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Dependence of chaotic behavior on the residual interaction in the odd-odd nucleus ^{106}Ag

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Dependence of the level-density fluctuations on the interaction strengths for the ^{106}Ag energy spectrum calculated in the interacting-boson-fermion-fermion model is investigated. Breaking of symmetry is accompanied by a rapid transition from a nearly Poissonian distribution to the intermediate pattern between Poisson and Gaussian-orthogonal-ensemble distributions. Fluctuation measures show sensitivity to details of nuclear dynamics. Possible connection with the degree of chaos is discussed.

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Quantum chaos is a term used to denote quantal phenomena which are characteristic of chaotic, nonlinear, behavior in the corresponding classical system [1–3]. Investigating fluctuations of the quantal energy levels in the Sinai-billiard problem, Bohigas, Giannoni, and Schmit [4] suggested that the Gaussian-orthogonal-ensemble (GOE) type of spectral distribution may imply manifestation of chaos in the classical system. In a semiclassical derivation of the spectral rigidity, Berry [5], concluded that the energy levels of quantum systems which correspond to classically regular and chaotic systems obey the Poisson and GOE (Wigner) statistics, respectively, and proposed the study of transitional cases between the two distributions [6–11]. Zhang *et al.* [12–14] have concluded that the existence of unbroken dynamical symmetry in a quantal system implies integrability, whereas breaking of the symmetry leads to nonintegrability and chaotic dynamics. Investigations of the relationship between dynamical symmetry, integrability, and quantum level statistics are presently an active field of research [15–22].

In nuclear physics, the density fluctuations of the realistic nuclear-level sequences including the ground-state region lie between the Poisson and GOE (Wigner) distributions [23–27]. Applying the dynamical-symmetry concept and mean-field theory [12–14] to nuclear many-body systems, Zhang and Feng [3] concluded that such systems are generally nonintegrable due to the complicated inherent interactions, but in certain regions of intrinsic parameters, where subdynamical symmetry, e.g., certain excitation modes dominate, the corresponding dynamics is regular.

In this paper we investigate possible manifestations of chaos in the odd-odd nucleus ^{106}Ag. The basis of our presentation is a realistic calculation of the energy spectrum and the electromagnetic properties within the framework of the interacting-boson-fermion-fermion model (IBFFM) [28–32], which satisfactorily reproduces the experimental properties of ^{106}Ag (Ref. [33]), and is characterized by the underlying SU(6) symmetry. Since the model Hamiltonian which will be given in Eqs. (1)–(14) considerably exceeds in complexity the examples which have been treated by the mean-field methods [3], we concentrate our attention on the analysis of the level-density fluctuations.

In the IBFFM [28–32], the odd-odd nucleus is described by the Hamiltonian

$$H_0=H_0^c+H_0^{qp}+H^{\text{int}} . \tag{1}$$

The first term in (1) is the collective Hamiltonian in the interacting-boson model (IBM) [33,34] expressed in terms of creation and annihilation operators for quadrupole bosons, $b_{2\mu}^\dagger$ and $b_{2\mu}$, respectively [36],

$$\begin{aligned}H_0^c=&h_1\hat{N}+h_2\{(b_2^\dagger b_2^\dagger)_0[(N-\hat{N})(N-\hat{N}-1)]^{1/2}+\text{H.c.}\}\\&+h_3[(b_2^\dagger b_2^\dagger\tilde{b}_2)_0(N-\hat{N})^{1/2}+\text{H.c.}]\\&+\sum_{L=0,2,4}h_{4L}[(b_2^\dagger b_2^\dagger)_L(\tilde{b}_2\tilde{b}_2)_L]_0 .\end{aligned} \tag{2}$$

Here,

$$\hat{N}=\sum_{\mu=-2}^{2}b_{2\mu}^\dagger b_{2\mu} \tag{3}$$

is the number operator for quadrupole bosons and

$$\tilde{b}_{2\mu}=(-)^\mu b_{2,-\mu} . \tag{4}$$

This Hamiltonian is constructed from the quadrupole boson generators of the SU(6) group, which are, expressed in Hollstein-Primakoff representation [35,37],

$$b_{2\mu}^\dagger(N-\hat{N})^{1/2},\ (N-\hat{N})^{1/2}\tilde{b}_{2\mu},\ \text{and}\ b_{2\mu}^\dagger\tilde{b}_{2\mu} . \tag{5}$$

Here N denotes the maximal number of quadrupole bosons. The second term in (1),

$$H_0^{qp}=\sum_{\rho,j_\rho}\mathcal{A}_{qp}(\rho,j_\rho)c_{j_\rho}^\dagger(\rho)c_{j_\rho}(\rho) , \tag{6}$$

describes independent quasiparticles in the shell-model configurations $j_\rho\equiv(n,l,j)_\rho$, with $\rho=\pi$ and $\rho=\nu$ denoting quasiprotons and quasineutrons, respectively. Operators $c_j^\dagger(\rho)$ and $c_j(\rho)$ are the creation and annihilation operators of the corresponding quasiparticle. Also, here we define

$$\tilde{c}_{j,m} = (-)^{j-m} c_{j,-m} \ , \tag{7}$$

with $-j \leq m \leq j$. The third, interaction term in (1) is given by

$$H^{\mathrm{int}} = \sum_{\rho} H^{\mathrm{int}}_{c,qp}(\rho) + H^{\mathrm{int}}(\pi\nu) \ . \tag{8}$$

The first term in (8) describes three types, dynamical, monopole, and exchange, of the quasiparticle-core interaction [36,38],

$$H^{\mathrm{int}}_{c,qp}(\rho) = H^{\mathrm{int}}_{\mathrm{dyn}}(\rho) + H^{\mathrm{int}}_{\mathrm{mon}}(\rho) + H^{\mathrm{int}}_{\mathrm{exch}}(\rho) \ , \tag{9}$$

$$H^{\mathrm{int}}_{\mathrm{dyn}}(\rho) = \Gamma_\rho \sum_{j_\rho, j'_\rho} \mathcal{A}^{\mathrm{dyn}}(j_\rho, j'_\rho)\{[c^\dagger_{j_\rho}(\rho)\tilde{c}_{j'_\rho}(\rho)]_2 Q_2\}_0 \ , \tag{10}$$

$$H^{\mathrm{int}}_{\mathrm{mon}}(\rho) = A_\rho \sum_{j_\rho} \mathcal{A}^{\mathrm{mon}}(j_\rho)\{[c^\dagger_{j_\rho}(\rho)c^\dagger_{j_\rho}(\rho)]_0[b^\dagger_2\tilde{b}_2]_0\} \ , \tag{11}$$

$$H^{\mathrm{int}}_{\mathrm{exch}}(\rho) = \Lambda_\rho \sum_{j_\rho, j'_\rho, j} \mathcal{A}^{\mathrm{exch}}(j_\rho, j'_\rho, j) \times :\{[c^\dagger_{j_\rho}(\rho)\tilde{b}_2]_j [\tilde{c}_{j_\rho}(\rho) b^\dagger_2]_j\}_0: \ , \tag{12}$$

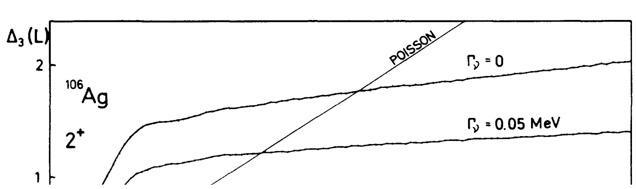
with

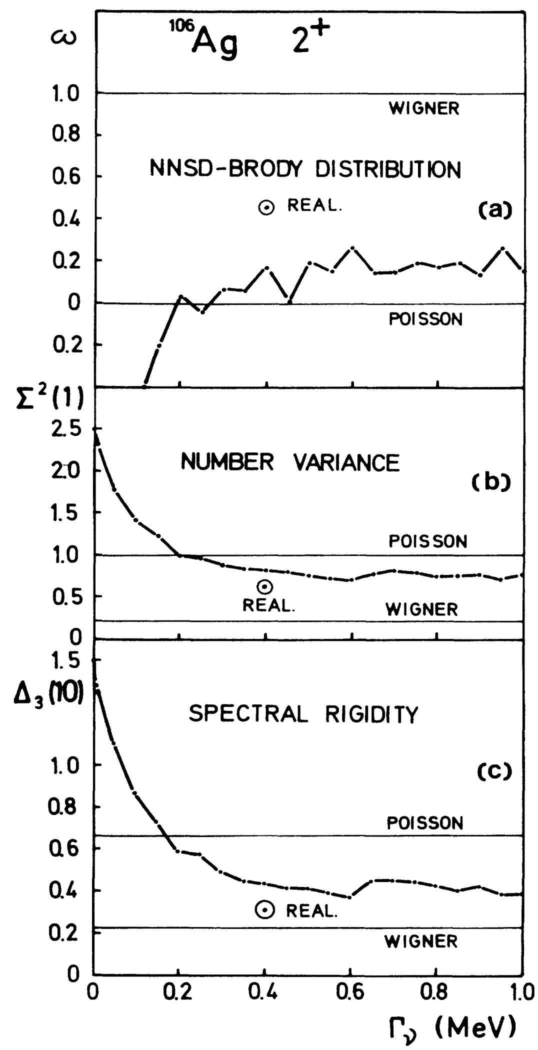
$$Q_{2\mu} = b^\dagger_{2\mu}(N-\hat{N})^{1/2} + (N-\hat{N})^{1/2}\tilde{b}_{2\mu} + \chi(b^\dagger_2\tilde{b}_2)_{2\mu} \ . \tag{13}$$

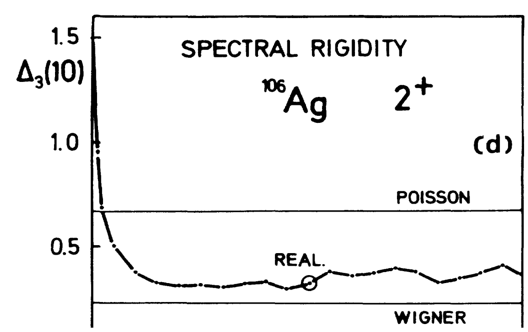
In the term $H^{\mathrm{int}}(\pi\nu)$, responsible for the quasiproton-quasineutron interaction, only the multipole-multipole interaction was taken into account,

$$H^{\mathrm{int}}_{\mathrm{mult}}(\pi\nu) = \sum_L \varepsilon_L \sum_{j_\pi, j_\nu, j'_\pi, j'_\nu} \mathcal{A}^{\mathrm{mult}}_L(j_\pi, j_\nu, j'_\pi, j'_\nu) \times \{[c^\dagger_{j'_\nu}(\nu)\tilde{c}_{j_\nu}(\nu)]_L \times [c^\dagger_{j'_\pi}(\pi)\tilde{c}_{j_\pi}(\pi)]_L\}_0 \tag{14}$$

with ε_2 and ε_4 different from zero. The detailed structure of the coefficients $\mathcal{A}$ can be found in Refs. [31,36]. The







values of parameters appearing in the terms (2) and (10)–(13) are determined from the properties of the neighboring even-even and odd-even nuclei.

We first diagonalize the Hamiltonian choosing the set of parameters which was adopted to describe the realistic ^{106}Ag spectrum [33]. In this special case the symmetry of the collective core is the SU(5) limit of SU(6) [34]. The resulting spectra of positive and negative parity are used to calculate the level-density-fluctuation measures [8,39–42]: the spectral rigidity Δ_3, the number variance Σ^2, and the nearest-neighbor-spacing distribution (NNSD) with the corresponding fit to the Brody function [39]. All of them give an intermediate distribution. The values obtained for the Brody parameter range from 0.116 for 5^+ to 0.486 for 0^+ and from 0.071 for 4^- to 0.822 for 0^-, for positive and negative parity states, respectively. To investigate the sensitivity of the level-density fluctuations to the interaction parameters, we chose the 2^+-level sequences (containing 282 levels). Typical results are plotted in Fig. 1. It shows the dependence of $\Delta_3(L)$ up to $L=50$ for the case where all the interaction strengths except Γ_ν of Eq. (10) are zero. Other parameters are the same as in the realistic calculation. Increased values of Γ_ν result in transition from the linear shape (steeper than $L/15$, because of the symmetry in the core [22]), with the typical saturation effect [5,8], to the logarithmic shape characteristic of GOE. Figures 2(a), 2(b), and 2(c) show comparison between the NNSD calculations, number variance, and spectral rigidity, respectively. For small Γ_ν, all three measures chosen here, the Brody parameter ω, number variance $\Sigma^2(1)$, and spectral rigidity $\Delta_3(10)$ [43], show a rapid transition from near-Poissonian to the intermediate values. At some values of Γ_ν discontinuities are noticed which can be traced to some special configuration of the strength parameters. This is observed at $\Gamma_\nu=0.627$ MeV, where the 2_2^+ and 2_3^+ states and nearly degenerate due to an avoided crossing, accompanied with the exchange of the character of the two states [44]. Figure 2(d) shows the variation of $\Delta_3(10)$ with the interaction parameter λ, chosen so that all the interaction parameters which differ from zero in the realistic calculation are varied proportionally and simultaneously. The transition from the over-Poissonian value to the intermediate region near $\lambda=0$ is even more rapid than in the case where Γ_ν was varied.

In conclusion, calculations of level-density fluctuation of the ^{106}Ag energy levels computed within the IBFFM show that (i) when changing the interaction parameters from zero to small values the level-density-fluctuation measures change rapidly from near-Poissonian values to the intermediate region between the Poisson and GOE statistics, indicating a sudden phase-transition-like change in the structure of the system; (ii) in the intermediate region they show irregularly oscillating behavior which indicates sensitivity to details of the nuclear structure. If the conjecture expressed in Refs. [4,5] is valid for the present case, then the results (i) and (ii) imply that changes in the interaction parameters induce first the rapid transition from regular to nearly chaotic behavior and then oscillations within the transitional region between regular and chaotic. Also, since the zero interaction strengths represent the case with unbroken dynamical symmetry, our results are in agreement with conclusions of Ref. [3] that breaking of dynamical symmetry is accompanied by the rapid transition from regular to chaotic regime, and that the degree of deviation from complete chaoticity is governed by variations within the set of interaction parameters, determining the complicated dynamics of the nuclear many-body problem.

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