Ansambli težinskih mreža

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Ansambli težinskih mreža

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Sažetak

Težinske mreže se koriste za modeliranje širokog spektra fenomena, od društvenih mreža, do širenja zaraznih bolesti. Kad struktura cijele mreže nije poznata, moramo pribjeći metodama rekonstrukcije koje identificiraju najmanje pristran ansambl mreža konzinstentan sa dostupnim informacijama. Izazovan slučaj koji često susrećemo, zbog privatnosti korisnika međubankovnih protoka, je taj da imamo samo lokalne (čvorne) informacije dostupne. Dok je naivan pristup ograničiti snagu svih čvorova, nedavni rezultati predlažu da su u težinskim mreža stupnjevi čvorova informativniji od jačine. U sklopu diplomskog rada implementirao sam, koristeći programski jezik Python i alate otvorenog koda, analitičku i nepristranu metodu koja rekonstruira mrežu u najkraćem mogućem vremenu, te ju testirao na primjeru trgovanja zemalja.

Weighted Network Ansambles

Abstract

Weighted networks are used to model a wide spectrum of phenomena, from social networks, to the spreading of contagious diseases. When the structure of the entire network is not known, we must resort to methods of reconstruction that identify the least biased nework ensemble consistent with the available information. A challenging case that we often encounter is, due to the privacy constraints of interbank flows, that we have only local (node specific) information available. While the naive approach is to constrain the strength of all nodes, recent results suggest that deegrees of nodes are more informative than the strengths. In this thesis, I have implemented a analytical method, using the Python programming language and open source tools, that finds the solution in the shortest time possible, and test edit on an example of intercountry trade.

Sadržaj

1. Uvod

U sklopu ovog diplomskog rada implementirao sam metodu rekonstruiranja mreža iznesenu u znanstvenom radu „*Enhanced reconstruction of weighted networks from strengths and degrees*" [1]. Rekonstrukcija mreža je sve bitnije polje, pogotovo u Big Data eri u kojoj se trenutačno nalazim, u kojoj su informacije koje su nam dostupne često nepostpuno, zbog zaštite privatnosti. Za implementaciju sam koristio programski jezik Python, i odgovarajuće biblioteke (Scipy, NumPy i NetworkX). U slijedećem poglavlju navodim najbitnije dijelove rada na kojem je baziran ovaj diplomski rad, te metodologiju kojom su došli do svojih rezultata. Nakon toga, implementiramo i provjeravamo iznesenu metodu rekonstrukcije mreža nad mrežom trgovanja zemalja u 2000. godini.

2. Pojačana rekonstrukcija težinskih mreža iz snaga i stupnjeva

Topologija mreža igra ključnu ulogu u širokom spektru fenomena kritične važnosti, od društvenih mreža do širenja zaraznih bolesti. Svi ti fenomeni su jako osjetljivi na topologiju mreže koja posreduje interakcijama. Ta osjetljivost znači da bi, bez obzira je li moguće imati potpuno empiričko stanje mreže, trebali optimalno koristiti nepotpune informacije koje imamo na raspolaganju i probati rekonstruirati najvjerojatniju mrežu, tj. ansambl vjerojatnih mreža. U Big Data eri ovakvi su problemi postali sve važniji jer dobivamo velike količine podataka koje su, zbog zaštite privatnosti, često agregirane prirode ili ne sadre potpuno informacije o mreži.

Najčešći slučaj je onaj u kojem znamo samo lokalne podatke o mreži, npr. u binarnoj mreži znamo broj veza (stupanj) jednog vrha, a ne identitet svih susjeda. Isto tako, u težinskim mrežama je puno lakše znati ukupnu snagu svih veza jednog vrha, nego težinu svake veze i identitet svih susjeda.

Rekonstrukcije mreže se mogu smatrati kao problem maksimizacije entropije, gdje su ograničenja predstavljena dostupnim informacijama, a maksimizacija entropije osigurava da je rekonstruirani ansambl mreža maksimalno nasumičan za ta ograničenja. Kada su dostupne samo lokalne informacije, znamo *O(N)* informacija (npr. stupnjeve vrhova), umjesto ukupnih $O(N^2)$ (identitet svih veza, tj. svi unosi u matrici susjednosti). To znači da nam je broj nepoznatih varijabli $O(N^2)$, tj istog reda veličine kao i ukupni broj varijabli.

Čak i kada znamo topologiju cijele mreže, često je potrebno rekonstruirati najvjerojatniju mrežu iz lokalnih svojstava da imamo mjerilo (nul model) za značajnost uzoraka višeg reda poput asortativnosti. Nul modeli filtriraju intrinzičnu i neizbježnu heterogenost vrhova, poput toga da popularniji ljudi imaju veći stupanj u društvenim mrežama. Najjednostavniji i najčešće korišteni nul model je konfiguracijski model (CM), definiran kao ansambl nasumičnih grafova sa danim vektorom stupnjeva svih vrhova.

Nedavno je pokazano da CM ima veliki problem sa pristranosti: teško je implementirati model na način da svaka mreža u rekonstruiranom ansamblu ima točnu vjerojatnost, te prosjek očekivanja u ansamblu nije pristran. Ovaj problem pristranosti u CM-u zahtjeva ne trivijalna rješenja koja su se pojavila tek nedavno. Kad implementiramo ta rješenja, mnoge binarne mreže su kvalitetno rekonstruirane samo iz vektora stupnjeva vrhova. U drugim slučajevima, rekonstuirane mreže se znatno razlikuju od stvarnih.

Taj rezultat ukazuje na prisutnost kompleksnijih uzoraka koji se ne mogu izvući samo iz niza stupnjeva mreže.

U radu na kojem je ovaj diplomski rad baziran, rješava se problem efektivne rekonstrukcije težinskih mreža iz lokalnih svojstava(stupnjeva i jačina svih vrhova). Autori rada prvo pokazuju da za rekonstrukciju težinskih mreža nije dovoljno koristiti istu metodologiju kao i kod binarnih mreža. Dakle, ne možemo ništa zaključiti iz samog vektora jačine, čak i za mreže koje, na binarnoj razini, možemo rekonstruirati iz vektora stupnjeva. Nakon toga, pretpostavljaju da je razlog za to da informacija o vektoru snage svih vrhova znači više od toga da samo sadržava ili poboljšava informacije dobivene vektorom stupnjeva, pošto su binarne informacije potpuno izgubljene kad se gledaju čisto težinske mjere. To vodi prema očekivanju da, za rekonstrukciju težinskih mreža, možemo poboljšati specifikaciju navođenjem jačine i težine. Nakon toga uvode analitičku i nepristranu tehniku rekonstrukcije nepristranih ansambala težinskih mreža iz vektora težina i jačina. Ta metoda dolazi do rješenja u najkraćem mogućem vremenu, bez da zahtjeva eksplicitno uzimanje uzorka rekonstruiranog grafa.

Primjenom te metode na više mreža, pokazano je da ta znatno poboljšava rekonstrukciju.

2.1 Naivna rekonstrukcija težinskih mreža

Najprirodnija generalizacija CM-a (eng. Configuration model) na težinske mreže je rekonstruiranje ansambla pomoću vektora težine, i nekad se naziva WCM (eng. Weightedconfiguration model). WCM je široko korišten kao rekonstrukcijska metoda i nul model za otkrivanje zajednica. U oba slučaja, ako (S_i) označava težinu vrha *i*, a *N* je broj vrhova, očekivana težina veze između vrhova *i* i *j* je napisana kao:

$$\langle w_{ij} \rangle = \frac{s_i s_j}{\sum_{m=1}^{N} s_m} \qquad (1)$$

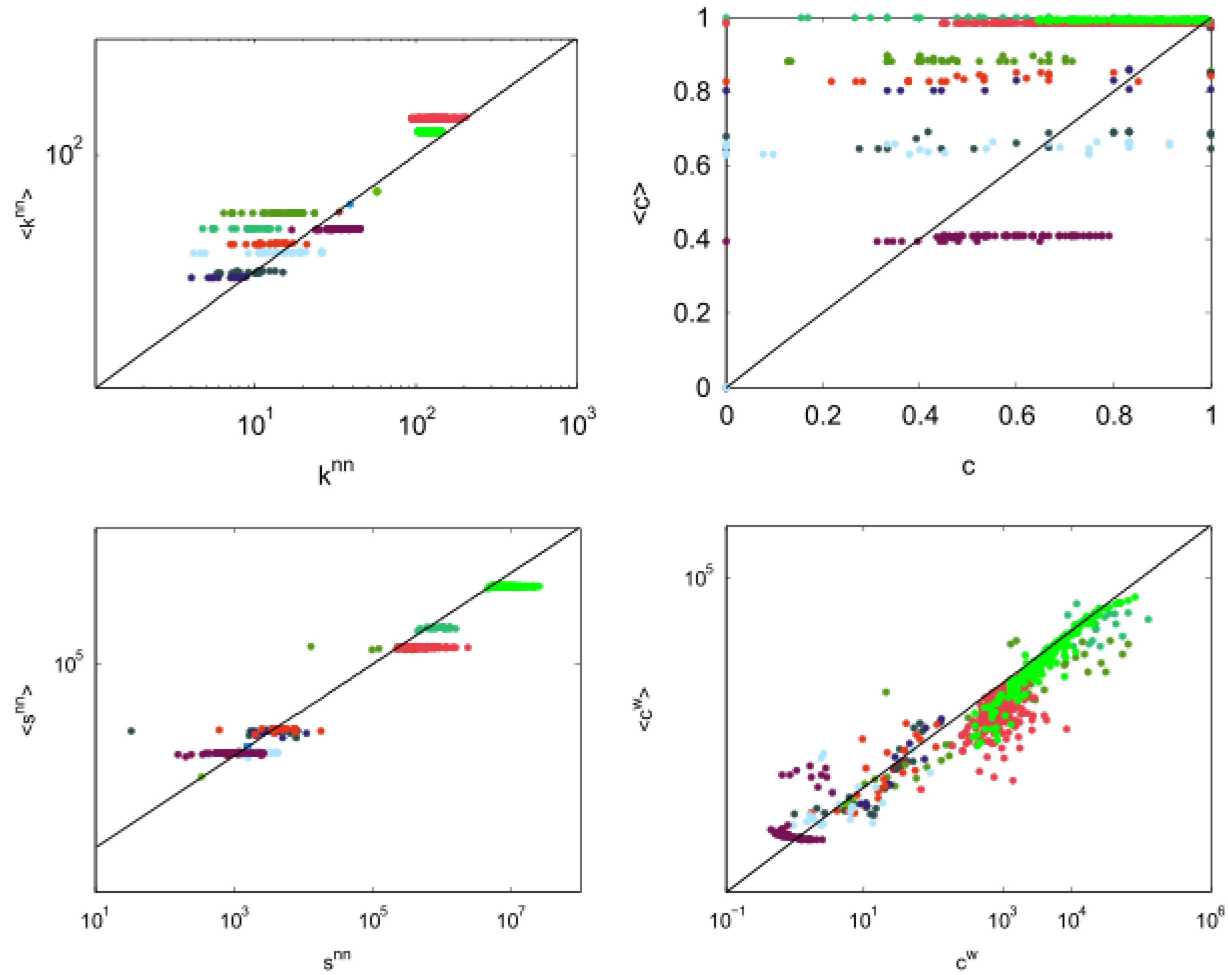
ili u malo različitom obliku ako je mreža usmjerena. Na primjer, gornji izraz predstavlja jednu od standardnih procedura za izvođenje veza između banaka iz ukupne izloženosti svake banke.

Nažalost, ovaj model je netočan, i razlikuje se od nepristranog izraza izvedenog rigoroznim pristupom maksimizacije entropije. U njemu, npr. nema indikacije o raspodijeli vjerojatnosti iz kojih je izvučen, makar je tretiran kao očekivana vrijednost. Dakle, nije moguće izvući očekivane vrijednosti topoloških svojstava koji su nelinearne funkcije težina. Taj problem je tek nedavno riješen uvođenjem analitičnog pristupa koji vodi do točnog izraza za vjerojatnost težine i bilo koju funkciju očekivane težine.

Drugo ograničenje WCM-a i dalje postoji čak i kad je model točno implementiran. Ovdje treba napomenuti da je motivacija za korištenje WCM-a kao generalizaciju CM-a za težinske mreže implicitna pretpostavka da jačina vrha je poboljšano svojstvo koje obuhvaća dodatne informacije koje dobijemo iz težine bridova grafa. Međutim, nedavni rezultati su pokazali da je vektor jačine mreže često manje informativan od vektora stupnjeva. Specifično, neka topološka svojstva stvarne težinske mreže se bolje reproduciraju koristeći CM nad binarnom projekcijom mreže, nego primjenjujući WCM na originalnu težinsku mrežu. Razlog je taj da vektor jačine daje jako lošu predikciju topoloških svojstava, pogotovo stupnjeva vrhova. Štoviše, WCM pri rekreaciji preferira stvaranje gustih mreža.

Dakle, da bi odredili valjanost metode rekonstrukcije mreže, nije dovoljno provjeravati svojstva koja ovise samo o težinama veza, nego i ona koja ovise o binarnoj topologiji.

Kao početni korak analize, autori rada provjeravaju svoje zaključke na rekonstrukcijama raznih mreža različitih svojstava. Koriste „*Italian Interbank Netwok*“ iz 1999. godine, tri „klasične“ društvene mreže, sedam hranidbenih mreža i za kraj agregirane WTW (eng. World Trade Web) podatke iz 2002. Godine.



Slika 2.1. Naivna rekonstrukcija mreža iz snage vrhova (WCM). U svakom kvadratu uspoređuju vrijednosti specifičnih mrežnih svojstava rekonsturirane mreže (svaka mreža je označena zasebnom bojom) (y-os) i originalne mreže (x-os) za sve vrhove slijedečih 12 mreža: Office social network, Research groupsocial network, Fraternitysocial network, MaspalomasLagoonfood web, Cheapskate Bay food web, CrystalRiver (control) food web, CrystalRiver food web, Michigan Lake food web, Mondego Estuary food web, Everglades Marshes food web, Italian Interbank network in year 1999. , aggregated World trade Web in year 2002. Promatrana svojstva: gore lijevo: average nearest neighbour degree, gore desno: binary clustering coefficient, dole lijevo: average nearest neighbour strength, dole desno: weighted clustering coefficient.

Na slici 2.1 se nalazi usporedba slijedećih svojstava na rekonstruiranoj i originalnoj mreži:

Average nearest neighbor degree:

Ovo svojstvo nam daje korelaciju između stupnjeva susjednih vrhova.

$$k_i^{nn}(\mathbf{W}) \equiv \frac{\sum_{j\neq i} a_{ij}k_j}{k_i} = \frac{\sum_{j\neq i}\sum_{k\neq j} w_{ij}^0 w_{jk}^0}{\sum_{j\neq i} w_{ij}^0} \qquad (2)$$

Gdje je k_i definiran kao:

$$\mathrm{k}_i = \sum_{j\neq i} a_{ij} = \sum_{j\neq i} w_{ij}^0 .$$

Clustering coeffiecient:

Ovo svojstvo nam daje udjele trokuta (tri povezana vrha) u mreži, tj. Koliko su susjedi nekog vrha blizu tome da tvore potpuni graf:

$$c_i(\mathbf{W}) = \frac{\sum_{j\neq i}\sum_{k\neq i,j} w_{ij}^0 w_{jk}^0 w_{ki}^0}{\sum_{j\neq i}\sum_{k\neq i,j} w_{ij}^0 w_{ki}^0} \qquad (3)$$

Odgovarajuća svojstva za težinske mreže su:

Average nearest neighbor strength:

$$s_i^{nn}(\mathbf{W}) \equiv \frac{\sum_{j\neq i} a_{ij}s_j}{k_i} = \frac{\sum_{j\neq i}\sum_{k\neq j} w_{ij}^0 w_{jk}}{\sum_{j\neq i} w_{ij}^0} \qquad (4)$$

Gdje je S_i definiran kao:

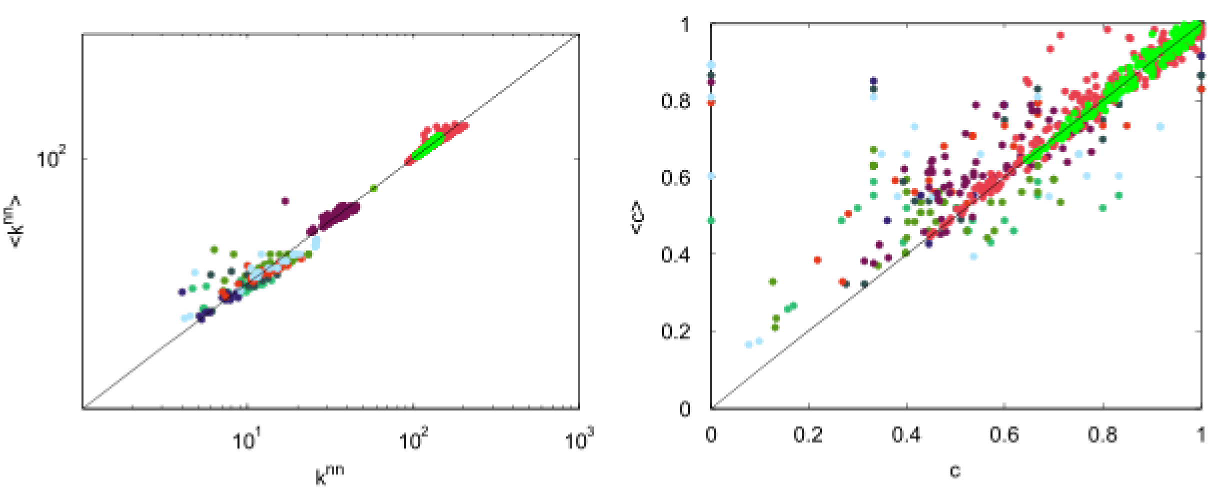
$$\mathrm{s}_i = \sum_{j\neq i} w_{ij} .$$

Te Weighted clustering coefficient:

$$c_i^w(\mathbf{W}) = \frac{\sum_{j\neq i}\sum_{k\neq i,j}(w_{ij}w_{jk}w_{ki})^{1/3}}{\sum_{j\neq i}\sum_{k\neq i,j}w_{ij}^0 w_{ki}^0} \qquad (5)$$

Svaka točka na slici 2.1 predstavlja jedan vrh svoje mreže. U dobro rekonstruiranoj mreži bi se rekonstruirana vrijednost poklapala sa stvarnom, dakle sve točke bi se nalazile na dijagonali. Kao što vidimo, većina rekonstruiranih vrijednosti za zadane mreže se nalazi na horizontalnim linijama, dake skoro su jednake jedne drugima i potpuno nepovezane sa mrežom koju pokušavaju rekonstruirati.

Tipična interpretacija ovih rezultata je ta da je rekonstrukcija mreža iz svojstva samih vrhova intrinzično problematična, vjerojatno zbog prisutnosti mehanizama više razine u formaciji stvarnih mreža. Štoviše, WCM se često koristi kao nul model za filtriranje lokalne heterogenosti vrhova u detekciji važnih uzoraka više razine, te tako interpretirajući razliku između stvarnih podataka i WCM kao znak postajanja na lokalnih uzoraka. Međutim, slijedeća slika pokazuje da su gornji rezultati i njihove klasične interpretacije potpuno obrnuti kada uzmemo u obzir CM.

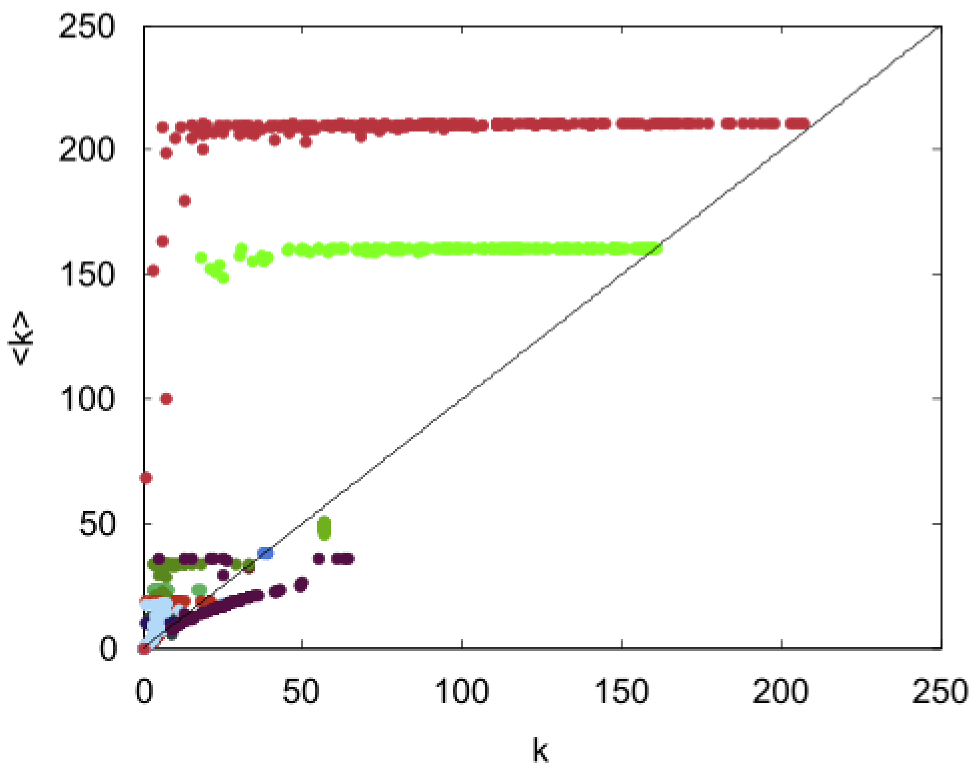


Slika 2.2. Rekonstrukcija binarne projekcije mreže iz stupnjeva vrhova (CM), koja pokazuje da binarna lokalna svojstva sadrže znatne informacije. Rekreirane mreže su iste kao i na slici 1. Na lijevoj slici se nalazi usporedba za *ANND*, a na desnoj za *BCC*.

2.2 Pretpostavka o ne reducibilnosti stupnjeva

Nadalje, u radu se predlaže da je loša rekonstrukcija postignuta WCM-om jer vektor snage odbacuje topološka svojstva mreže, najviše stupnjeve. Štoviše, tvrde da je su stupnjevi osnovno strukturalno svojstvo težinskih mreža, i da se ne mogu izvesti iz snage vrhova,pa su barem jednako bitni kao jačina.

U dosadašnjem dijelu autori su pokazali nedostatke WCM u rekonstrukciji stvarnih mreža, ali nisu još isprobali performanse CM-a kada ga se primjeni na binarne projekcije istih mreža. Iz slike 2.2 i slike 2.1. vidimo da je CM uspješniji u rekonstrukciji binarnih projekcija mreža od WCM-a, te tako pokazujemo da je pretpostavka da su vrijednosti izračunate nad originalnom težinskom mrežom više informativne od odgovarajućih vrijednosti izračunatih na binarnom projekcijom kriva!



Slika 2.3. Rekonstrukcija stupnjeva vrhova iz snage vrhova, pokazuje da sama težinska lokalna svojstva nisu jako informativna. Uspoređuju se rekonstruirane i originalne vrijednosti za prijašnjih 12. mreža.

Autori tvrde da su stupnjevi fundamentalno lokalno svojstvo težinskih mreža, i da ih se ne može svesti na informaciju o snagama. Dakle, nedostatak WCM-a je taj da zanemaruje vektor stupnjeva vrhova mreže.

Potrebno je i razjasniti da se pod ne reducibilnostne misli na numeričke vrijednosti snaga i stupnjeva, već na različite funkcionalne uloge koje te vrijednosti igraju u određivanju i ograničavanju strukture mreže. Štoviše, snage i stupnjevi su tipično jako korelirani u stvarnim mrežama, što znači da se iz vrijednosti jedne razumno može zaključiti vrijednost druge. Međutim, ono što je nama bitno je dublja vrsta nesvodljivosti, na koju nailazimo kada specifikacija snaga i stupnjeva ograničava mrežu na fundamentalno drukčiji način od specifikacije samo jednog od ta dva svojstva.

Ujedno, ništa ne garantira da je jaka korelacija između stupnjeva i snaga očuvana pri specifikaciji samo snaga. Iz navedenog proizlazi da, općenito, WCM ne reproducira točno vektor stupnjeva stvarnih mreža.

Problem s time je što nije predložen zadovoljavajući način implementiranja takve metode za analizu stvarnih mreža. Povrh toga, nije definiran rigorozni kriterij za utvrđivanje je li dodavanje vektora stupnjeva kao dodatno ograničenje redundantno (pre-prilagodba).

U slijedećem dijelu rada, autori definiraju brz i nepristran pristup realiziranja ECM-a (EnhancedConfiguration Model), te kriterije za provjeru njegove valjanosti. U ovom radu ćemo se zaustaviti nakon definicije ECM-a, pošto nismo koristili te kriterije za provjeru točnosti rekonstruiranog ansambla.

2.3 ECM

U ovom dijelu rada se razvija formalizam koji implementira ECM (EnhancedConfiguration Model) na analitički, ne pristran i brzi način. Autori razmatraju samo slučaj ne usmjerenih mreža, iako je generalizacija na usmjeren slučaj jednostavna. Formalno, ansambl težinskih mreža sa N vrhova se može karakterizirati skupom $\{W\}N\times N$ matrica sa prikladnom vjerojatnosti $P(W)$. Na svakoj mreži W, jačina je definirana kao $s_i(W) = \sum_{j\neq i} w_{ij}$ i stupanj je definiran kao

$k_i(W) \equiv \sum_{j\neq i} w_{ij}^0$. Pretpostavljamo da je svaki w_{ij} ne negativni cijeli broj (sa konvencijom $0^0 = 0$).

Autori započinju sa sažetkom korisnih rezultata koji su već dostupni[2]. Ti rezultati potpuno opisuju strukturalnu korelaciju težinskih mreža,demonstriraju da ponašanje nul modela težinskih mreža odudara od prijašnjih pretpostavki, te pokazuju da je potrebno redefinirati težinska svojstva mreža. Tražimo vjerojatnost, koja, osim sto je normalizirana, osigurava da je očekivani stupanj i snaga svakog vrha ograničena, i ostavlja ansambl maksimalno nasumičnim u drugim točkama. To se postiže zahtijevajući da *P(W)* maksimizira Shannonovu entropiju $S \equiv -\sum_W P(W) \ln P(W)$ s ograničenjem na očekivani vektor stupnjeva i jačine. Fundamentalni rezultat ove ograničene maksimizacije je vjerojatnost

$$P(\mathbf{W}|\vec{x},\vec{y}) = \prod_{i<j} q_{ij}(w_{ij}|\vec{x},\vec{y}) \qquad (6)$$

gdje su x i y dva *N*-dimenzionalnaLangrangeova multiplikatora koja kontroliraju očekivane stupnjeve i jačine (sa $x_i \geq 0$ i $0 \leq y_i < 1$za sve *i*), a

$$q_{ij}(w|\vec{x},\vec{y}) = \frac{(x_i x_j)^{\Theta(w)}(y_i y_j)^w (1 - y_i y_j)}{1 - y_i y_j + x_i x_j y_i y_j} \qquad (7)$$

je vjerojatnost da veza težine *w* postoji između vrhova *iij*. U gornjem izrazu vrijedi da je $\Theta(x) = 1$ ako je $x > 0$ i $\Theta(x) = 0$ ako nije.

Jednadžba (7) definira miješanu Bose-Fermi distribuciju gdje, zbog prisutnosti *Θ(w)*, uspostavljenje jedinke težine ima različitu "cijenu" od pojačavanja postojeće veze. Ovo svojstvo dolazi od prisutnosti binarnih i težinskih ograničenja, i čini ECM potencijalno prikladnim za modeliranje stvarnih mreža. Međutim, do ovog rada nije predložena niti jedna metoda za implementiranje ECM-a za empiričke analize.

Da bi to postigli, primjenjujemo pristup maksimalne vjerodostojnosti na model. Razmatramo stvarnu mrežu *W** čiji su stupnjevi i jačine poznate. Logaritamska vjerojatnost ECM-a je definirana jednadžbama 6 i 7 je:

$$\mathcal{L}(\vec{x},\vec{y}) \equiv \ln P(\mathbf{W}^*|\vec{x},\vec{y}) = \sum_{i<j} \ln q_{ij}(w^*_{ij}|\vec{x},\vec{y}) = \sum_{i=1}^{N}\left(k^*_i \ln x_i + s^*_i \ln y_i\right) + \sum_{i<j} \ln\left(\frac{1-y_iy_j}{1-y_iy_j+x_ix_jy_iy_j}\right) \tag{8}$$

Sad tražimo specifične parametre *x** i *y** koji maksimiziraju *L(x,y).* Izravan izračun, sličan jednostavnijima koje smo susreli u drugim nul modelima, pokazuje da se *x** i y* mogu dobiti kao realna rješenja slijedećih *2N* jednadžbi:

$$\langle k_i \rangle = \sum_{j\neq i} \frac{x_ix_jy_iy_j}{1-y_iy_j+x_ix_jy_iy_j} = k^*_i \quad \forall i \tag{9}$$

$$\langle s_i \rangle = \sum_{j\neq i} \frac{x_ix_jy_iy_j}{(1-y_iy_j)(1-y_iy_j+x_ix_jy_iy_j)} = s^*_i \quad \forall i \tag{10}$$

Iz toga izlazi da su vrijednosti *x** i *y** točno one koje osiguravaju da se očekivani nizovi stupnjeva i jačina podudaraju sa promatranim nizovima *k** i*s****,** što rješava naš početni problem.

Ovaj rezultat pokazuje da vrijednosti *x** i *y** imaju sve informacije potrebne za rekonstrukciju mreže.

U ostatku rada, autori navode način rješavanja *2N* sparenih jednadžbi i provjeru dobivenih rezultata, koji se drukčiji od implementacije i provjere provedene u toku ovog diplomskog rada.

3. Korišteni alati

U sklopu ovog diplomskog rada, korišten je programski jezik Python, te pripadajuće biblioteke koje sadrže razne alate koji pojednostavljuju i ubrzavaju modeliranje podataka i analizu nad istima. Sve navedene biblioteke se distribuiraju sa BSD licencom, što znači da je izvorni kod javno dostupan, te ga slobodno možemo mijenjati, nadopunjavati, te širiti dalje.

Programski Python je odabran za implementaciju ovog rada ne samo zbog jednostavnosti korištenja i velike količine gotovih biblioteka, nego i zbog njegove široke upotrebe u nastavi kao prvi programski jezik s kojim se učenici susreću. Više o tome u poglavlju 7.

3.1 Scipy

Scipy je set znanstvenih i numeričkih alata za programski jezik Python. Javno je dostupan i otvorenog je koda. Trenutačno podržava specijalne funkcije, integraciju, alate za paralelno programiranje, te mnoge druge značajke[3].

Scipy je nastao iz potrebe da se ujedine mnoge znanstvene metode koje su razni znanstvenici implementirali, te da se prekine tadašnja rascjepanost znanstvene zajednice s obzirom na to koji alati se koriste za obrade podataka i rješavanje numeričkih problema. Najčešće se koristi za:

- Optimizaciju postojećeg koda
- Naprednu analizu podataka
- Kao baza znanja implementiranih metoda i standardnih načina rješavanja problema
- Pomoć pri interaktivnom radu sa podacima, najčešće uz IPython

U sklopu ovog diplomskog rada je korišten algoritam Basinhopping iz ove biblioteke za rješavanje *2N* jednadžbi.

3.1.1 BasinHopping

Basin-hopping je stohastički algoritam koji pokušava naći globalni minimum glatke skalarne funkcije jedne ili više varijabli. Taj algoritam u svom trenutačnom obliku su opisali David Wales i Jonathan Doye[6].

Basin-hopping je iterativan, i svaki ciklus je sastavljen od slijedećih koraka:

- nasumično mijenjanje koordinata
- lokalna minimizacija
- prihvaćanje ili odbijanje novih koordinata s obzirom na vrijednost minimizirane funkcije

Koristi Metropolis kriterij standardnih Monte Carlo algoritama, iako postoje razne druge opcije. Ova metoda se pokazala jako korisnom u raznim problemima u fizici i kemiji.

Za stohastičku globalnu optimizaciju ne postoji način određivanja je li nađen pravi globalni minimum, tako da kao provjeru konzistentnosti, algoritam pokrećemo iz raznog broja raznih početnih točaka da bi osigurali da smo našli najmanji globalni minimum. Iz tog razloga, Basin-hopping se ne vrti dok ne nađe neko rješenje, nego za zadani broj iteracija.

3.1.1.1 Odabir veličine koraka

To je krucijalan parametar i jako ovisi o problemu koji rješavamo. Idealno bi trebao biti usporediv sa tipičnom udaljenosti između lokalnih minimuma funkcije koju optimiziramo. Algoritam će cijelo vrijeme mijenjati veličinu koraka u pokušaju da nađe optimalnu vrijednost, ali to može trajati velik broj iteracija. Brže dobijemo rezultate ako stavimo neku logičnu vrijednost za veličinu koraka.

3.1.1.2 Odabir Temperature

T je "temperatura" koja se koristi sa Metropolis kriterijem. Basin-hopping koraci se prihvaćaju sa vjerojatnosti 1 ako nova vrijednost funkcije manja od stare vrijednost. Za najbolje rezultate, T bi trebao biti usporediv sa tipičnom razlikom u vrijednosti funkcije u lokalnim minimumima.

3.1.1.3 Definicija funkcije u modulu scipy.optimize:

scipy.optimize.basinhopping(func, x0, niter=100, T=1.0, stepsize=0.5, minimizer_kwargs=None, take_step=None, accept_test=None, callback=None, interval=50, disp=False, niter_success=None)

Parametri:

- func: funkcija koju treba optimizirati, oblika f(x, *args). Dodani parametri mogu biti proslijeđeni u rječniku minimizer_kwargs
- x0: početni pokušaj rješenja
- niter: broj basinhopping iteracija
- T : "temperatura" kao kriterij za prihvaćanje ili odbijanje. Više "temperature" znače da su veći skokovi u vrijednosti funkcija prihvaćeni. Za najbolje rezultate trebala bi biti usporediva sa razmakom u vrijednosti između lokalnih minimuma
- stepsize: veličina početnog koraka za korištenje u nasumičnim pomacima
- minimizer_kwargs: dodatni argumenti oblika ključ-vrijednost koji se prosljeđuju funkciji scipy.optimize.minimize

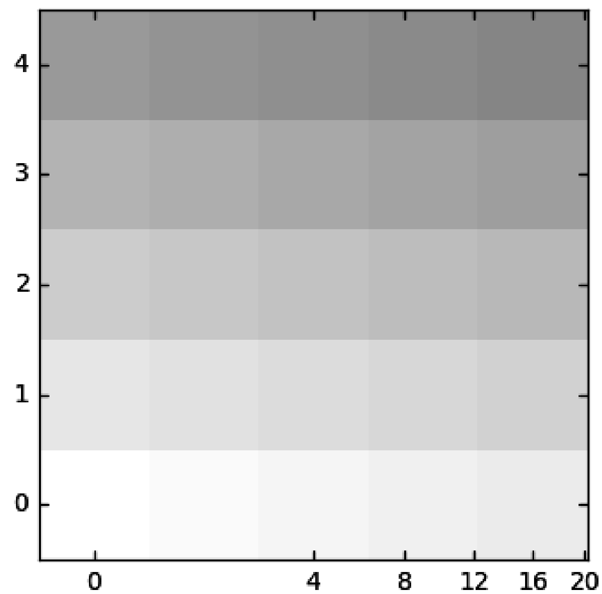
- take_step: funkcija koja mijenja zadanu funkciju za uzimanje koraka. Zadana funkcija uzima nasumične korake.
- accept_test: definiramo test koji određuje hoćemo li prihvatit ili odbaciti uzeti korak. Koristi se uz Metropolis test baziran na "temperaturi" T.
- callback: Funkcija koja se zove za sve nađene minimume. Najčešće se koristi za spremanje najmanjih N nađenih minimuma.
- interval: interval koji određuje koliko često će se mijenjati veličina koraka
- disp: staviti da je True ako želimo detaljan ispis statusa izvršavanja funkcije
- niter_success: broj koraka nakon kojih se treba zaustaviti ako se kandidat za globalni minimum nije promijenio.
- res: rezultat optimizacije. U sebi sadrži listu rješenja, vrijednost funkcije na rješenju, i poruku koja govori zašto je algoritam završio.

3.2 Numpy

Numpy je biblioteka za numeričko računanje sa programskim jezikom Python. Osim mnogih alata za linearnu algebra, generiranje nasumičnih brojeva, integriranje sa postojećim C/C++, MATLAB i Fortran kodom, te sadrži u sebi jako efikasne implementacije višedimenzionalnih lista, koje većinom služe kao način implementiranja matrica[4]. U sklopu ovog diplomskog rada, vidjeli smo do 100 puta brže izvršavanje koda pri korištenju Numpy struktura podataka.

3.3 Matplotlib

Matplotlib je biblioteka za 2D prikaz podataka u programskom jeziku Python. Omogućuje iscrtavanje podataka u raznim formatima, te ga je lako integrirati u Python skripte te koristiti sa Jupyter alatom za interaktivan rad[5]. U sebe integrira već postojeće alata poput wxPythona, Qt I GTK+-a. Značajka koju mnogi znanstvenici cijene je uporaba LaTeX-a odnosno umetanje formula i teksta formatiranih LaTeX-om. Kao i svi dosad navedeni alati, otvorenog je koda i potpuno besplatan.



Slika 3.1 Primjer iscrtavanja rešetke. Preuzeto sa www.matplotlib.com/examples

U sklopu ovog rada koristili smo ga za iscrtavanje 2D matrica originalnih i rekonstruiranih mreža.

3.4. NetworkX

NetworkX je biblioteka za stvaranje, manipulaciju i proučavanje strukture, dinamike, i funkcija kompleksnih mreža. Za učitavanje mreža koristi Pythonovu strukturu podataka "rječnik", a za numeričku i grafičku analizu koristi već navedene biblioteke (NumPy, Scipy, Matplotlib). U sebi sadrži mnoge funkcije najčešće operacije nad mreže, poput dodavanja i brisanja veza i vrhova, te računanja raznih svojstava mreže.

3.5 Jupyter

Jupyter "bilježnica" je web aplikacija koja omogućava jednostavan i brz razvoj aplikacija i skripti. Najčešće se koristi za čišćenje, procesiranje i transformaciju podataka, te numeričke simulacije, strojno učenje i statističko modeliranje.

Svoju popularnost duguje tome da je olakšao vizualizaciju i dijeljenje koda s drugim ljudima, u jako jednostavnom formatu. Podržava preko 40 programskih jezika, među kojima su Python, R, Julia i Scala, neki od najpopularnijih jezika za obradu podataka.

4. Priprema podatka

U sklopu ovog diplomskog, koristio sam podatke World Wide Trade Web-a. Podaci se nalaze u slijedećem obliku:

Acra numa acrb numb year expab eabo impab iabo

USA 2 CAN 20 1948 1946 0 1794 0

USA 2 CAN 20 1949 1959 0 1743 0

USA 2 CAN 20 1950 2025 0 2101 0

USA 2 CAN 20 1951 2674 0 2466 0

USA 2 CAN 20 1952 2976 0 2601 0

USA 2 CAN 20 1953 3165 0 2669 0

USA 2 CAN 20 1954 2950 0 2574 0

USA 2 CAN 20 1955 3383 0 2887 0

USA 2 CAN 20 1956 4130 0 3156 0

USA 2 CAN 20 1957 4017 0 3187 0

USA 2 CAN 20 1958 3540 0 3262 0

Na prva četiri mjesta su skraćena imena zemalja i njihovi identifikacijski brojevi. Nakon toga slijedi godina trgovanja, a zadnja četiri mjesta su rezervirana za količinu uvoza i izvoza u milijunima dolara.

Iz njih konstruiramo mrežu trgovanja između zemalja u 2000. godini, te tu mrežu pokušavamo rekonstruirati pomoću ECM-a.

Postoje razni načini kreiranja grafova u NetworkX-u, od ručnog upisivanja do nasumičnog generiranja vrhova i rubova. Za naš slučaj najjednostavnije je iščitati graf iz *.edgelist* datoteke, u kojoj se podaci nalaze u slijedećem formatu:

USA CAN 199537.5

USA BHM 685.5

USA HAI 447.75

USA DOM 4909.5

USA JAM 997.5

USA TRI 1422.25

USA BAR 210.0

USA DMA 23.472

USA GRN 22.0185

USA SLU 59.1985

USA SVG 28.82875

USA AAB 50.337

USA SKN 41.731

USA MEX 129762.75

USA BLZ 152.75

USA GUA 2296.5

USA HON 2878.75

Prve dvije stavke su imena zemalja, tj. vrhova u našoj mreži, a na trećem mjestu se nalazi težina veze između njih.

Da bi podatke pretvorili u odgovarajući format, prvo prođemo kroz početne podatke zadržavajući samo nama bitne informacije(imena zemalja, te količinu uvoza i izvoza), te te podatke zapisujemo u datoteku “*temp.txt*”:

```
In [1]: file = open('temp.txt', 'w+')
        with open('trade_dd.asc') as f:
            for line in f:
                words = line.split()
                if words[4] == '2000' and float(words[5])\
                != 0 and float(words[7]) != 0:
                    file.write(words[0] + ' ' + words[2] \
                + ' ' + str(float(words[5]) + float(words[7]))\
                + '\n')
        file.close()
```

Slika 4.1 Prvi dio koda za formatiranje podataka

U toj datoteci su podaci u formatu koji nam je potreban za učitavanje mreže, ali su duplicirani. Na primjer, za vezu težine 20 između Hrvatske i Slovenije, imali bi slijedeće zapise:

HRV SLO 20

SLO HRV 20

Što bi pri učitavanju grafa rezultiralo dobrim brojem vrhova, točnim vezama, ali bi sve težine bile duplo veće. Da bi se riješili duplikacije, učitavamo sve linije temp.txt datoteke u listu (s tim da je svaka linija ujedno i lista), uspoređujemo sve članove međusobno za jednakost, te izbacujemo duplikate:

```
In [3]: tempList = []
        resultList = []

        with open('temp.txt') as f:
            for line in f:
            tempList.append(line)

        for i in tempList:
            x = i.split(' ')
            for j in tempList:
                y = j.split(' ')
                if x[0] == y[1] and x[1] == y[0]:
                    zbroj = float(x[2]) + float(y[2])
                    resultList.append(x[0] + ' ' + x[1]
                                      \+ ' ' + str(zbroj/4.0))

        file = open('result.edgelist', 'w+')
        for i in resultList:
        file.write(i + '\n')
        file.close()
```

Slika 4.2 Izbacivanje duplikata iz mreže

Rezultat zapisujemo u datoteku „*result.edgelist*“. Sada kada imamo dobro formatirane podatke, potrebno je testirati rezultate naše pripreme podataka.

4.1 Učitavanje mreže i provjera obrade

Da bi iz datoteke učitali podatke u graf, koristimo slijedeće naredbe:

```
In [*]: import networkx as nx
        edgelist = raw_input("Apsolutna putanja do edgelista\n")
        G = nx.read_weighted_edgelist(edgelist, nodetype = str)

        Apsolutna putanja do edgelista
```

Slika 4.3 Učitavanje mreže iz .edgelist datoteke

Funkcija *read_weighted_edgelist* prima dva parametra. Prvi je putanja do naše mreže a drugi je kakav tip podataka se nalazi u datoteci. Pošto mi imamo običnu tekstualnu datoteku stavili smo za vrijednost parametar *nodetype* vrijednost str, no tu bi funkcija prihvaćala razne stvari, npr. int za brojeve, img za slike, itd.

NetworkX koristi Pythonovu strukturu podataka „rječnik”(eng. dictionary, hashmap), koji je skupina ključ-vrijednost argumenata, za modeliranje mreža. Svaki ključ je jedan od vrhova mreže, a vrijednost ključa je novi rječnik u kojem su ključevi vrhovi s kojima ima vezu (susjedi), a vrijednosti su atributi tih veza (u našem slučaju, težina).

Sad kad imamo svoju mrežu, prvo ju testiramo za točnost. Za to smo odabrali provjeriti imamo li točan broj vrhova, i ručno provjeriti težinu neko broj veza u originalnoj datoteci.

```
In [1]: assert(187, len(G.nodes()))

Out[1]: True
```

Slika 4.4 Provjera broja vrhova u mreži

Funkcija G.nodes() vraća listu svih vrhova u mreži, a *len()* duljinu te liste, te preko funkcije assert provjeravamo ima li ih 187 (broj zemalja u našim početnim podacima).

Isto tako ručno u našim podacima izračunamo težinu veze između nekih zemalja, te provjeravamo odgovaraju li težinama u našem grafu. Ispod se nalazi provjera veze između Sjedinjenih Američkih Država i Kanade.

```
In [2]: assert(3470, G['USA']['CAN']['weight'])

Out[2]: True
```

Slika 4.5 Provjera točnosti težine veza u grafu.

Gornji dio koda pristupi ključu "USA" u rječniku G, te u rječniku "USA" pristupi ključu "CAN", i nakon toga u rječniku "CAN" uzmi vrijednost "weight". Tu vrijednost uspoređuje sa ručno izračunatom vrijednošću iz originalnih podataka.

5. Riješavanje 2N jednadžbi

Sada prelazimo na rješavanje *2N* jednadžbi 8 i 9 da nađemo *x* i *y* svakog od vrhova u našoj mreži. Prvo učitavamo potrebne module i našu mrežu:

```
In [1]: from __future__ import division
        import networkx as nx
        from scipy.optimize import *
        import numpy as np
        import random
        G = nx.read_weighted_edgelist('bolji.edgelist',\
                                       nodetype = str)
```

Slika 5.1 Učitavanje mreže

Nakon toga započinjemo proces generiranja naših početnih pretpostavki za krajnja rješenja. Basinhopping (algoritam koji koristimo za rješavanje *2N* jednadžbi) zahtjeva da mu na početku damo listu svojih "pokušaja" rješenja. Za *x* smo odlučili uzeti stupanj vrha podijeljen sa najvećim stupnjem u mreži, a za *y* nasumičan broj između 0 i 1. Svi naši pokušaju su u intervalu između 0 i 1, jer želimo da pokušaj bude što bliži stvarnom rješenju, a očekujemo da će krajnje vrijednosti x i y biti između 0 i 1.

Prolazimo kroz listu stupnjeva mreže, te najveći spremamo u varijablu maxDeg:

```
In [3]: maxDeg = 0
        for i in G.nodes():
            #nalazimo najveci stupanj u mrezi
            if G.degree(i) > maxDeg:
                maxDeg = G.degree(i)
```

Slika 5.2. Nalaženje najvećeg stupnja u mreži

Nakon toga generiramo praznu listu veličine 374 (2 mjesta za svaki vrh grafa), te u njoj na odgovarajuća mjesta stavljamo naše pokušaje za vrijednosti x i y tog vrha:

```
In [4]: guessList = np.empty([G.number_of_nodes()*2])
        for index, value in enumerate(G.nodes()):
            #Za svaki vrh ubaci njegov degree/maxDegree i
            #nasumični broj između 0 i 1 u listu
            guessList[index*2] = G.degree(value)/maxDeg
            guessList[index*2+1]= random.random()
```

Slika 5.3 generiranje liste pokušaja

Sada smo spremni opisati svoju funkciju koja opisuje naših *2N* ne linearnih jednadžbi. Prvi korak je konstruiranje prazne liste koja odgovara po veličini rješenju naše jednadžbe. Drugi korak je ubacivanje stupnjeve i jačine vrhova naše mreže u tu listu. Treći korak je uzeti iz naše liste pokušaja odgovarajuće vrijednosti za svaki vrh, te izračunati potrebne dijelove jednadžbe, iterirajući kroz listu vrhove te preskačući trenutačni vrh. Zadnji korak je naći element koji ima najveću pogrešku (najviše odstupa od nule), i vratiti ga kao rješenje funkcije.

```
In [5]: def myFunk(guess):
            #1
            equations = np.empty([G.number_of_nodes()*2])
            for index,node in enumerate(G.nodes()):
                #2.
                equations[index*2] = - G.degree(node)
                equations[index*2-1] = - G.degree(node,\
                                        weight = 'weight')


            #3.
            for index,node in enumerate(G.nodes()):
                x = guess[index*2]
                y = guess[index*2+1]
                for indexTwo, nodeTwo in enumerate(G.nodes()):
                    if nodeTwo == node:
                        continue
                    xx = guess[indexTwo*2]
                    yy = guess[indexTwo*2+1]
                    equations[index*2] += (x*xx*y*yy)/\
                                    (1-y*yy + x*xx*y*yy)
                    equations[index*2+1] += (x*xx*y*yy)/\
                                    ((1-y*yy)*(1-y*yy+x*xx*y*yy))
            #4.
            maxErr = 0
            for i in equations:
                if abs(i) > maxErr:
                    maxErr = abs(i)

            return maxErr
```

Slika 5.4 Definicija *2N* jednadžbi koje pokušavamo riješiti

Potom definiramo novu funkciju imena "*test_prihvacanja*" koji basinhopping koristi kao provjeru svojih koraka (uz Metropolis test prihvaćanja rješenja). Ona provjerava jesu li svi članovi liste pozitivni i manji od 1, jer su to jedine vrijednosti koje prihvaćamo. Ako rješenja zadovoljavaju naše uvijete, korak koji je uzet u toj iteraciji algoritma se prihvaća (funkcija vraća True), ako ne zadovoljava (funkcija vraća False), odbacuje se uzeti korak. Popis argumenata se sastoji od vrijednosti funkcije te uzetom koraku u prošloj i trenutnoj iteraciji Basinhopping algoritma:

```python
In [6]: def test_prihvacanja(f_new, x_new, f_old, x_old):
          for index, value in enumerate(x_new):
            if (index % 2 == 0) and (value < 0):
              return False
            if (index % 2 != 0) and ((value < 0) or (value >= 1)):
              return False
          return True
```

Slika 5.5 test prihvaćanja rezultata

Nakon toga je samo preostalo pozvati scipy funkciju basinhopping i izvršiti naš program.

```python
In [7]: basinhopping(myFunk, guessList, disp = True, \
                     accept_test = test_prihvacanja, \
                     stepsize = 0.0001)
```

Slika 5.6 pozivanje basinhoppping funkcije

Program pokrećemo iz terminala naredbom:

nohuppython imePrograma.py > riješenje.txt

Nohup garantira da će se program izvršavati i nakon što prekinemo vezu sa serverom, a znak “>” preusmjerava ispis programa (rekonstruiranu mrežu) u datoteku „*riješenje.txt*“.

Program pokrećemo *x* puta, gdje je *x* željena veličina našeg ansambla težinskih mreža.

Izvršavanje naše funkcije traje u prosjeku 0.4 sekunde, no zbog velikog broja nasumičnih peturbacija koje izvodi Basinhopping, naša funkcija je pozivana više milijuna puta, tako da je izvršavanje programa trajalo sedam dana.

6. Provjera rezultata rekonstrukcije

Rezultat našeg programa je lista u kojoj se nalaze x i y vrijednosti svakog vrha u rekonstruiranoj mreži. Za provjeru našeg rješenja, prvo konstruiramo praznu mrežu H, u nju dodajemo sve vrhove iz originalne mreže, te učitavamo naše rješenje u varijablu resultList:

```
In [1]: H = nx.Graph()
        for i in G.nodes():
            H.add_node(i)
```

Slika 6.1 Učitavanje vrhova iz originalne mreže u rezultantnu mrežu

Nakon toga, za sve postojeće veze iz originalne mreže, izračunamo vjerojatnost postajanja te veze u rekonstruiranoj mreži pomoću jednadžbe 7:

```
In [2]: for index, i in enumerate(G.nodes()):
            for index2, j in enumerate(G.nodes()):
                if G.has_edge(i, j):
                    x = resultList[index]
                    xx = resultList[index2]
                    y = resultList[index+1]
                    yy = resultList[index2+1]
                    w = G[i][j]['weight']
                    weight = ((x*xx)**(y*yy)**(w)*(1-y*yy))/\
                            (1 - y*yy + x*xx*y*yy)*1.0
                    H.add_edge(i, j)
                    H[i][j]['weight'] = weight
                else:
                    x = resultList[index]
                    xx = resultList[index2]
                    y = resultList[index+1]
                    yy = resultList[index2+1]
                    weight = (1 - y*yy)/\
                        (1 - y*yy + x*xx*y*yy)*1.0
                    H.add_edge(i, j)
                    H[i][j]['weight'] = weight
```

Slika 6.2 Dodavanje težina u rekreiranu mrežu

Sada konstruiramo matricu susjednosti iz našeg originalnog grafa(Gmatrix). Matrica susjednosti je matrica dimenzija *nxn* čiji su elementi nule i jedinice. Koriste se za jednostavan prikaz mreža. Jedinice znače da postoji veza između vrhova, nula da ne postoji. Istovremeno konstruiramo i matricu vjerojatnosti iz naše rekonstruirane mreže(Hmatrix). Na odgovarajuća mjesta stavljamo vjerojatnosti, koje odgovaraju vjerojatnostima da veza postoji u našem rekonstruiranom grafu.

```
In [3]: w, h = 187, 187
        Gmatrix = [[0 for x in range(w)] for y in range(h)]
        Hmatrix = [[0 for x in range(w)] for y in range(h)]
        for index, i in enumerate(G.nodes()):
            for index2, j in enumerate(G.nodes()):
                if G.has_edge(i,j):
                    Gmatrix[index][index2] = 1
                    Hmatrix[index][index2] = H[i][j]['weight']
                else:
                    Gmatrix[index][index2] = 0
                    Hmatrix[index][index2] = H[i][j]['weight']
```

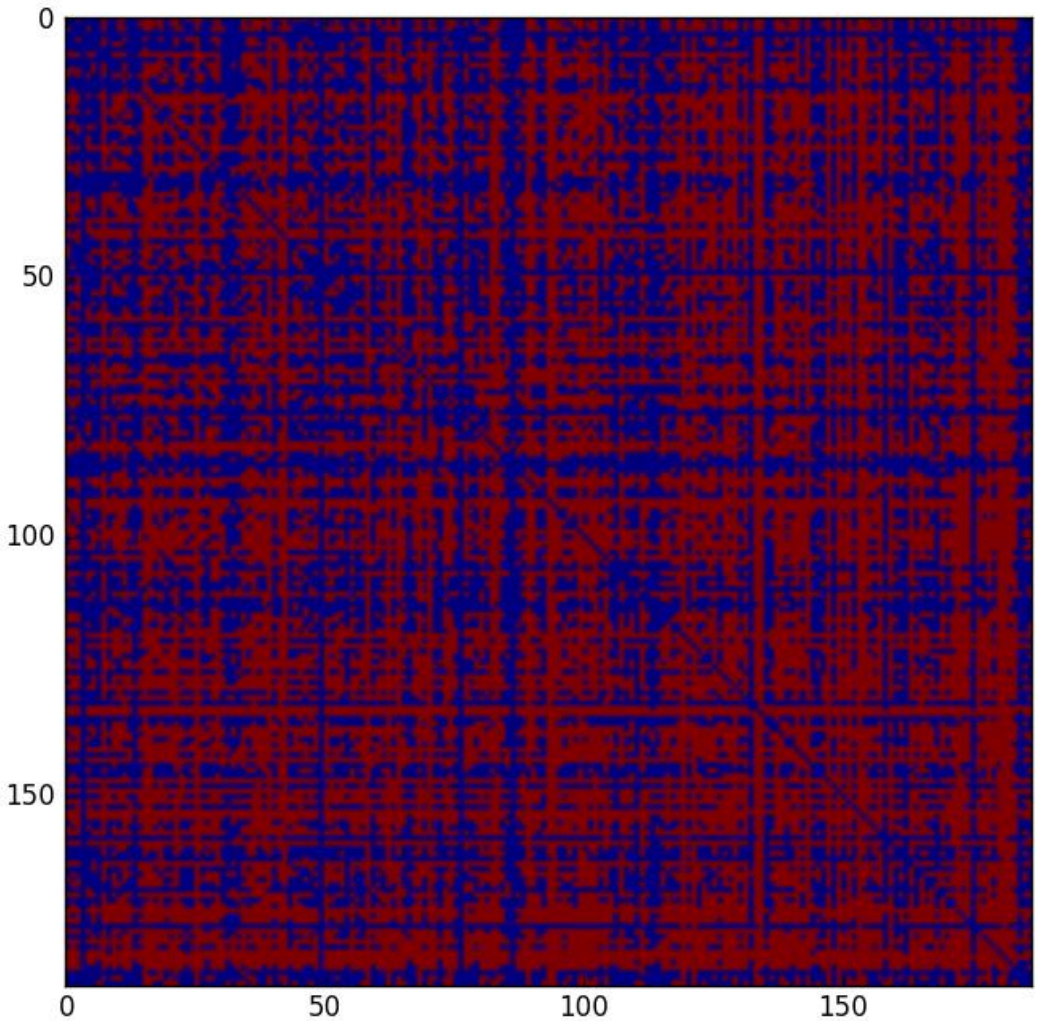
Slika 6.3 Generiranje matrica susjednosti

Kada dobijemo obje matrice, računamo njihovu sličnost preko sume korijena razlike kvadrata svakog polja u matricama.

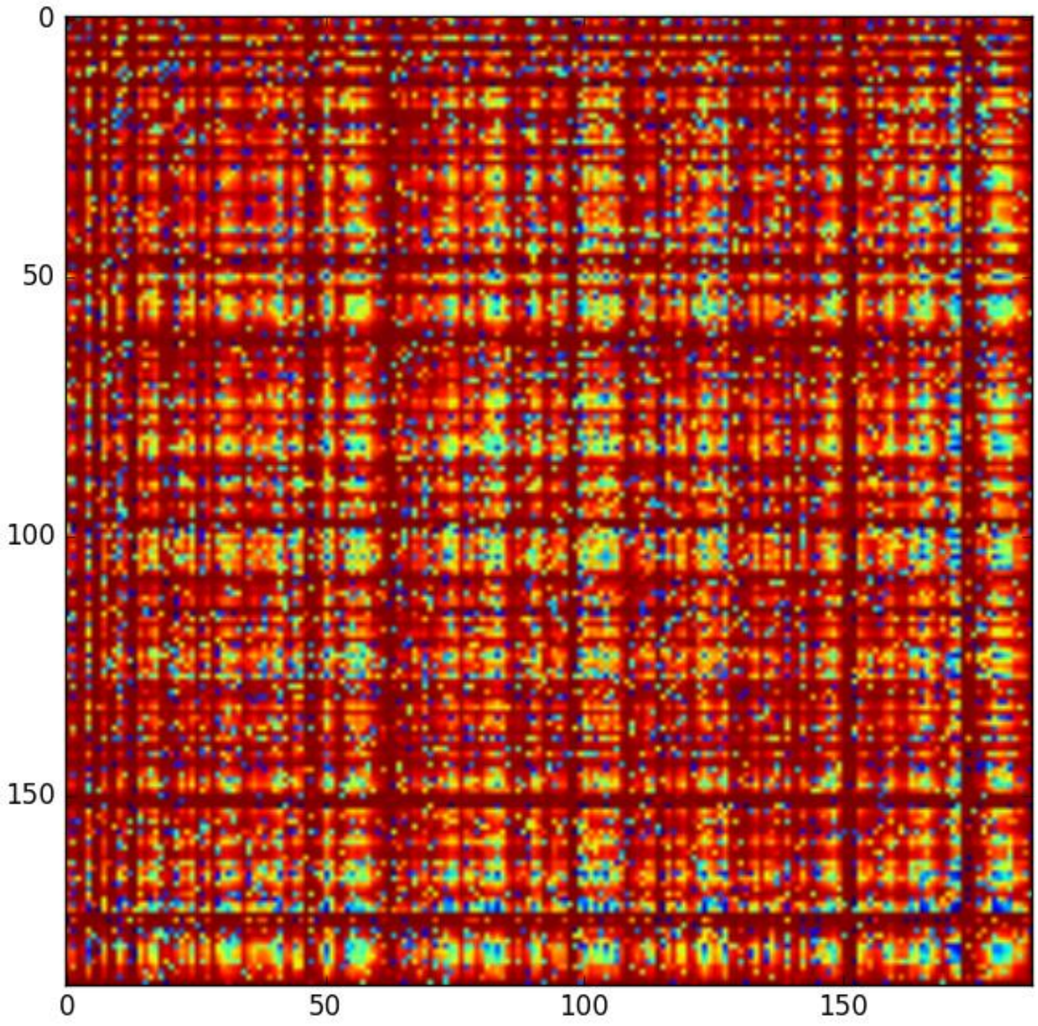
```
In [4]: similarity = 0
        for i in range(187):
            for j in range(187):
                similarity = similarity + pow(Gmatrix[i][j]\
                                              - Hmatrix[i][j],2)
```

Slika 6.4 Računanje sličnosti matrica susjednosti originalne i rekonstruirane mreže

Proces ponavljamo za sva rješenja koja smo dobili, te vidimo da konzistentno dobivamo udaljenost matrica rekonstruirane i originalne mreže od ~130, što je na matrici od 34969 članova (maksimalna udaljenost 34969) jako mala udaljenost. No to nije dovoljno da bi smo mogli tvrditi da smo dobro rekonstruirali mrežu. Kao dodatan korak provjere, na isti način provjeravamo međusobnu udaljenost naših rješenja, te dobivamo da su sva naša rješenja međusobno udaljena za ~31. To znači da smo uspješno našli minimum funkcije koju smo rješavali, te da naš ansambl težinskih mreža dobro rekonstruira originalnu mrežu. Za lakšu vizualizaciju, crtamo "*heatmap*" jednog od rješenja i originalne matrice susjednosti.



Slika 6.5 „Heatmap" originalne mreže



Slika 6.6 „Heatmap“ rekonstruirane mreže

7. Python i cijeloživotno učenje

Korištenje IT tehnologija poput „pametnih“ mobitela i interneta je svakodnevnica gotovo svih učenika(i građanina) Hrvatske. Kako tehnologija prodire sve dublje u svakodnevnicu prosječnog učenika, potrebno je i povećati njihovu razinu znanja o tome kako ona funkcionira. U tu svrhu, sve se češće i sve ranije se započinje sa strukturiranim učenjem Informatike u osnovnim i srednjim školama, te u sklopu nje učenje programiranje.

Europski kvalifikacijski okvir(EQF), iz kojeg je nastao i Hrvatski kvalifikacijski okvir (HKO) definira osposobljavanje učenika za cjeloživotno učenje. Pojedinac stječe znanja, vještine, samostalnost i odgovornost te se na taj način poboljšava i napreduje u karijeri, a društvo dobiva pojedince koji se stalno razvijaju te stečena znanja i vještine koriste za daljnji doprinos društvu.

Ključne kompetencije za cjeloživotno učenje propisane u EQF-u su:

1. Komunikacija na materinskom jeziku
2. Komunikacija na stranom jeziku
3. Matematička kompetencija i temeljne kompetencije u prirodnim znanostima i tehnologiji
4. Digitalna kompetencija
5. Kompetencija učenja
6. Društvene i građanske kompetencije
7. Smisao za inicijativu i poduzetništvo
8. Kulturološka senzibilizacija i izražavanje.

Učenje informatike, pa tako i programiranje u Pythonu, doprinosi postizanju 2., 3., 4. I 5. kompetencije. Python se pokazao kao jako dobar izbor za prvi programski jezik za učenike osnovne i srednje škole, a u nastavku poglavlja ćemo pokušati obrazložiti zašto.

7.1 Python kao prvi programski jezik

Python je programski jezik opće upotrebe, što znači da se može koristiti za implementaciju praktički bilo čega, što se i odražava u stvarnosti. Jedan je od tri najkorištenija programska jezika na svijetu, kako za osobnu upotrebu na samostalnim projektima tako i u raznim poljima u industriji.

Nameće se pitanje zašto baš Python kao prvi programski jezik?

Python je dizajniran da bude *jednostavan* i *zabavan* za upotrebu! Zabava je odličan motivator, a u Pythonu je lako napraviti prototipe i alate za koju god svrhu ga koristimo. Zbog činjenice da je napravljen da bude jednostavan za korištenje i učenje, pojavila se veliča količina javno dostupnog materijala za pomoć pri samostalnom učenju, te razne knjige i alati namijenjeni nastavnicima.

Iako je Python programski jezik visoke razine, jako je fleksibilan, i nema striktna pravila poput programskih jezika Java ili C. To omogućuje izgled sintakse koji je jako sličan engleskom jeziku što olakšava pisanje novih programa i razumijevanje postojećih. Lakoća korištenja Pythona ujedno olakšava težak zadatakmotiviranja učenika i objašnjavanja apstraktnih koncepata potrebnih za savladavanje osnova programiranja.

Uz sve to, odličan razlog za izbor Pythona kao programskog jezika koji ćemo predavati u školama, je i mogućnost, što nakon što prođemo osnove programiranja (osnovni algoritmi, strukture podataka, kontrola izvršavanja programa, itd.), prilagođavanja individualnim željama naših učenika. Pošto je programiranje zapravo proces stvaranja, možemo pustiti učenike da stvaraju ono što ih zanima.

Ako ih zanima programiranje video igara, imamo biblioteku PyGame.

Ako žele raditi web aplikacije, postoje okviri Django i Flask.

Ako žele raditi robote, postoji platforma raspberrypi za robotiku, koja se specijalizira specifično za učenje programiranja u robotici.

Ako naši učenici pak matematički nastrojeni, biblioteke koje smo već naveli (NumPy i SciPy) im stoje na raspolaganju.

Jedan od glavnih ciljeva svih gore navedenih alata je približiti programiranje početnicima, olakšati im prve korake, te ih motivirati za daljnji rad. Za sve njih isto tako postoji velika količina javno dostupnog materijala napisanog specifično za učenike osnovnih i srednjih škola.

Uz to, bitno je napomenuti i da se Python ne koristi samo za učenje programiranja, nego je u širokoj upotrebi u industriji, te tako pomaže našim učenicima da nađu svoj prvi posao.

Primjeri korištenja Pythona u industriji:

- analiziranje podataka

- skripte za automatiziranje održavanja servera
- penetracijsko testiranje u polju cyber sigurnosti
- razvoj web servisa

7.2 Primjer korištenja biblioteke PyGame u nastavi

Primjer sata koji slijedi je pogodan učenicima osnovne i srednje škole. Predviđen je blok-sat, u obliku vježbi, da se dozvoli učenicima da se upoznaju sa bibliotekom uz pomoć nastavnika te zatim u parovima primjenjuju stečeno znanje. Obrazovni ishodi postignuti tijekom ovih sati su pisanje osnovnih igara u Pythonu, te implementacija prethodno stečenog algoritamskog znanja. Funkcionalni obrazovni ishodi su razvijanje sposobnosti uočavanja problema, razvijanje apstraktnog načina razmišljanja, povezivanje ranije stečenih znanja te razvijanje sposobnosti primjene usvojenog znanja. Odgojni ishodi su razvijanje odgovornosti, temeljitosti i sistematičnosti, uvažavanje tuđih mišljenja i zaključaka, te prilagođavanje radu u skupini.

Potrebno predznanje za ovaj sat je osnovno znanje o kontroli izvedbe programa (petlje i if/else grananje), te definiranje funkcija i klasa u Pythonu.

S učenicima prolazimo kroz primjere korištenja biblioteke. U prvom primjeru crtamo sliku bubamare na bijeloj ploči.

Prvo trebamo u naš program uključiti PyGame i sys biblioteke. Varijabla WHITE inicijalizira strukturu podataka zvanu *"tuple",* koja sadrži tri broja. Svaki broj označava koliko će koje boje biti na našoj pozadini (crvena, bijela, plava), te koliko će te boje biti (u razmaku od nula do 255). Sve boje na maksimumu nam daju bijelu pozadinu, dok za npr. crnu bi sve boje stavili na vrijednost 0. Preko varijable DISPLAYSURF inicijaliziramo praznu ploču. Brojevi koje smo proslijedili funkciji set_mode označavaju širinu i visinu naše pozadine (600,800) i dubinu iste (0, 32).

Varijabla tile u sebi sadrži sliku bubamare (možemo ju zamijeniti proizvoljnom slikom, dok god je u istom direktoriju u kojem pokrećemo program).

Za kraj, pokrećemo petlju koja se vrti dok ju ručno ne ugasimo, i za svaki prolaz kroz petlju ponovno crta našu bubamaru na zadanoj poziciji na ploči. Metoda DISPLAYSURF.blit uzima kao parametre varijablu koja sadrži sliku koju želimo prikazati, te koordinate na kojima želimo iscrtati sliku.

In [1]:

```
import pygame, sys

from pygame.locals import *

pygame.init()
WHITE = (255, 255, 255)

DISPLAYSURF = pygame.display.set_mode((600, 800), 0, 32)
pygame.display.set_caption('My Bug')
tile = pygame.image.load('Enemy Bug.png')

while True:
    DISPLAYSURF.fill(WHITE)
    DISPLAYSURF.blit(tile, (100, 200))

    for event in pygame.event.get():
        if event.type == QUIT:
            pygame.quit()
            sys.exit()

    pygame.display.update()
```

Slika 7.1 Kod za prikaz bubamare na bijeloj površini

Slika 7.2 Rezultat izvršavanja prvog primjera

Za drugi primjer, demonstrirati ćemo kako micati bubamaru po našoj površini. Dodajemo varijable koje kontroliraju x i y koordinate bubamare (tileX i tileY). Uz to dodajemo varijable FPS i fpsClock, koje određuju koliko puta u sekundi ćemo crtati bubamaru na pozadini. Unutar naše petlje, za svako izvršavanje petlje pomičemo bubamaru udesno, te ako dođe do ruba, vraćamo ju na početak.

```
In [ ]: FPS = 33
        fpsClock = pygame.time.Clock()

        tileX = 100
        tileY = 600

        while True:
            DISPLAYSURF.fill(WHITE)

            if tileX >= 0:
                tileX += 5
                if tileX == 700:
                    tileX = 0

            DISPLAYSURF.blit(tile, (tileX, tileY))

            for event in pygame.event.get():
                if event.type == QUIT:
                    pygame.quit()
                    sys.exit()

            pygame.display.update()
            fpsClock.tick(FPS)
```

Slika 7.3 Primjer programa za micanje bubamare

Sada su naši učenici spremni rješavati zadatke. Predviđeno je da se učenici podijele u parove, i zajedno rješavaju zadatke. Jedan učenik je „*pilot*“, a drugi „*kontrolor*“. Jedan piše program, a drugi ga navodi i govori mu kako misli da bi se trebao riješiti zadatak. Ova tehnika se zove „*pair programming*“, te se često koristi i na poslu pri poučavanju novih zaposlenika. Nakon svakog uspješno riješenog zadatka, učenici mijenjaju uloge.

Zadaci koje bi učenici trebali riješiti su:

1. Postaviti bubu u gornji desni kut ploče i te ju micati dolje
2. Postaviti bubu u donji lijevi kut, te ju dovesti do gornjeg desnog kuta micanjem po dijagonali

3. Postaviti bubu u gornji lijevi kut, te ju (micanjem po rubu ploče) dovesti do donjeg desnog kuta
4. Micati bubu u krug po ploči, u smjeru kazaljke na satu
5. Micati bubu u krug po ploči, suprotno od smjera kazaljke na satu
6. Nacrtati dvije bube koje se gibaju u krug po ploči, u suprotnim smjerovima. Kada se bube susretnu, trebaju se okrenuti bez dodirivanja i promijeniti smjer gibanja.

8. Zaključak

Metodom pojačane rekonstrukcije mreža smo zaista dobili dobar rezultat, te potvrdili rezultate dobivene u članku. Zajedničnom specifikacijom jačine i stupnjeva vrhova dobivamo jaču specifikaciju mreže koju pokušavamo rekonstruirati, nego navođenjem samo stupnjeva ili jačine. Isto tako, svođenje problema rekonstrukcije na riješavanje *2N* jednadžbi je jako pojednostavnilo rekonstruiranje mreža, pošto svi alati i programski jeziki koji se koriste za konstrukciju mreža imaju u sebi alate za riješavanje jednadžbi.

9. Literatura

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Dodaci

A. Ansambl Težinskih mreža

Ovdje prilažem rezultate rekonstrukcije mreže, tj. Ansamble težinskih mreža.

```
x: array([ 0.51285669,  0.81010154,  0.4847616 ,  0.61871273,  0.55650146,
    0.71160973,  0.00549451,  0.01058584,  0.37393765,  0.17583421,
    0.380366  ,  0.67268121,  0.22195918,  0.95926578,  0.84190446,
    0.6591858 ,  0.31930388,  0.2263406 ,  0.61020139,  0.19991571,
    0.546135  ,  0.99403803,  0.67157479,  0.5912964 ,  0.440183  ,
    0.25384886,  0.11175779,  0.84736177,  0.41333303,  0.48514049,
    0.89591568,  0.06194588,  0.81992644,  0.60305671,  0.56811303,
    0.9521878 ,  0.71003633,  0.65669695,  0.44692263,  0.8807171 ,
    0.55007311,  0.23567169,  0.85195963,  0.17127153,  0.46184974,
    0.17083283,  0.42888271,  0.11091019,  0.51741736,  0.49239296,
    0.96703297,  0.02332415,  0.49150123,  1.00047431,  0.38523794,
    0.28705884,  0.69811348,  0.28125555,  0.79152007,  0.16685457,
    0.41239919,  0.21085079,  0.17800314,  0.91790923,  0.31443193,
    0.49483108,  0.30924871,  0.7604192 ,  0.66701413,  0.94076291,
    0.61662974,  0.68661378,  0.84771025,  0.78143035,  0.68162996,
    0.11443475,  0.63311325,  0.48975297,  0.91851626,  0.41802441,
    0.33640996,  0.62766169,  0.81505449,  0.79378979,  0.95086073,
    0.125062  ,  0.30800359,  0.08881998,  0.56811303,  0.94416373,
    0.59371787,  0.10316604,  0.53390087,  0.39952754,  0.47408387,
    0.7630352 ,  0.46858937,  0.73925616,  0.00705091,  0.6729204 ,
```

0.68318636, 0.82706325, 0.45086073, 0.138572 , 0.56230724,

0.85300018, 0.53877282, 0.23372029, 0.22651985, 0.5682222 ,

0.62212424, 0.62620894, 0.40721597, 0.3494328 , 0.49450549,

0.0255867 , 0.95760036, 0.80946833, 0.28053106, 0.15687526,

0.54032922, 0.87361227, 0.51161157, 0.27611584, 0.94598879,

0.5504332 , 0.47988966, 0.81179898, 0.95117201, 0.26125743,

0.46734425, 0.25389238, 0.20879121, 0.00206573, 0.56106212,

0.354665 , 0.90690469, 0.13132235, 0.45179457, 0.64344911,

0.93468849, 0.29069988, 0.43406593, 0.04167629, 0.22745369,

0.97612325, 0.70391926, 0.35740282, 0.79120879, 0.00669136,

0.39653824, 0.55134179, 0.00642835, 0.52048729, 0.6379852 ,

0.42417956, 0.39010989, 0.02243154, 0.42463332, 0.70823792,

0.69873604, 0.46031646, 0.44567751, 0.35197308, 0.89010989,

0.02108998, 0.94049428, 0.43764503, 0.50580579, 0.15160678,

0.25855304, 0.1361094 , 0.9569778 , 0.47891855, 0.00642835,

0.46730598, 0.34708769, 0.50901306, 0.46827809, 0.59041238,

0.92950527, 0.50199175, 0.79857098, 0.88724894, 0.41333303,

0.52778576, 0.31474321, 0.66276234, 0.97470643, 0.9728865 ,

0.94567751, 0.30274783, 0.62761875, 0.53338469, 0.53452343,

0.67857183, 0.65446871, 0.26308199, 0.53970666, 0.63410729,

0.89197757, 0.86590115, 0.71584211, 0.73845915, 0.73139179,

0.17164066, 0.88492666, 0.06854694, 0.43075039, 0.93547801,

0.8357874 , 0.33339351, 0.46921193, 0.99466212, 0.21065889,

0.7816999 , 0.79338775, 0.95438942, 0.38523794, 0.36301246,

0.45635524, 0.19657977, 0.20972505, 0.44454364, 0.79120879,

0.01124764, 0.3635712 , 0.47999963, 0.15084189, 0.9951793 ,

0.25948688, 0.60674291, 0.59496299, 0.7829713 , 0.64835165,

0.04571461, 0.45086073, 0.04931118, 0.63736264, 0.01645924,

0.99543933, 0.33384498, 0.84221574, 0.71284942, 0.9569778 ,

0.5555425 , 0.56593407, 0.01253847, 0.63891904, 0.61286425,

0.67800314, 0.98753701, 0.82086028, 0.97802944, 0.37424893,

0.2744457 , 0.84065934, 0.02251398, 0.71490827, 0.40166405,

0.94505495, 0.050587 , 0.41302175, 0.47490279, 0.74880915,

0.66938341, 0.10532944, 0.43864957, 0.97439515, 0.92573624,

0.98963355, 0.23060415, 0.21459699, 0.08788527, 0.40296658,

0.81903208, 0.84646513, 0.08552122, 0.8412819 , 0.40782923,

0.59371787, 0.29220108, 0.62181296, 0.45025897, 0.93499977,

0.44104442, 0.7299131 , 1.3983951 , 0.97315003, 0.30032832,

0.31443193, 0.55343164, 0.34097062, 0.18911452, 0.29763714,

0.41934931, 0.49606189, 0.77395702, 0.26498138, 0.61099037,

0.9624723 , 0.32650493, 0.57298497, 0.71624658, 0.94536623,

0.0645445 , 0.36481632, 0.87571676, 1.00217896, 0.97919738,

0.91426687, 0.99559481, 0.61144651, 0.66297224, 0.55681274,

0.90299779, 0.84221574, 0.69616696, 0.11662974, 0.63428909,

0.91239919, 0.06102779, 0.33547612, 0.20987785, 0.39591568,

0.09604297, 0.13280197, 0.42674473, 0.88648306, 0.87803929,

0.46890065, 0.8425003 , 0.71584211, 0.71510405, 0.53970666,

0.54902416, 0.50549451, 0.09203436, 0.21615339, 0.81994576,

0.78695941, 0.63237606, 0.43499977, 0.48430579, 0.66120834,

0.80836485, 0.97408387, 0.74195537, 0.80831487, 0.34751956,

0.03846154, 0.05147568, 0.78115362, 0.45871398, 0.65571383,

0.90595144, 0.93987172, 0.18148152, 0.54551244, 0.66791796,

0.96734425, 0.21609712, 1.00062256, 0.18417575, 0.90659341,

0.03410145, 0.74787531, 0.17920083, 0.1227468 , 0.85832205,

```
        0.37518277,  0.68841034,  0.38523794,  0.1883518 ])
x: array([ 0.50451978,  0.4850831 ,  0.47580137,  0.88156213,  0.55933918,
        0.2922034 ,  0.00454026,  0.59582833,  0.39207891,  0.2020887 ,
        0.37310778,  0.78219143,  0.22128877,  0.65884522,  0.85717513,
        0.7262707 ,  0.34780581,  0.91925458,  0.59336366,  0.41974182,
        0.54806842,  0.93637355,  0.66780768,  0.29132151,  0.42263122,
        0.24521053,  0.13377539,  0.1340672 ,  0.39931475,  0.94725995,
        0.90569243,  0.0693912 ,  0.82380722,  0.03291432,  0.54619238,
        0.42750853,  0.71589267,  0.12758553,  0.48334254,  0.88958807,
        0.56461245,  0.01894811,  0.87029234,  0.63464064,  0.45943879,
        0.54582306,  0.40415999,  0.03289084,  0.51344437,  0.25879969,
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