PHYSICAL REVIEW D **84,** 016006 (2011)

η' multiplicity and the Witten-Veneziano relation at finite temperature

S. Benić,[1] D. Horvatić,[1] D. Kekez,[2] and D. Klabučar[1,*]
[1]Physics Department, Faculty of Science, University of Zagreb, Bijenička c. 32, Zagreb 10000, Croatia
[2]Rugjer Bošković Institute, Bijenička c. 54, Zagreb 10000, Croatia
(Received 3 May 2011; published 21 July 2011)

We discuss and propose the minimal generalization of the Witten-Veneziano relation to finite temperatures, prompted by STAR and PHENIX experimental results on the multiplicity of η' mesons. After explaining why these results show that the zero-temperature Witten-Veneziano relation cannot be straightforwardly extended to temperatures T too close to the chiral restoration temperature T_{Ch} and beyond, we find the quantity which should replace, at $T > 0$, the Yang-Mills topological susceptibility appearing in the $T = 0$ Witten-Veneziano relation, in order to avoid the conflict with experiment at $T > 0$. This is illustrated through concrete T-dependences of pseudoscalar meson masses in a chirally well-behaved, Dyson-Schwinger approach, but our results and conclusions are of a more general nature and, essentially, model-independent.

DOI: 10.1103/PhysRevD.84.016006 PACS numbers: 11.10.St, 11.10.Wx, 12.38.−t, 24.85.+p

I. INTRODUCTION

The ultrarelativistic heavy-ion collider facilities like RHIC at BNL and LHC at CERN strive to produce a new form of hot QCD matter. The experiments show [1,2] that it has very intricate properties and presents a big challenge especially for theoretical understanding. While above the (pseudo)critical temperature $T_c \sim 170$ MeV this matter is often called the quark-gluon plasma (QGP), it cannot be a perturbatively interacting quark-gluon gas (as widely expected before RHIC results [1,2]) until significantly higher temperatures $T \gg T_c$. Instead, the interactions and correlations in the hot QCD matter are still strong (e.g., see Refs. [3,4]) so that its more recent and more precise name is *strongly coupled QGP* (sQGP) [4]. One of its peculiarities seems to be that strong correlations in the form of quark-antiquark ($q\bar{q}$) bound states and resonances still exist [3,5] in the sQGP well above T_c. In the old QGP paradigm, even deeply bound charmonium ($c\bar{c}$) states such as J/Ψ and η_c were expected to unbind at $T \approx T_c$, but lattice QCD simulations of mesonic correlators now indicate they persist till around $2T_c$ [6,7] or even above [8]. Similar indications for light-quark mesonic bound states are also accumulating from lattice QCD [9] and from other methods [3,10,11]. This agrees well with the findings on the lattice (e.g., see Ref. [12] for a review) that for realistic explicit chiral symmetry breaking (ChSB), i.e., for the physical values of the current quark masses, the transition between the hadron phase and the phase dominated by quarks and gluons, is not an abrupt, singular phase transition but a smooth, analytic crossover around the *pseudo* critical temperature T_c. It is thus not too surprising that a clear experimental signal of, e.g., deconfinement, is still hard to find and identify unambiguously.

The most compelling signal for production of a new form of QCD matter, i.e., sQGP, would be a restoration—in hot and/or dense matter—of the symmetries of the QCD Lagrangian which are broken in the vacuum. One of them is the [$SU_A(N_f)$ flavor] chiral symmetry, whose dynamical breaking results in light, (almost-)Goldstone pseudoscalar (P) mesons—namely the octet $P = \pi^0, \pi^\pm, K^0, \bar{K}^0, K^\pm, \eta$, as we consider all three light-quark flavors, $N_f = 3$. The second one is the $U_A(1)$ symmetry. Its breaking by the non-Abelian axial Adler-Bell-Jackiw anomaly ("gluon anomaly" for short) makes the remaining pseudoscalar meson of the light-quark sector, the η', much heavier, preventing its appearance as the ninth (almost-) Goldstone boson of dynamical chiral symmetry breaking (DChSB) in QCD.

The first experimental signature of a partial restoration of the $U_A(1)$ symmetry seems to have been found in the $\sqrt{s_{NN}} = 200$ GeV central Au + Au reactions at RHIC. Namely, Csörgő *et al.* [13,14] analyzed combined data of PHENIX [15] and STAR [16] collaborations very robustly, through six popular models for hadron multiplicities, and found that at 99.9% confidence level, the η' mass, which in the vacuum is $M_{\eta'} = 957.8$ MeV, is reduced by at least 200 MeV inside the fireball. It is the sign of the disappearing contribution of the gluon axial anomaly to the η' mass, which would drop to a value readily understood together with the (flavor-symmetry-broken) octet of $q\bar{q}'$ ($q, q' = u, d, s$) pseudoscalar mesons. This is the issue of the "return of the prodigal Goldstone boson" predicted [17] as a signal of the $U_A(1)$ symmetry restoration.

Another related but less obvious issue to which we want to draw attention, concerns the status, at $T > 0$, of the famous Witten-Veneziano relation (WVR) [18,19]

*klabucar@oberon.phy.hr
Corresponding author.
Senior Associate of Abdus Salam ICTP, Trieste, Italy.

$$M_{\eta'}^2 + M_\eta^2 - 2M_K^2 = \frac{6\chi_{\rm YM}}{f_\pi^2} \tag{1}$$

between the η', η and K-meson masses $M_{\eta',\eta,K}$, pion decay constant f_π, and Yang-Mills (YM) topological susceptibility $\chi_{\rm YM}$. WVR was obtained in the limit of large number of colors N_c [18,19]. It is well satisfied at $T = 0$ for $\chi_{\rm YM}$ obtained by lattice calculations (e.g., [20–23]). Nevertheless, the T-dependence of $\chi_{\rm YM}$ is such [24] that the straightforward extension of Eq. (1) to $T > 0$ [24], i.e., replacement of all quantities[1] therein by their respective T-dependent versions $M_{\eta'}(T)$, $M_\eta(T)$, $M_K(T)$, $f_\pi(T)$, *and* $\chi_{\rm YM}(T)$, leads to a conflict with experiment [13,14]. Since this extension of Eq. (1) to $T > 0$ was studied in Ref. [24] before the pertinent experimental analysis [13,14], one of the purposes of this paper is to revisit the implications of the results of Ref. [24] for WVR at $T > 0$, and demonstrate explicitly that they are practically model-independent. The other, more important purpose is to propose a mechanism which can enable WVR to agree with experiment at $T > 0$.

II. THE RELATIONS CONNECTING TWO THEORIES, QCD AND YM

Both issues pointed out before Eq. (1) and around it are best understood in a model-independent way if one starts from the chiral limit of vanishing current quark masses ($m_q = 0$) for all three light flavors, $q = u, d, s$. Then not only pions and kaons are massless, but is also η, which is then (since the situation is also SU(3)-flavor-symmetric) a purely SU(3)-octet state, $\eta = \eta_8$. In contrast, η' is then purely singlet, $\eta' = \eta_0$; since the divergence of the singlet axial quark current $\bar{q}\gamma^\mu\gamma_5\frac{1}{2}\lambda^0 q$ is nonvanishing even for $m_q = 0$ due to the gluon anomaly, the η' mass squared receives the anomalous contribution $\Delta M_{\eta'}^2$ ($= \lambda^4/f_{\eta'}^2$ in the notation of Ref. [17]) which is nonvanishing even in the chiral limit

$$\frac{\lambda^4}{f_{\eta'}^2} = \Delta M_{\eta_0}^2 = \Delta M_{\eta'}^2 = \frac{6\chi_{\rm YM}}{f_\pi^2} + O\left(\frac{1}{N_c}\right). \tag{2}$$

However, λ^4 and $f_{\eta'}$ are known accurately[2] only in the large N_c limit. There, in the leading order in $1/N_c$, λ^4 is given by the YM (i.e., "pure glue") topological susceptibility $\chi_{\rm YM}$ times $2N_f = 6$ [18,19], and the "η' decay constant" $f_{\eta'}$ is the same as f_π [27]. Thus, keeping only the leading order in $1/N_c$, the last equality is WVR in the chiral limit.

The consequences of Eq. (2) remain qualitatively the same realistically away from the chiral limit. This will soon become clear on the basis of, e.g., Eq. (3) below. Namely, due to DChSB in QCD, for relatively light current quark masses m_q ($q = u, d, s$), the $q\bar{q}'$ bound-state pseudoscalar meson masses (including the *nonanomalous* parts of the η' and η masses) behave as

$$M_{q\bar{q}'}^2 = \text{const}(m_q + m_{q'}), \qquad (q, q' = u, d, s). \tag{3}$$

The pseudoscalar mesons (including η') thus obtain relatively light nonanomalous contributions $M_{q\bar{q}'}$ to their masses M_P, allowing them to reach the empirical values. That is, instead of the eight strictly massless Goldstone bosons, π^0, $\pi^\pm$, K^0, $\bar{K}^0$, $K^\pm$, and η are relatively light almost-Goldstones. Among them, in the limit of isospin symmetry ($m_u = m_d$), only η now receives also the gluon-anomaly contribution since the explicit SU(3) flavor breaking between the nonstrange (NS) u, d-quarks and s-quarks causes the mixing between the isoscalars η and η'. For $m_q \neq 0$, Eq. (2) is replaced by the usual WVR (1) containing also the nonanomalous contributions to meson masses. Nevertheless, these contributions largely cancel due to the approximate SU(3) flavor symmetry and to DChSB (i.e., Eq. (3)).

This can be seen assuming the usual SU(3) $q\bar{q}$ content of the pseudoscalar meson nonet with well-defined isospin[3] quantum numbers, in particular, the isoscalar ($I = 0$) octet and singlet etas, $\eta_8 = (u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6}$, $\eta_0 = (u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3}$, whose mixing yields the physical particles η and η'. Since the *nonanomalous* parts of the η_0 and η_8 masses squared, $M_{\eta_0}^2$ and $M_{\eta_8}^2$, are respectively $M_{00}^2 \approx \frac{2}{3}M_K^2 + \frac{1}{3}M_\pi^2$ and $M_{88}^2 \approx \frac{4}{3}M_K^2 - \frac{1}{3}M_\pi^2$ (see, e.g., Ref. [30]), and since $M_{\eta_8}^2 + M_{\eta_0}^2 = M_\eta^2 + M_{\eta'}^2$, the nonanomalous parts of the η and η' masses are canceled by $2M_K^2$ in WVR (1). Another way of seeing this is expressing the nonanomalous parts of $M_\eta^2 + M_{\eta'}^2 = M_{\eta_8}^2 + M_{\eta_0}^2$ by Eq. (3). Thus again $M_\eta^2 + M_{\eta'}^2 - 2M_K^2 \approx \Delta M_{\eta_0}^2$, showing again that already WVR's chiral-limit-nonvanishing part (2) reveals the essence of the influence of the gluon anomaly on the masses in the η'-η complex. This is important also for the presently pertinent finite-T context because thanks to this, below it will be shown model-independently that WVR (1) containing the YM topological susceptibility $\chi_{\rm YM}$ implies T-dependence of η' mass in conflict with the recent experimental results [13,14].

Namely, the gluon-anomaly contribution (2) is established at $T = 0$ but it is not expected to persist at high

[1]Throughout this paper, all quantities are for definiteness assumed at $T = 0$ unless their T-dependence is specifically indicated in formulas or in the text.

[2]Also note that a unique "η' decay constant" $f_{\eta'}$ is, strictly speaking, not a well-defined quantity, as *two* η' decay constants are actually needed: the singlet one, $f_{\eta'}^0$, and the octet one, $f_{\eta'}^8$; *e.g.*, see an extensive review [25] or the short Appendix of Ref. [26].

[3]Vanishing of the anomaly at sufficiently high T opens the possibility of studying the interesting scenario of maximal isospin violation at high T [28,29], but as the effects of the small difference between m_u and m_d are not important for the present considerations, we stick to the isospin limit throughout the present paper.

temperatures. Ultimately, η' should also become a massless Goldstone boson at sufficiently high T, where $\chi_{\rm YM}(T) \rightarrow 0$. However, according to WVR, $\Delta M_{\eta'}(T)$ falls only for T where $f_\pi(T)^2$ does not fall faster than $6\chi_{\rm YM}(T)$, as stressed in Ref. [24].

The WVR's chiral-limit version (2) manifestly points out the ratio $\chi_{\rm YM}(T)/f_\pi(T)^2$ as crucial for the anomalous η' mass, but the above discussion shows that this remains essentially the same away from the chiral limit.

In the present context, it is important for practical calculations to go realistically away from the chiral limit, in which the chiral restoration is a sharp phase transition at its critical temperature $T_{\rm Ch}$ where the chiral-limit pion decay constant vanishes very steeply, i.e., as steeply as the chiral quark condensate. In contrast, for realistic explicit ChSB, i.e., m_u and m_d of several MeV, this transition is a *smooth crossover* (e.g., see Ref. [12]). For the pion decay constant, this implies that $f_\pi(T)$ still falls relatively steeply around *pseudo* critical temperature $T_{\rm Ch}$, but less so than in the chiral case, and even remains finite, enabling the usage of WVR (1) for the temperatures across the chiral and $U_A(1)$ symmetry restorations.

WVR is very remarkable because it connects two different theories: QCD with quarks and its pure-gauge, YM counterpart. The latter, however, has much higher characteristic temperatures than QCD with quarks: the ‘‘melting temperature’’ $T_{\rm YM}$ where $\chi_{\rm YM}(T)$ starts to decrease appreciably was found on lattice to be, for example, $T_{\rm YM} \approx 260$ MeV [31,32] or even higher, $T_{\rm YM} \approx 300$ MeV [33]. In contrast, the pseudocritical temperatures for the chiral and deconfinement transitions in the full QCD are lower than $T_{\rm YM}$ by some 100 MeV or more (e.g., see Ref. [12]) due to the presence of the quark degrees of freedom.

This difference in characteristic temperatures, in conjunction with $\chi_{\rm YM}(T)$ in WVRs (1) and (2) would imply that the (partial) restoration of the $U_A(1)$ symmetry (understood as the disappearance of the anomalous η_0/η' mass) should happen well after the restoration of the chiral symmetry. But, this contradicts the RHIC experimental observations of the reduced η' mass [13,14] *if* WVRs (1) and (2) hold unchanged also close to the QCD chiral restoration temperature $T_{\rm Ch}$, around which $f_\pi(T)$ decreases still relatively steeply[4] [24] for realistic explicit ChSB, thus leading to the increase of $6\chi_{\rm YM}(T)/f_\pi(T)^2$ and consequently also of $M_{\eta'}$.

There is still more to the relatively high resistance of $\chi_{\rm YM}(T)$ to temperature: not only does it start falling at rather high $T_{\rm YM}$, but $\chi_{\rm YM}(T)$ found on the lattice is falling with T *relatively* slowly. In some of the applications in the past (e.g., see Refs. [34,35]), it was customary to simply rescale a temperature characterizing the pure-gauge, YM sector to a value characterizing QCD with quarks. (For example, Refs. [34,35] rescaled $T_{\rm YM} = 260$ MeV found by Ref. [31] to 150 MeV). However, even if we rescale the critical temperature for melting of the topological susceptibility $\chi_{\rm YM}(T)$ from $T_{\rm YM}$ down to $T_{\rm Ch}$, the value of $6\chi_{\rm YM}(T)/f_\pi(T)^2$ still increases a lot [24] for the pertinent temperature interval starting already below $T_{\rm Ch}$. This happens because $\chi_{\rm YM}(T)$ falls with T more slowly than $f_\pi(T)^2$. (It was found [24] that the rescaling of $T_{\rm YM}$ would have to be totally unrealistic, to less than 70% of $T_{\rm Ch}$, in order to achieve sufficiently fast drop of the anomalous contribution that would allow the observed enhancement in the η' multiplicity.)

These WVR-induced enhancements of the η' mass for $T \sim T_{\rm Ch}$ were first noticed in Ref. [24]. This reference used a concrete dynamical model (with an effective, rank-2 separable interaction, convenient for computations at $T \geq 0$) [36] of low-energy, nonperturbative QCD to obtain mesons as $q\bar{q}'$-bound states in Dyson-Schwinger (DS) approach [37–39], which is a bound-state approach with the correct chiral behavior (3) of QCD. Nevertheless, this concrete dynamical DS model was used in Ref. [24] to get concrete values for only the nonanomalous parts of the meson masses, but was essentially *not* used to get model predictions for the mass contributions from the gluon anomaly, in particular $\chi_{\rm YM}(T)$. On the contrary, the anomalous mass contribution was included, in the spirit of $1/N_c$ expansion, through WVR (1). Thus, the T-evolution of the η'-η complex in Ref. [24] was not dominated by dynamical model details, but by WVR, i.e., the ratio $6\chi_{\rm YM}(T)/f_\pi(T)^2$. Admittedly, $f_\pi(T)$ was also calculated within this model, causing some *quantitative* model dependence of the anomalous mass in WVR, but this cannot change the qualitative observations of Ref. [24] on the η' mass enhancement. Namely, our model $f_\pi(T)$, depicted as the dash-dotted curve in Fig. 1, obviously has the right crossover features [12]. It also agrees qualitatively with $f_\pi(T)$'s calculated in other realistic dynamical models [10,38]. Various modifications were tried in Ref. [24] but could not reduce much the η' mass enhancement caused by this ratio, let alone bring about the significant η' mass reduction found in the RHIC experiments [13,14].

One must therefore conclude that either WVR breaks down as soon as T approaches $T_{\rm Ch}$, or that the T-dependence of its anomalous contribution is different from the pure-gauge $\chi_{\rm YM}(T)$. We will show that the latter alternative is possible, since WVR can be reconciled with experiment thanks to the existence of another relation which, similarly to WVR, connects the YM theory with full QCD. Namely, using large-N_c arguments, Leutwyler and Smilga derived [27], at $T = 0$,

$$\chi_{\rm YM} = \frac{\chi}{1 + \chi \frac{N_f}{m\langle\bar{q}q\rangle_0}} (\equiv \tilde{\chi}), \qquad (4)$$

[4] Relative to decay constants of mesons containing a strange quark; e.g., compare $f_{s\bar{s}}(T)$ of the unphysical $s\bar{s}$ pseudoscalar with $f_\pi(T)$ in Fig. 1.

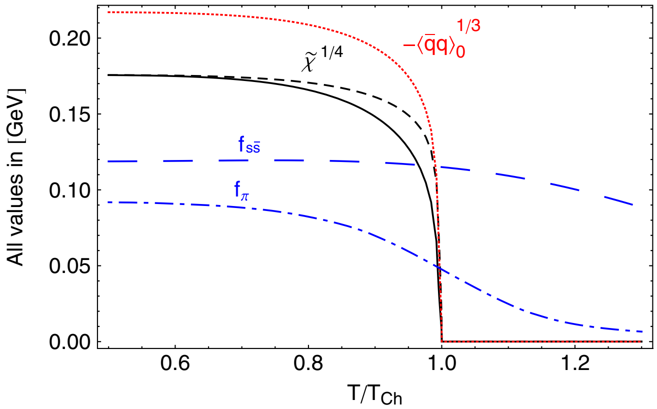

FIG. 1 (color online). The relative-temperature dependences, on $T/T_{\rm Ch}$, of $\tilde{\chi}^{1/4}$, $\langle\bar{q}q\rangle_0^{1/3}$, f_π and $f_{s\bar{s}}$, i.e., the $T/T_{\rm Ch}$-dependences of the quantities entering in the anomalous contributions to various masses in the η'-η complex—see Eq. (10) and formulas below it. The solid curve depicts $\tilde{\chi}^{1/4}$ for $\delta = 0$ in Eq. (9), and the short-dashed curve is $\tilde{\chi}^{1/4}$ for $\delta = 1$. At $T = 0$, the both $\tilde{\chi}$'s are equal to $\chi_{\rm YM} = (0.1757\ {\rm GeV})^4$, the weighted average [24] of various lattice results for $\chi_{\rm YM}$. The dotted (red) curve depicts $-\langle\bar{q}q\rangle_0^{1/3}$, the dash-dotted (blue) curve is f_π, and the long-dashed (blue) curve is $f_{s\bar{s}}$.

the relation (in our notation) between the YM topological susceptibility $\chi_{\rm YM}$, and the full-QCD topological susceptibility χ, the *chiral-limit* quark condensate $\langle\bar{q}q\rangle_0$, and m, the harmonic average of N_f current quark masses m_q. That is, m is N_f times the reduced mass. In the present case of $N_f = 3$, $q = u, d, s$, so that

$$\frac{N_f}{m} = \sum_{q=u,d,s} \frac{1}{m_q}. \tag{5}$$

Equation (4) is a remarkable relation between the two pertinent theories. For example, in the limit of all very heavy quarks ($m_q \to \infty$, $q = u, d, s$), it correctly leads to the result that $\chi_{\rm YM}$ is equal to the value of the topological susceptibility in *quenched* QCD, $\chi_{\rm YM} = \chi(m_q = \infty)$. This holds because χ is by definition the vacuum expectation value of a gluonic operator, so that the absence of quark loops would leave only the pure-gauge, YM contribution. However, the Leutwyler-Smilga relation (4) also holds in the opposite (and presently pertinent) limit of light quarks. This limit still presents a problem for getting the full-QCD topological susceptibility χ on the lattice [40], but we can use the light-quark-sector result [27,41]

$$\chi = -\frac{m\langle\bar{q}q\rangle_0}{N_f} + \mathcal{C}_m, \tag{6}$$

where $\mathcal{C}_m$ stands for corrections of higher orders in small m_q, and thus of small magnitude. The leading term is positive (as $\langle\bar{q}q\rangle_0 < 0$), but $\mathcal{C}_m$ is negative, since Eq. (4) shows that $\chi \le \min(-m\langle\bar{q}q\rangle_0/N_f, \chi_{\rm YM})$.

Although small, $\mathcal{C}_m$ should not be neglected, since $\mathcal{C}_m = 0$ would imply, through Eq. (4), that $\chi_{\rm YM} = \infty$. Instead, its value (at $T = 0$) is fixed by Eq. (4):

$$\mathcal{C}_m = \mathcal{C}_m(0) = \frac{m\langle\bar{q}q\rangle_0}{N_f}\left(1 - \chi_{\rm YM}\frac{N_f}{m\langle\bar{q}q\rangle_0}\right)^{-1}. \tag{7}$$

All this starting from Eq. (4) has so far been at $T = 0$. If the left- and right-hand side of Eq. (4) are extended to $T > 0$, it is obvious that the equality cannot hold at arbitrary temperature $T > 0$. The relation (4) must break down somewhere close to the (pseudo)critical temperatures of the full QCD ($\sim T_{\rm Ch}$) since the pure-gauge quantity $\chi_{\rm YM}$ is much more temperature-resistant than the right-hand side, abbreviated as $\tilde{\chi}$. The quantity $\tilde{\chi}$, which may be called the effective susceptibility, consists of the full QCD quantities χ and $\langle\bar{q}q\rangle_0$, the quantities of full QCD with quarks, characterized by $T_{\rm Ch}$, just as $f_\pi(T)$. As $T \to T_{\rm Ch}$, the chiral quark condensate $\langle\bar{q}q\rangle_0(T)$ drops faster than the other DChSB parameter in the present problem, namely $f_\pi(T)$ for realistically small explicit ChSB. (See Fig. 1 for the results of the dynamical model adopted here from Ref. [24], and, e.g., Refs. [10,38] for analogous results of different DS models). Thus, the troublesome mismatch in T-dependences of $f_\pi(T)$ and the pure-gauge quantity $\chi_{\rm YM}(T)$, which causes the conflict of the temperature-extended WVR with experiment at $T \gtrsim T_{\rm Ch}$, is expected to disappear if $\chi_{\rm YM}(T)$ is replaced by $\tilde{\chi}(T)$, the temperature-extended effective susceptibility. The successful zero-temperature WVR (1) is, however, retained, since $\chi_{\rm YM} = \tilde{\chi}$ at $T = 0$.

Extending Eq. (6) to $T > 0$ is something of a guesswork as there is no guidance from the lattice for $\chi(T)$ (unlike $\chi_{\rm YM}(T)$). Admittedly, the leading term is straightforward as it is plausible that its T-dependence will simply be that of $\langle\bar{q}q\rangle_0(T)$. Nevertheless, for the correction term $\mathcal{C}_m$ such a plausible assumption about the form of T-dependence cannot be made and Eq. (7), which relates YM and QCD quantities, only gives its value at $T = 0$. We will therefore explore the T-dependence of the anomalous masses using the following Ansatz for the $T \ge 0$ generalization of Eq. (6):

$$\chi(T) = -\frac{m\langle\bar{q}q\rangle_0(T)}{N_f} + \mathcal{C}_m(0)\left[\frac{\langle\bar{q}q\rangle_0(T)}{\langle\bar{q}q\rangle_0(T=0)}\right]^\delta, \tag{8}$$

where the correction-term T-dependence is parametrized through the power δ of the presently fastest-vanishing (as $T \to T_{\rm Ch}$) chiral order parameter $\langle\bar{q}q\rangle_0(T)$.

The $T \ge 0$ extension (8) of the light-quark χ, Eq. (6), leads to the $T \ge 0$ extension of $\tilde{\chi}$:

$$\tilde{\chi}(T) = \frac{m\langle\bar{q}q\rangle_0(T)}{N_f}\left(1 - \frac{m\langle\bar{q}q\rangle_0(T)}{N_f\mathcal{C}_m(0)}\left[\frac{\langle\bar{q}q\rangle_0(T=0)}{\langle\bar{q}q\rangle_0(T)}\right]^{\delta}\right). \tag{9}$$

We now use $\tilde{\chi}(T)$ in WVR instead of $\chi_{\rm YM}(T)$ used by Ref. [24]. This gives us the temperature dependences of the masses in the η-η' complex, such as those in Fig. 2 illustrating the cases $\delta = 0$ and $\delta = 1$.

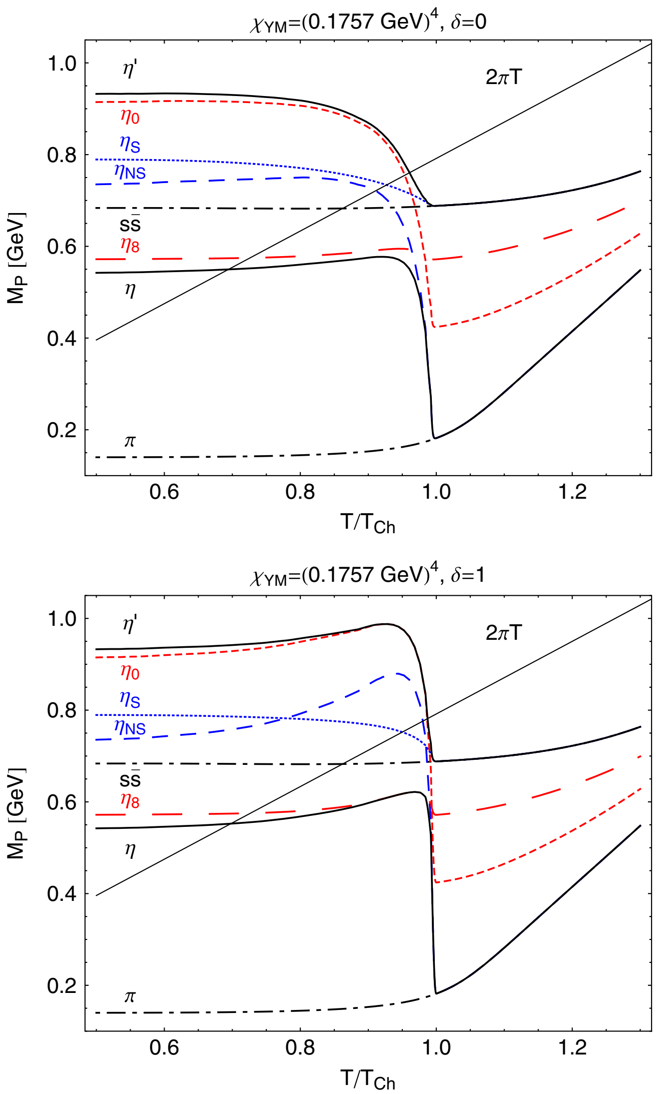

FIG. 2 (color online). The relative-temperature dependence, on $T/T_{\rm Ch}$, of the pseudoscalar meson masses for two $\tilde{\chi}(T)$, namely, Eq. (8) with $\delta = 0$ (upper panel) and with $\delta = 1$ (lower panel). The meaning of all symbols is the same on the both panels: the masses of η' and η are, respectively, the upper and lower solid curve, those of the pion and nonanomalous $s\bar{s}$ pseudoscalar are, respectively, the lower and upper dash-dotted curve, M_{η_0} and M_{η_8} are, respectively, the short-dashed (red) and long-dashed (red) curve, $M_{\eta_{NS}}$ is the medium-dashed (blue), and M_{η_S} is the dotted (blue) curve. The straight line $2\pi T$ is twice the lowest Matsubara frequency.

It is clear that $\tilde{\chi}(T)$ (9) blows up as $T \to T_{\rm Ch}$ if the correction term there vanishes faster than $\langle\bar{q}q\rangle_0(T)$ *squared*. Thus, varying δ between 0 and 2 covers the cases from the T-independent correction term, to (already experimentally excluded) enhanced anomalous masses for δ noticeably above 1, to even sharper mass blow-ups for $\delta \to 2$ when $T \to T_{\rm Ch}$. On the other hand, it does not seem natural that the correction term vanishes faster than the fastest-vanishing order parameter $\langle\bar{q}q\rangle_0(T)$. Indeed, already for the same rate of vanishing of the both terms ($\delta = 1$), one can notice in Fig. 2 the start of the precursors of the blow-up of various masses in the η'-η complex as $T \to T_{\rm Ch}$ although these small mass bumps are still experimentally acceptable. Thus, in Fig. 2 we depict the $\delta = 1$ case, and $\delta = 0$ (T-independent correction term) as the other acceptable extreme. Since they turn out to be not only qualitatively, but also quantitatively so similar that the present era experiments cannot discriminate between them, there is no need to present any "in-between results," for $0 < \delta < 1$.

Next we turn to completing the explanation how the above-mentioned results in Fig. 2 were obtained.

Using $\tilde{\chi}(T)$ in WVR instead of $\chi_{\rm YM}(T)$ used by Ref. [24], does not change anything at $T = 0$, where $\tilde{\chi}(T) = \chi_{\rm YM}(0)$, which remains an excellent approximation even well beyond $T = 0$. Nevertheless, this changes drastically as T approaches $T_{\rm Ch}$. For $T \sim T_{\rm Ch}$, the behavior of $\tilde{\chi}(T)$ is dominated by the T-dependence of the chiral condensate, tying the restoration of the $U_A(1)$ symmetry to the chiral symmetry restoration.

As for the nonanomalous contributions to the meson masses, we use the same DS model (and parameter values) as in Ref. [24], since it includes both DChSB and correct QCD chiral behavior as well as realistic explicit ChSB. That is, all nonanomalous results (M_π, f_π, M_K, f_K, as w ell as $M_{s\bar{s}}$ and $f_{s\bar{s}}$, the mass and decay constant of the unphysical $s\bar{s}$ pseudoscalar meson) in the present paper are, for all T, taken over from Ref. [24]. We used this same model also for computing the chiral quark condensate $\langle\bar{q}q\rangle_0$, including its T-dependence displayed in Fig. 1.

This defines completely how the results displayed in Fig. 2 were generated. For details, see Ref. [24] (and Ref. [30] for M_{η_0} and M_{η_8}). Here we list only the formulas which, in conjunction with Fig. 1, enable the reader to understand easily the T-dependences of the masses in Fig. 2: The theoretical η' and η mass eigenvalues are

$$M^2_{\eta'}(T) = \frac{1}{2}[M^2_{\eta_{NS}}(T) + M^2_{\eta_S}(T) + \Delta_{\eta\eta'}(T)], \tag{10}$$

$$M^2_{\eta}(T) = \frac{1}{2}[M^2_{\eta_{NS}}(T) + M^2_{\eta_S}(T) - \Delta_{\eta\eta'}(T)], \tag{11}$$

where $\Delta_{\eta\eta'} \equiv \sqrt{[M^2_{\eta_{NS}} - M^2_{\eta_S}]^2 + 8\beta^2X^2}$,

$$\beta = \frac{1}{2+X^2}\frac{6\tilde{\chi}}{f_\pi^2}, \quad X \equiv \frac{f_\pi}{f_{s\bar{s}}},$$
$$M_{\eta_{NS}}^2 = M_\pi^2 + 2\beta, \quad M_{\eta_S}^2 = M_{s\bar{s}}^2 + \beta X^2,$$
$$M_{\eta_0}^2 = M_{00}^2 + \frac{1}{3}(2+X)^2\beta, \quad M_{\eta_8}^2 = M_{88}^2 + \frac{2}{3}(1-X)^2\beta,$$
$$M_{88}^2 = \frac{2}{3}M_{s\bar{s}}^2 + \frac{1}{3}M_\pi^2, \quad M_{00}^2 = \frac{1}{3}M_{s\bar{s}}^2 + \frac{2}{3}M_\pi^2.$$

In all expressions after Eq. (11), the T-dependence is understood.

In both cases considered for the topological susceptibility (8) [$\delta = 0$, i.e., the constant correction term, and $\delta = 1$, i.e., the strong T-dependence $\propto \langle \bar{q}q\rangle_0(T)$ of both the leading and correction terms in $\chi(T)$], the results are consistent with the experimental findings on the decrease of the η' mass of Csörgő *et al.* [13,14].

III. SUMMARY, DISCUSSION AND CONCLUSIONS

In the light of the recent experimental results on the η' multiplicity in heavy-ion collisions [13,14], we revisited the earlier theoretical work [24] concerning the thermal behavior of the η'-η complex following from WVR straightforwardly extended to $T > 0$. We have confirmed the results of Ref. [24] on WVR where the ratio $\chi_{\rm YM}(T)/f_\pi(T)^2$ dominates the T-dependence, and clarified that these results are practically model-independent. It is important to note the difference between our approach and those that attempt to give model predictions for topological susceptibility, such as Refs. [42,43]. By contrast, in Refs. [24] and here, as well as earlier works [26,30,44,45] at $T = 0$, a DS dynamical model is used (as far as masses are concerned) to obtain only the nonanomalous part of the light pseudoscalar meson masses (where the model dependence is however dominated by their almost-Goldstone character), while the anomalous part of the masses in the η'-η complex is, through WVR, dictated by $6\chi_{\rm YM}/f_\pi^2$. In this ratio, $f_\pi(T)$ is admittedly model-dependent in quantitative sense, but other realistic models yield qualitatively similar crossover behaviors [46] of $f_\pi(T)$ for $m_q \neq 0$, as exemplified by our Fig. 1, and Fig. 2 in Ref. [24], and by Fig. 6 in Ref. [10]. Such $f_\pi(T)$ behaviors are also in agreement with the T-dependence expected of the DChSB order parameter on general grounds: a pronounced falloff around $T_{\rm Ch}$—but exhibiting, in agreement with lattice [12], a smooth crossover pattern for nonvanishing explicit ChSB, a crossover which gets slower with growing m_q [e.g., compare $f_\pi(T)$ with $f_{s\bar{s}}(T)$ in Fig. 1]. In contrast to the QCD topological susceptibility χ, the YM topological susceptibility and even its T-dependence $\chi_{\rm YM}(T)$, including its "melting" temperature $T_{\rm YM}$, can be extracted [24] reasonably reliably from the lattice [22,31]. Thus, it was not modeled in Ref. [24]. Hence our assertion that the results of Ref. [24] unavoidably imply that the straightforward extension of WVR to $T > 0$ is falsified by experiment [13,14], especially if one recalls that even the sizeable T-rescaling [34,35] $T_{\rm YM} \to T_{\rm Ch}$ was among the attempts to control the η' mass enhancement [24].

Nevertheless, we have also shown that there is a plausible way to avoid these problems of the straightforward, naive extension of WVR to $T > 0$, and this is the main result of the present paper. Thanks to the existence of another relation, Eq. (4), connecting the YM quantity $\chi_{\rm YM}$ with QCD quantities χ and $\langle \bar{q}q\rangle_0$, it is possible to define a quantity, $\tilde{\chi}$, which can meaningfully replace $\chi_{\rm YM}(T)$ in finite-T WVR. It remains practically equal to $\chi_{\rm YM}$ up to some 70% of $T_{\rm Ch}$, but beyond this, it changes following the T-dependence of $\langle \bar{q}q\rangle_0(T)$. In this way, the successful zero-temperature WVR is retained, but the (partial) restoration of $U_A(1)$ symmetry [understood as the disappearing contribution of the gluon anomaly to the η' (η_0) mass] is naturally tied to the restoration of the $SU_A(3)$ flavor chiral symmetry and to its characteristic temperature $T_{\rm Ch}$, instead of $T_{\rm YM}$.

It is very pleasing that this fits in nicely with the recent *ab initio* theoretical analysis using functional methods [47], which finds that the anomalous breaking of $U_A(1)$ symmetry is related to DChSB (and confinement) in a self-consistent manner, so that one cannot have one of these phenomena without the other.

Of course, the most important thing is that this version of the finite-T WVR, obtained by $\chi_{\rm YM}(T) \to \tilde{\chi}(T)$, is consistent with experiment [13,14] for all reasonable strengths of T-dependence [$0 \leq \delta \lesssim 1$ in Eq. (8)]. Namely, the both tablets in Fig. 2 show, first, that η' mass close to $T_{\rm Ch}$ suffers the drop of more than 200 MeV with respect to its vacuum value. This satisfies the minimal experimental requirement abundantly. Second, Fig. 2 shows an even larger drop of the η_0 mass, to some 400 MeV, close to the "best" value of the in-medium η' mass (340 MeV, albeit with large errors) obtained by Csörgő *et al.* [13,14]. This should be noted because the η_0 mass inside the fireball is possibly even more relevant. Namely, although it is, strictly speaking, not a physical meson, η_0 is the state with the $q\bar{q}$ content closest to the $q\bar{q}$ content of the physical η' *in the vacuum*. Thus, among the isoscalar $q\bar{q}$ states inside the fireball, η_0 has the largest projection on, and thus the largest amplitude to evolve, by fireball dissipation, into an η' in the vacuum.

ACKNOWLEDGMENTS

D. H., D. Kl. and S. B. were supported through the project No. 0119-0982930-1016, and D. Ke. through the project No. 098-0982887-2872 of the Ministry of Science, Education and Sports of Croatia. D. H. and D. Kl. acknowledge discussions with R. Alkofer. The support by CompStar network is also acknowledged.

[1] J. Adams *et al.* (STAR), Nucl. Phys. **A757**, 102 (2005).
[2] B. Muller and J. L. Nagle, Annu. Rev. Nucl. Part. Sci. **56**, 93 (2006).
[3] E. V. Shuryak and I. Zahed, Phys. Rev. D **70**, 054507 (2004).
[4] M. A. Stephanov, Proc. Sci., LAT2006 (2006) 024 [arXiv:hep-lat/0701002].
[5] D. B. Blaschke and K. A. Bugaev, Fiz. B **13**, 491 (2004), http://fizika.phy.hr/fizika_b/bv04/b13p491.htm.
[6] S. Datta, F. Karsch, P. Petreczky, and I. Wetzorke, Nucl. Phys. B, Proc. Suppl. **119**, 487 (2003).
[7] M. Asakawa and T. Hatsuda, Phys. Rev. Lett. **92**, 012001 (2004).
[8] E. Shuryak, Nucl. Phys. **A774**, 387 (2006).
[9] F. Karsch *et al.*, Nucl. Phys. **A715**, 701c (2003).
[10] P. Maris, C. D. Roberts, S. M. Schmidt, and P. C. Tandy, Phys. Rev. C **63**, 025202 (2001).
[11] M. Mannarelli and R. Rapp, Phys. Rev. C **72**, 064905 (2005).
[12] Z. Fodor and S. D. Katz, arXiv:0908.3341.
[13] T. Csorgo, R. Vertesi, and J. Sziklai, Phys. Rev. Lett. **105**, 182301 (2010).
[14] R. Vertesi, T. Csorgo, and J. Sziklai, Phys. Rev. C **83**, 054903 (2011).
[15] S. S. Adler *et al.* (PHENIX Collaboration), Phys. Rev. Lett. **93**, 152302 (2004).
[16] J. Adams *et al.* (STAR Collaboration), Phys. Rev. C **71**, 044906 (2005).
[17] J. I. Kapusta, D. Kharzeev, and L. D. McLerran, Phys. Rev. D **53**, 5028 (1996).
[18] E. Witten, Nucl. Phys. **B156**, 269 (1979).
[19] G. Veneziano, Nucl. Phys. **B159**, 213 (1979).
[20] B. Lucini, M. Teper, and U. Wenger, Nucl. Phys. **B715**, 461 (2005).
[21] L. Del Debbio, L. Giusti, and C. Pica, Phys. Rev. Lett. **94**, 032003 (2005).
[22] B. Alles, M. D'Elia, and A. Di Giacomo, Phys. Rev. D **71**, 034503 (2005).
[23] S. Durr, Z. Fodor, C. Hoelbling, and T. Kurth, J. High Energy Phys. 04 (2007) 055.
[24] D. Horvatic, D. Klabučar, and A. E. Radzhabov, Phys. Rev. D **76**, 096009 (2007).
[25] T. Feldmann, Int. J. Mod. Phys. A **15**, 159 (2000).
[26] D. Kekez, D. Klabučar, and M. D. Scadron, J. Phys. G **26**, 1335 (2000).
[27] H. Leutwyler and A. V. Smilga, Phys. Rev. D **46**, 5607 (1992).
[28] D. Kharzeev, R. D. Pisarski, and M. H. G. Tytgat, Phys. Rev. Lett. **81**, 512 (1998).
[29] D. E. Kharzeev, R. D. Pisarski, and M. H. G. Tytgat, arXiv:hep-ph/0012012.
[30] D. Kekez and D. Klabučar, Phys. Rev. D **73**, 036002 (2006).
[31] B. Alles, M. D'Elia, and A. Di Giacomo, Nucl. Phys. **B494**, 281 (1997).
[32] G. Boyd *et al.*, Nucl. Phys. **B469**, 419 (1996).
[33] C. Gattringer, R. Hoffmann, and S. Schaefer, Phys. Lett. B **535**, 358 (2002).
[34] K. Fukushima, K. Ohnishi, and K. Ohta, Phys. Rev. C **63**, 045203 (2001).
[35] J. Schaffner-Bielich, Phys. Rev. Lett. **84**, 3261 (2000).
[36] D. Blaschke, G. Burau, Yu. L. Kalinovsky, P. Maris, and P. C. Tandy, Int. J. Mod. Phys. A **16**, 2267 (2001).
[37] For reviews, see, e.g., Refs. [38,39].
[38] C. D. Roberts and S. M. Schmidt, Prog. Part. Nucl. Phys. **45**, S1 (2000).
[39] R. Alkofer and L. von Smekal, Phys. Rep. **353**, 281 (2001).
[40] T. DeGrand and S. Schaefer, arXiv:0712.2914.
[41] P. Di Vecchia and G. Veneziano, Nucl. Phys. **B171**, 253 (1980).
[42] G. A. Contrera, D. Gomez Dumm, and N. N. Scoccola, Phys. Rev. D **81**, 054005 (2010).
[43] P. Costa, M. C. Ruivo, C. A. de Sousa, H. Hansen, and W. M. Alberico, Phys. Rev. D **79**, 116003 (2009).
[44] D. Klabučar and D. Kekez, Phys. Rev. D **58**, 096003 (1998).
[45] D. Kekez and D. Klabučar, Phys. Rev. D **65**, 057901 (2002).
[46] E.g., within DS approach, compare our $f_\pi(T)$ and the one in Ref. [10] and in reviews such as Ref. [38].
[47] R. Alkofer, Proc. Sci., FACESQCD (2011) 030 [arXiv:1102.3166].