\log_{10}ft$ values are smaller than the experimental values. Some previous works introduced overall normalization factors to account for the absolute values of the β-transition rates [3]. In contrast to this approach, which is common in shell model calculations, we do not introduce any adjustable parameters in the β-decay operators. If one takes the transition operators without normalization parameters, then the difference between the calculated and experimental values is caused by the transition matrix elements, which in our case have to be overestimated. This may indicate that other components are admixed in the wave functions (for example, those involving intruder states), which would decrease the amplitudes of the present IBFM2 components, leading to an increase of the theoretical $\log_{10}ft$ values. If this is the case, then the differences between the IBFM2 and experimental $\log_{10}ft$ values have to be larger for ^{71}Ge and ^{73}Ge (which are neighbors of ^{72}Ge where the intruder 0^+ level is the first excited state) than for ^{69}Ge. Our calculations indicate this trend.

Barea *et al.* introduced to the particle transfer operator the additional d-boson number conserving terms [29]

$$\sum_{j',J} \phi_{jj'}^J [(a_{j'}^\dagger \times d^\dagger)^{(J)} \times \tilde{d}]_m^{(j)}. \tag{34}$$

The coefficients $\phi_{jj'}^J$ are determined so that the matrix elements of the fermion operator $C_j^\dagger$ and the IBFM transfer operator between states of seniority $v=2$ and $v=3$ in the corresponding spaces are equal. As a result of including

these terms, the transfer operator of Eq. (17) is modified as

$$A_m^{\dagger(j)} = \frac{u_j}{K_j'} a_{jm}^\dagger - \sum_{j'} \frac{v_j \beta_{j'j}}{K_j' K} \left(\frac{10}{N(2j+1)} \right)^{1/2} s^\dagger [\tilde{d} a_{j'}^\dagger]_m^{(j)}$$

$$+ \frac{1}{K_j'} \sum_{j'J} \phi_{jj'}^J [[a_{j'}^\dagger, d^\dagger]^{(J)} \tilde{d}]_m^{(j)}, \tag{35}$$

with the same normalization conditions (31) and (32) The results of the calculation with this transfer operator are shown by the symbol $\diamond$ in Fig. 6. The effect of the additional terms is small. The overall normalization factor K_j' may be reducing the effects of the additional terms. In fact, this factor was introduced in a phenomenological study of transfer reactions in order to compensate for the effect of the truncation higher-order terms [30].

V. CONCLUSIONS

In a systematic calculation, we have performed an IBFM2 analysis of negative-parity states in $A = 69$, 71, and 73 iso-topes of As and Ge. Nuclei in this mass region display a complex structure. The overall agreement between the experimental data and IBFM2 calculated properties is very good for the excitation energies and electromagnetic transition properties. For the β-decay rates that involve matrix elements with wave functions of these nuclei, providing a fine test of the nuclear model, the calculated values are satisfactory but for most decays the calculated $\log_{10} ft$ values are smaller than the experimental values. The effect of the additional d-boson number conserving term in the particle transfer operator, in the case of nuclei considered here, was small. This may indicate that other components in the wave functions, like those involving intruder states, are admixed in the wave functions of low-lying states in odd-mass nuclei in this mass region.

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