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VECTOR MESONIC PHASE AND THE CHIRAL BAG MODEL

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The mesonic sector of the standard chiral bag model was enlarged to include the vector and axial vector components. New model openly displays the current field identities. It's predictions are close to the older model. This seems to be the consequence of the chiral invariance and of the PCAC and CVC constraints. Particle masses, the axial-vector coupling constant, the proton magnetic moment and the charge radius have been calculated.

1. Introduction

An approximate picture of QCD results, which is provided by the chiral bag model (CM)[1)], can be enriched by introduction of the $J = 1$ excitations in the mesonic phase[2)]. Such an operation has been already carried out[3)] by using a hedgehog ansatz. In this paper the solutions will be obtained by methods developed for the chiral bag model[1)] which neglects the non-linearities in the pion Lagrangian L_3. Such methods find the static pion "field" in terms of quark (anti-quark) operators showing explicitly that one is dealing with the pionic phase of the quark "matter".

The vector (axial-vector) static "fields" in this approach are obtained in quite analogous form. Their operator character is due to quark (antiquark) operators. One is again dealing with the vector (axial-vector) phase of the quark model. The terminology such as: pions, vector mesons, etc. will be used as the short code for various mesonic phases of our model. As one deals with a static model, our pions, vector mesons etc. are static "fields" which create (or annihilate) quark (anti-quark) pairs.

The vectors (axial vectors) in the mesonic sector of the model lead to vector (axial-vector) currents which have the current-field the current-field identity (CFI) form[4] outside bag. Inside bag, in the quark sector the currents are the usual bilinear combinations of quark spinors[5]. With this particular form of the vector meson dominance (VMD) one has in the same model combined physics of sixties with physics of the eighties[2]. At the same time such formalism openly displays underlaying quark structure of the fundamental theory i.e. of QCD.

Model is especially convenient for the calculation of weak processes where an effective electroweak Hamiltonian expressed in terms of quark fields has to be combined with QCD based strong renormalization.

Both the partial conservation of axial vector current (PCAC) and the conservation of vector current (CVC) play an important role in the development of the model. They appear as constraints which do fix the model parameters and select the physically acceptable solutions. It turns out that leads to the virtual elimination of axial vector a_1 phase through the PCAC imposed dynamical equality. It is well known that such feature appears in analogous models[2]. a_1 plays a minor role in NN interaction where its contribution is almost completely masked by the $\rho\pi$ continuum. a_1 is not a sharp resonance and its parameters are not uniquely established[6]. The vector phase gives contributions which show some similarity with the chiral-bag model results[1].

Alternatively one can say that CM contains, hidden in its pion phase, appropriate vectorial properties.

The VCM contains just one parameter more than CM model. This is the vector (axial vector) "field" coupling $\hat{g}$. Due to that and to the axial vector phase contribution to the hadron masses (7.1) the VCM parameters R (7.11) and ω (7.2) differ from the CM ones, as shown in Table 3. The theoretical results for the axial vector coupling g_A, for the proton magnetic moment μ and for the mean-square charge radius $\langle r^2 \rangle$ depend on these parameters. As shown in Tables 4, 5 and 6, this leads to model dependent predictions.

In both models, CM and VCM, the value of g_A is wrong by about 50%. Morever the theoretical values differ by only few percents from model to model. This is well known result[7], which has it's counterpart in the skyrmion phenomenology where g_A comes out to low by almost factor 2. In the CM and VCM picture it probably reflects inadequate description of the Goldstone boson character of pion.

In the description of electromagnetic form factors VCM, despite its open display of VMD, does not offer any striking differences with CM. This is due to PCAC and CVC constraints.

2. *VCM Lagrangians*

The usual CM Lagrangian density

$$L_{CM} = \mathrm{i}\bar{\psi}^{\mu}\gamma^{\mu}\partial_{\mu}\psi\Theta_{\nu} - \frac{1}{2}\bar{\psi}U\psi\delta_s + L_{\pi}(1-\Theta_{\nu}) = L_{\psi}\Theta_{\nu} + L_s\delta_s + L_{\pi}(1-\Theta_{\nu}) \quad (2.1)$$

is enlarged by chiral invariant inclusion of vector (axial vector) meson fields outside the bag. Inside the bag with radius R, which in (2.1) is symbolized by $\Theta_\nu = (R-r)$, the form of LCM is unchanged. Same goes for the surface coupling term with $\delta_s = \delta(R-r)$. However outside of the bag the SU(2) valued chiral (pion) field:

$$U = \exp(\mathrm{i}\vec{\tau}\vec{\pi}/f_n) \tag{2.2}$$

has covariant derivaties in the outside region:

$$D_\mu U = \partial_\mu U - \mathrm{i}gA_\mu^L U + \mathrm{i}gUA_\mu^R\,. \tag{2.3}$$

A_{PP} contains the outside mesonic phase analogous to the physical $\rho(\rho_\mu)$ and $A_\mu^{L(R)}$ fields

$$A_\mu^L = \frac{\rho_\mu + A_\mu}{\sqrt{2}}, \quad A_\mu^R = \frac{\rho_\mu - A_\mu}{\sqrt{2}}\,. \tag{2.4}$$

That leads to

$$L_\pi = \frac{f_\pi^2}{4}\,\mathrm{Tr}\,D_\mu U^+ D^\mu U\,. \tag{2.5}$$

The full VCM Lagrangian density contains also terms corresponding to ρ^μ and A^μ "fields"

$$L_V = \left[-\frac{1}{4}\vec{\rho}_{\mu\nu}\cdot\vec{\rho}^{\mu\nu} - \frac{1}{4}\vec{A}_{\mu\nu}\cdot\vec{A}^{\mu\nu} + \frac{m_0^2}{2}(\vec{\rho}_\mu\cdot\vec{\rho}^\mu + \vec{A}_\mu\cdot\vec{A}^\mu)\right]. \tag{2.6}$$

Here

$$\begin{aligned}\vec{\rho}_{\mu\nu} &= \partial_\mu\vec{\rho}_\nu - \partial_\nu\vec{\rho}_\mu - g(\vec{\rho}_\mu\times\vec{\rho}_\nu + \vec{A}_\mu\times\vec{A}_\nu)\,,\\ \vec{A}_{\mu\nu} &= \partial_\mu\vec{A}_\nu - \partial_\nu\vec{A}_\mu - \hat{g}(\vec{\rho}_\mu\times\vec{A}_\nu - \vec{\rho}_\nu\times\vec{A}_\mu)\,,\\ \rho_\mu &= \vec{\tau}\cdot\vec{\rho}_\mu/\sqrt{2};\quad A_\mu = \vec{\tau}\cdot\vec{A}_\mu/\sqrt{2}\,.\end{aligned} \tag{2.7}$$

The VCM Lagrangian

$$L_{VCM} = L_\psi\Theta_\nu + L_s\delta_s + (1-\Theta_\nu)[L_\pi + L_V] \tag{2.8}$$

is chiral gauge invariant only in its outside, $1-\Theta_\nu$ sector. Alternatively one might say that quark field φ inside the bag is not subject to the local gauge transformation. In that respect one might say that chiral gauge invariance does not hold for the complete L_{VCM} even when there is no mass term (i.e. $m_0 = 0$ in L_V (2.6)). With this term included one is using the massive Yang-Mils (MYM) scheme[3)] in which gauge invariance is broken. However MYM scheme is related, through the Stueckelberg transformations to the hidden local symmetry (HLS) scheme[3)]. In general the meson Lagrangians are derivable by bosonization techniques from a fermion theory[2,3,8)].

It should be mentioned that quark meson couplings are restricted to the pion-quark coupling L_s (2.1). Vector "fields" are coupled to quarks indirectly, via their coupling to "pions" in (2.5). In principle the interactions $\bar{\psi}\Gamma^\mu\psi V_\mu^{L,R}$ at the bag boundary can be constructed. However such surface coupling would require unusually dimensioned $[(\mathrm{mass})^{-1}]$ coupling constant. Furthermore they would lead to great, practically unsurmountable, difficulties when imposing CVC constraint.

In order to find static solutions for "fields" appearing in L_{VCM} (2.8) one has to make certain approximations. The U matrix operator (2.2) has to be expanded in leading orders off f_π^{-1}keeping only first few terms. After some manipulations this results in:

$$L = \mathrm{i}\bar{\Psi}\gamma_\mu\partial^\mu\Psi\Theta_\nu - \frac{1}{2}\Psi\exp(\mathrm{i}\frac{\vec{\tau}\cdot\vec{\pi}}{f_\pi}\gamma_5)\Psi\delta_s + (1-\widetilde{\Theta}_\nu)\Big[-\frac{1}{4}\vec{\rho}_{\mu\nu}\cdot\vec{\rho}^{\mu\nu} - \frac{1}{4}\vec{A}_{\mu\nu}\cdot\vec{A}^{\mu\nu}$$

$$+\frac{m_0^2}{2}(\vec{\rho}_\mu\cdot\vec{\rho}^\mu + \vec{A}_\mu\cdot\vec{A}^\mu) + \frac{1}{2}\partial_\mu\vec{\pi}\partial^\mu\vec{\pi} + \frac{1}{2}(\hat{g}f_\pi)^2\vec{A}_\mu\cdot\vec{A}^\mu - f_\pi g\hat{g}(\vec{A}_\mu\times\vec{\rho}^\mu)\vec{\pi}$$

$$-\hat{g}f_\pi\vec{A}^\mu\partial_\mu\vec{\pi} - g(\partial_\mu\vec{\pi}\times\vec{\pi})\vec{\rho}^\mu\Big]. \tag{2.9}$$

The standard variation procedure leads that to the boundary conditions

$$\mathrm{i}\gamma_\mu n^\mu\Psi = \left(1+\mathrm{i}\frac{\vec{\tau}\vec{\pi}}{f_\pi}\gamma_5\right)\Psi\,,$$

$$n_\mu\partial^\mu\vec{\pi} = \frac{\mathrm{i}}{f_\pi}\Psi\vec{T}\gamma_5\Psi + \hat{g}f_\pi n^\mu\vec{A}_\mu\,, \tag{2.10}$$

$$n^\mu\vec{A}_{\nu\mu} = 0\,, \qquad n^\mu\vec{\rho}_{\nu\mu} = 0.$$

Inside the bag the quark field is determined by

$$\mathrm{i}\gamma_\mu\partial^\mu\Psi = 0\,. \tag{2.11}$$

Outside the bag meson "fields" must satisfy following equations:

$$\Box\vec{\pi} = -f_\pi g\hat{g}\vec{A}_\mu\times\vec{\rho}^\mu - 2g\vec{\rho}^\mu\times\partial_\mu\vec{\pi}\,,$$

$$\partial^\mu\vec{A}_{\nu\mu} - (m_0^2 + (\hat{g}f_\pi)^2\vec{A}_\nu) - g\vec{A}^\mu\times\vec{\rho}_{\nu\mu} - \hat{g}\vec{\rho}^\mu\times\vec{A}_{\nu\mu}$$

$$-g\hat{g}f_\pi\vec{\pi}\times\rho_\nu + \hat{g}f_\pi\partial_\nu\vec{\pi} = 0\,, \tag{2.12a}$$

$$\partial^\mu \vec{\rho}_{\nu\mu} - m_0^2 \vec{\rho}_\nu - g\vec{\rho}^\mu \times \vec{\rho}_{\nu\mu} - \hat{g}\vec{A}^\mu \times \vec{A}_{\nu\mu} + g\hat{g}f_\pi \vec{\pi} \times \vec{A}_\nu + g\partial_\nu \vec{\pi} \times \vec{\pi} = 0 .$$

They are further simplified in order to find leading order results. One uses

$$(\Box + \mu_\pi^2)\vec{\pi} = 0 ,$$

$$\partial^\mu \vec{A}_{\nu\mu} + \hat{g}f_\pi \partial_\nu \vec{\pi} - (m_0^2 + f_\pi^2 \hat{g}^2)\vec{A}_\nu = 0 , \tag{2.12b}$$

$$\partial^\mu \vec{\rho}_{\nu\mu} - m_0^2 \vec{\rho}_\nu + g\partial_\nu \vec{\pi} \times \vec{\pi} + f_\pi g\hat{g}\vec{\pi} \times \vec{A}_\nu = 0 .$$

Here vector fields have linearised forms

$$\vec{\rho}_{\mu\nu} \to \partial_\mu \vec{\rho}_\nu - \partial_\nu \vec{\rho}_\mu , \qquad \vec{A}_{\mu\nu} \to \partial_\mu \vec{A}_\nu . - \partial_\nu \vec{A}_\mu \tag{2.12c}$$

Additional conditions for PCAC[7] and CVC will be discussed and defined below. They will fix all undetermined constants which appear in general solutions of equations (1.12).

3. *Currents and conservations*

The "fields" in the Lagrangian (2.9) transform under chiral group as

$$\delta\Psi = -\mathrm{i}T^i \omega^i \gamma_5 \Psi ,$$

$$\delta\vec{\pi} = \vec{\omega} f_\pi ,$$

$$\delta\vec{\rho}_\mu = \vec{\omega} \times \vec{A}_\mu , \tag{3.1}$$

$$\delta\vec{A}_\mu = \vec{\omega} \times \vec{\rho}_\mu + \frac{1}{\hat{g}}\partial_\mu \vec{\omega} ,$$

and

$$\delta\Psi = \mathrm{i}T^i \omega^i \Psi ,$$

$$\delta\vec{\pi} = -\vec{\omega} \times \vec{\pi} ,$$

$$\delta\vec{\rho}_\mu = \vec{\omega} \times \vec{\rho}_\mu + \frac{1}{g}\partial_\mu \vec{\omega} , \tag{3.2}$$

$$\delta\vec{A}_\mu = \vec{\omega} \times \vec{A}_\mu ,$$

respectively. The first set of infinitesimal transformations determines[9] the axial vector current

$$\vec{J}^A_\mu = \Psi\gamma_\mu\gamma_5\vec{T}\Psi\Theta_\nu + \frac{m_0^2}{\hat{g}}\vec{A}_\mu(1-\widetilde{\Theta}_\nu)\,, \tag{3.3}$$

while the second set leads to the vector current

$$\vec{J}^V_\mu = \Psi\gamma_\mu\vec{T}\Psi\Theta_\nu + \frac{m_0^2}{g}\vec{\rho}_\mu(1-\widetilde{\Theta}_\nu)\,. \tag{3.4}$$

The divergence of the axial-vector current has a redundant surface term, shown below in brackets

$$\partial^\mu\vec{J}^A_\mu = (\bar{\Psi}\gamma_\mu\gamma_5\vec{T}\Psi - \frac{m_0^2}{\hat{g}}\vec{A}_\mu)n^\mu\delta_s - \frac{m_0^2}{m_0^2+f_\pi^2\hat{g}^2}f_\pi\mu_\pi^2\vec{\pi}(1-\widetilde{\Theta}_\nu)\,. \tag{3.5}$$

The second term on r.h.s. of (3.5) has the form required by PCAC. It tells that PCAC is a long-range effect represented by the pionic phase $\vec{\pi}$, for $r > R$. The unwanted term leads to an additional constraint

$$\mathrm{i}\bar{\Psi}T^\alpha\gamma_5\Psi = \frac{m_0^2}{\hat{g}}n^\mu A^{\alpha\mu}\,, \tag{3.6}$$

which will actually help to determine the axial vector phase (or "field") as shown in the next section.

The conservation of the vector current

$$\partial^\mu\vec{J}^V_\mu = \left[\bar{\Psi}\gamma_\mu\vec{T}\Psi n^\mu - \frac{m_0^2}{g}\vec{\rho}_\mu n^\mu\right]\delta_s + \frac{m_0^2}{g}\partial^\mu\vec{\rho}_\mu(1-\widetilde{\Theta}_\nu) \tag{3.7}$$

produces two constraints for the vector phase:

$$\bar{\Psi}\gamma_\mu\vec{T}\Psi n^\mu|_{r=R} = \frac{m_0^2}{g}n^\mu\vec{\rho}_\mu|_{r=R} \tag{3.8}$$

and

$$\partial^\mu\vec{\rho}_\mu = 0, \quad r > R\,. \tag{3.9}$$

As shown in Section 5 these conditions lead to the unique determination of the vector phase. However it should be mentioned that (3.7), and thus (3.8) and (3.9), can be implemented only for the average value

$$\langle H|\partial^\mu J^V_\mu|H\rangle = 0\,. \tag{3.10}$$

Here $|H\rangle$ symbolizes some hadron state, as for example (see (6.7) below) proton state $|p\rangle$. Both forms (3.3) and (3.4) are the ones to be expected by CFI or VMD hypotheses[4]. It is natural that VMD holds outside the bag radius R, as this is the region in which, in our model, the mesonic phase represents an approximation to the full QCD dynamics. However, it will turn out that PCAC and CVC bring VCM results close to the old CM ones.

4. *Quark and mesonic phases*

The coupled equations (2.10)-(2.12) can be approximatively resolved by successive steps which are analogous to the procedure[1,5] which was used in CM. Thus the quark field Ψ and the pion "field" (or better pionic condensate) are found first. Their analytical forms are very similar to the CM ones.

For the quark field one uses

$$\Psi_f = \frac{N}{\sqrt{4\pi}} \sum_m \left\{ \begin{bmatrix} \epsilon_+ F(pr) \\ \epsilon_- \mathrm{i}\vec{\sigma}\vec{r}_0 G(pr) \end{bmatrix} \chi_m b_{mf} + \begin{bmatrix} \epsilon_- G(pr)\vec{\sigma}\vec{r}_0 \\ \epsilon_+ \mathrm{i} F(pr) \end{bmatrix} \chi_m d^+_{mf} \right\}. \qquad (4.1a)$$

Here $f = u,\ d$ denotes quark flavours, b annihilates particles and d^+ creates antiparticles. Inside the bag ($r < R$) functions F and G are spherical Bessel functions

$$F = j_0\,, \qquad G = j_1\,. \qquad (4.1b)$$

The pionic phase is determined by

$$\vec{\pi}^\alpha = k_0(\mu_\pi r)\chi^+_m \chi^-_m (d_{mf_1} b^-_{mf_2} + b^+_{mf_1} d^+_{mf_2}) D_0$$

$$+ k_1(\mu_\pi r)\chi^+_m \vec{\sigma}\vec{r}_0 \chi^-_m b^+_{mf_1} (b^-_{mf_2} + d_{mf_1} d^+_{mf_2}) D_1\,,$$

$$D_0 = \frac{N\widetilde{N} m_A^2}{4\pi f_\pi m_0^2} \frac{\exp(\mu_\pi R) R^2}{1+\mu_\pi R} \left[\widetilde{\epsilon}_+ \epsilon_+ \widetilde{j}_0(\omega) j_0(\omega) + \widetilde{\epsilon}_- \epsilon_- \widetilde{j}_1(\omega) j_1(\omega)\right], \qquad (4.2)$$

$$D_1 = \frac{N\widetilde{N} m_A^2}{8\pi f_\pi m_0^2} \frac{\exp(\mu_\pi R)\mu_\pi^2 R^3}{1+\mu_\pi R + \frac{1}{2}\mu_\pi^2 R^2} \left[\widetilde{\epsilon}_+ \epsilon_- \widetilde{j}_0(\omega) j_1(\omega) + \widetilde{\epsilon}_- \epsilon_+ \widetilde{j}_1(\omega) j_0(\omega)\right].$$

The functions k_0 and k_1 are described in Appendix. The quantities p and $\epsilon_\pm$, which appear in both (4.1) and (4.2), are defined by

$$\epsilon_\pm = 1\,, \qquad p = \frac{\omega}{R}\,. \qquad (4.3)$$

When one calculates some transitions (as for example $K \to 2\pi$ decays) the quarks which form mesonic phase belong either to the initial or to the final bag. The quantities with the wiggle: $\tilde{\epsilon}$, $\tilde{j}_0$, $\tilde{j}_1$, etc. refer to the final bag. The expression

(4.2) satisfies the boundary condition (2.10b), including the identity ($A_\mu = \partial_\mu \pi$), explained below.

The boundary condition (2.10a) determines the frequency ω (4.3). Its approximate form is

$$-F_f(R) + G_f(R) A_{H/f} = 0\,,$$

$$G_f(R) = \frac{F_f(R)}{\omega + m_f R}\left[1 - \frac{\sqrt{\omega^2 - (mR)^2}}{tg\sqrt{\omega^2 - (m_f R)^2}}\right],$$

$$A_{H/f} = 1 + \frac{1}{n}\rho_{H/f}\,, \qquad (4.4a)$$

$$\rho = \frac{\omega_0}{48\pi f_M^2 R^2}\,\frac{1}{\omega_0 - 1}\,\frac{1 + \mu R}{1 + \mu R + \frac{1}{2}\mu^2 R^2}\,\widetilde{\Sigma}_{H/f}\,,$$

$$\widetilde{\Sigma} = \sum_{ij} \langle H|\vec{\sigma}_i \vec{\sigma}_j \vec{\tau}_i \vec{\tau}_j|H\rangle\,.$$

In the expression (4.4) n is the number of valence quarks which determine the hadron state $|H\rangle$. Some useful values of the matrix element $\widetilde{\Sigma}$ are displayed in Table 1.

TABLE 1.

H	N	Δ	ρ	ω	π
$\widetilde{\Sigma}$	57	33	16	24	16

Matrix elements $\widetilde{\Sigma}$.

The substitution of (4.1 b) into (4.4) gives the transcendental equation for ω

$$\mathrm{tg}\,\sqrt{\omega^2 - (m_f R)^2} = \frac{\sqrt{\omega^2 - (m_f R)^2}}{1 + (\omega + m_f R)/A}\,, \qquad (4.4b)$$

The bag radius R is selected together with other model parameters by a varionational procedure described in Sect. 7 below.

The factors D_0 and D_1 which appear in (4.2) are obtained by combining the boundary conditions (2.10b) and (3.6). In the static limit this leads to

$$\frac{\mathrm{d}}{\mathrm{d}r}\pi^\alpha = \frac{m_0^2 + f_\pi^2 \hat{g}^2}{m_0^2}\,\frac{1}{f_\pi}\,\bar{\Psi} T^\alpha \gamma_5 \Psi\,. \qquad (4.5)$$

The right hand side of (4.5) is evaluated by using the quark field (4.1), thus determining D_0 and D_1.

5. *Axial-vector mesonic phase*

The mesonic phase of VCM is determined by the coupled equations (2.12b) and by the boundary conditions (2.10). Outside of the bag the axial vector "field" in particular has to satisfy the equation

$$\partial^\mu A^a_{\nu\mu} + \hat{g} f_\pi \partial_\nu \pi^a - (m_0^2 + f_\pi^2 \hat{g}^2) A^a_\nu = 0\,. \tag{5.1}$$

By taking the derivative of (4.1) (and ignoring the step function at the boundary) one obtains

$$\partial^\mu A^a_\mu = \frac{\hat{g} f_\pi}{m_0^2 + f_\pi^2 \hat{g}^2} \mu_\pi^2 \pi^a\,, \tag{5.2}$$

which agrees with (3.5) and (3.6). In the static case this goes into

$$(\nabla^2 - m_A^2)\vec{A}^a - \hat{g} f_\pi \left(1 - \frac{\mu\pi^2}{m_A^2}\right) \vec{\nabla}\pi^a = 0\,, \tag{5.3}$$

$$m_A^2 = m_0^2 + \hat{g}^2 f_\pi^2\,.$$

Pion "field" which contains two quark (antiquark) operators appears as the inhomogeneous term (5.3). A general solution of that equation is

$$\vec{A}^a = \vec{A}^a_H + \vec{A}^a_P\,. \tag{5.4}$$

Here the vector refers to the spatial spin $J = 1$ properties, while the index a corresponds to the SU(2) isospin group. The solution of the homogeneous equation (5.1) has a general form

$$\vec{A}^a_H(\vec{x}) = \hat{a}^a T^0_{01} k_1(m_A r) + b^a T^\nu_{01} \sigma^+_\nu k_0(m_A r) + \hat{c}^a T^\nu_{12} \sigma^+_\nu k_2(m_A r)\,. \tag{5.5}$$

The spherical tensors $T^M_{J\lambda}$ and the functions k_λ are defined in Appendix. The coefficients $\hat{a}^a$, b^a and $\hat{c}^a$ must contain quark (antiquark) operators in order to satisfy the equation (5.3) and the boundary conditions (2.10) and (3.6).

A particular solution of the inhomogeneous equation (5.3) is

$$\vec{A}^a_P = \hat{g} f_\pi \left(1 - \frac{\mu_\pi^2}{m_A^2}\right) \int\limits_{r>R} \mathrm{d}^3 x' G(\vec{x}, \vec{x}') \vec{\nabla}_{\vec{x}'} \pi^a(\vec{x} \tag{5.6}$$

The explicit form of the Green's function $G(\vec{x}, \vec{x}')$ is given in Appendix while the pionic phase π^a is determined by (4.2).

The axial-vector field (5.4) has to satisfy the conditions (2.10b, c), (3.5) and (3.6). The PCAC constrained, which is obtained as a combination of conditions

(3.5) and (3.6) puts a stringent condition on either $\vec{A}_H$ or $\vec{A}_P$ "fields". For a static axial-vector phase one must have

$$\vec{\nabla}_{\vec{x}'}\vec{A}^a(\vec{x}) = -\hat{g}f_\pi \frac{\mu_\pi^2}{m_A^2}\pi^a(\vec{x})\,. \tag{5.7}$$

After some calculation one finds

$$\vec{\nabla}_{\vec{x}}\vec{A}^a_H(\vec{x}) = \hat{a}^a m_A Y_0^0 k_0(m_A r) + \frac{m_A}{\sqrt{2\pi}}k_1(m_A r)\left[\hat{c}^a - \frac{\hat{b}}{\sqrt{2}}\right]\vec{\sigma}\vec{r}_0\,,$$

$$\vec{\nabla}_{\vec{x}}\vec{A}^a_H(\vec{x}) = -\hat{g}f_\pi\frac{\mu_\pi^2}{m_A^2}\pi^a(\vec{x})$$

$$+g f_\pi M_A \mu_\pi R^2 d_0 k_0(m_A r)\chi^+_m\chi_{\tilde{m}}\hat{A}_{m\tilde{m}}\Big[i_0(m_A R)k_1(\mu_\pi R)$$

$$+\frac{\mu_\pi}{m_A}i_1(m_A R)k_0(\mu_\pi R)\Big] \tag{5.8a}$$

$$+\hat{g}f_\pi m_A m_\pi r^2 k_1(m_A r)\chi^+_m\vec{\sigma}\vec{r}_0\chi_{\tilde{m}}\hat{B}^a_{m\tilde{m}}\frac{d_1}{3}\Big[i_1(m_A R)(k_0(\mu_\pi R)+2k_2(\mu_\pi R)$$

$$+\frac{\mu_\pi}{m_A}k_1(\mu_\pi R)(i_0(m_A R)+2i_2(m_A R))\Big]\,.$$

Introducing this into the relation (5.7) and using the results (4.2) one finds

$$\frac{\hat{a}^a}{\sqrt{4\pi}} = -\hat{g}f_\pi\mu_\pi R^2 D_0\hat{A}_{m\tilde{m}}\chi^+_m\chi_{\tilde{m}}\Big[i_0(m_A R)k_1(\mu_\pi R)+\frac{\mu_\pi}{m_A}i_1(m_A R)k_0(\mu_\pi R)\Big]\,,$$

$$\hat{c}^a - \frac{\hat{b}^a}{\sqrt{2}} = -\sqrt{2\pi}\hat{g}f_\pi\mu_\pi R^2\chi^+_m :: \chi_{\tilde{m}}\hat{B}m\tilde{m}\frac{d_1}{3}\Big[i_1(m_A R)(k_0(\mu_\pi R) \tag{5.8b}$$

$$+2k_2(\mu_\pi R)) + \frac{\mu_\pi}{m_a}k_1(\mu_\pi R)(i_0(m_A R)+2i_2(m_A R))\Big]\,.$$

Here

$$\hat{A}^a_{m\tilde{m}} = d_{mf_1}b_{\tilde{m}f_2} + b^+_m d_{\tilde{m}f_2}\,, \qquad \hat{B}_{m\tilde{m}} = b_m f_1^+ b_{\tilde{m}f_2} + d_{mf_1}d^+_{\tilde{m}f_2}\,. \tag{5.8c}$$

Instead of working with quantities $\hat{a}$, $\hat{b}$ and $\hat{c}$ it is simpler to use

$$\frac{\hat{a}^a}{\sqrt{4\pi}} = \frac{\hat{a}}{\sqrt{4\pi}}\hat{A}_{m\tilde{m}}\chi^+_m\chi^+_m\hat{c}^a - \frac{\hat{b}^a}{\sqrt{2}} = \left(c^a - \frac{b^a}{\sqrt{2}}\right)\chi^+_m :: \chi_{\tilde{m}}\hat{B}^a_{m\tilde{m}}\,. \tag{5.8d}$$

By invoking the condition (2.6) one can find the equality

$$\frac{NN}{4\pi}\left(\tilde{\epsilon}_+\epsilon_-\tilde{j}_0(\omega)j_1(\omega)+\tilde{\epsilon}_-\epsilon_+\tilde{j}_1(\omega)j_0(\omega)\right)\frac{\hat{g}}{m_0^2}$$

$$=\hat{g}f_\pi\left(1-\frac{\mu_\pi^2}{m_A^2}\right)m_Am_\pi\left[i_0(m_AR)I_0(R)\frac{D_1}{3}+\frac{2}{3}i_2i_2(m_Ar)I_2(R)d_1\right] \tag{5.9}$$

$$+\frac{b}{\sqrt{4\pi}}k_0(m_AR)-ck_2(m_AR)\frac{2}{\sqrt{8\pi}}\,.$$

Here

$$I_0(R)=\int\limits_R^\infty \mathrm{d}r\,r^2k_0(m_Ar)k_0(\mu_\pi R)=\frac{\exp(-(m_A+\mu_\pi)R)}{m_A\mu_\pi(m_A+\mu_\pi)}\,,$$

$$I_1(R)=\int\limits_R^\infty \mathrm{d}r\,r^2k_1(m_Ar)k_1(\mu_\pi R)\frac{\exp(-(m_A+\mu_\pi)R)}{m_A^2\mu_\pi^2R}+\frac{\exp(-(m_A+\mu_\pi)R)}{m_A\mu_\pi(m_A+\mu_\pi)}\,, \tag{5.10}$$

$$I_2(R)=\int\limits_R^\infty \mathrm{d}r\,r^2k_2(m_Ar)k_2(\mu_\pi R)\,.$$

Thus all coefficients a, b and c are completely determined by various boundary conditions.

By using (5.4) together with (5.5), (5.6), (5.9) and (5.10) one obtains the equality

$$\frac{m_0^2}{\hat{g}}A_\mu=f_\pi\partial_\mu\tilde{\pi}\,. \tag{5.11}$$

This is an operator equality, which is valid for any $r<R$. It simplifies the calculation of hadron internal energies (i.e. of hadron masses). The boundary conditions (2.10b) and (3.6) are then trivially compatible. They had obviously compelled the axial vector and pion mesonic phase to satisfy the relation (5.11).

6. Vector mesonic phase

The equation (2.12a) for the vector mesonic phase is considerably simplified by the identity (5.11). Its new form contains only one inhomogeneous term, i.e.

$$\Delta\vec{\rho}_i-m_0^2\vec{\rho}_i+g\frac{m_0^2}{m_A}\partial_i\vec{\pi}\times\vec{\pi}=0\,. \tag{6.1}$$

Here the constraint (3.9) has been already included. Furthermore the constraint (3.8) in combination with (2.10a) leads to

$$\vec{\pi} \times \frac{\mathrm{i}}{f_\pi} \bar{\Psi} \vec{T} \gamma_5 \Psi - \frac{m_o^2}{g} \eta^\mu \vec{\rho}_\mu = 0 | r = R \,. \tag{6.2a}$$

The solution of the homogeneous part of equation (6.1) has the general form

$$\vec{\rho}^{\,H,c}(x) = -\epsilon^{cab} \Big[I^{ab} T_{01}^{0} k_1(m_1 r) + J^{ab} \big[j T_{21}^{\mu+\nu} k_1(m_1 r) + k T_{23}^{\mu+\nu} k_3(m_0 r) \big] \Big] . \tag{6.3}$$

Here the tensors T_{JA}^{M} are defined in Appendix while I^{ab} and J^{ab} symbolise the required spin and isospin contents. The constraint (3.9) leads to

$$\vec{\nabla} \vec{\rho}^{\,H,c}(\vec{x}) = -\frac{1}{\sqrt{\pi}} \epsilon^{cab} I^{ab} m_0 k_0(m_0 r)$$

$$-m_0 \epsilon^{cab} J^{ab} k_2(m_0 r) Y_1^{\mu+\nu}(\Omega) \left[-\sqrt{\frac{2}{5}} j + \sqrt{\frac{3}{5}} k \right] . \tag{6.4}$$

A particular solution of the full inhomogeneous equation is

$$\vec{\nabla}_{\vec{x}} \rho^{P,c}(\vec{x} = -g \epsilon^{cab} \frac{m_o^2}{m_A^2} \int \mathrm{d}^3 y \vec{\nabla}_{\vec{x}} G(\vec{x}, \vec{y}) \pi^a(\vec{y}) \,. \tag{6.5}$$

Here

$$G(\vec{x}, \vec{x}') = G_0(\vec{x}, \vec{x}') + \gamma G_1(\vec{x}, \vec{x}') \,, \tag{6.6a}$$

with

$$G_0(\vec{x}, \vec{x}') = m \sum_{lm} Y_l^m(\Omega) Y_l^{m*}(\Omega') i_l(m r_<) k_l(m r_>) \,,$$

$$G_1(\vec{x}, \vec{x}') = m \sum_{lm} Y_l^m(\Omega) Y_l^{m*}(\Omega') \gamma_l i_l(m r_<) k_l(m r_>) \,,$$

$$\vec{\nabla}_{\vec{x}} G_0(\vec{x}, \vec{x}') = -\vec{\nabla}_{\vec{x}'} G_0(\vec{x}, \vec{x}') \,, \tag{6.6b}$$

$$\vec{\nabla}_{\vec{x}} G_1(\vec{x}, \vec{x}') = \vec{\nabla}_{\vec{x}'} G_1(\vec{x}, \vec{x}') \,.$$

The CVC constraint will be implemented as an expectation value between proton (neutron) states

$$\langle p | \nabla \vec{\rho}(\vec{x}) | p \rangle = 0 \,. \tag{6.7}$$

In the approximation used in this paper the nucleon states contain valence quarks only. This means that in the pionic phases π^a, which appear in (6.5), one has to keep only quark operators, i. e.

$$\pi = D_1 k_1(\mu_\pi r)(\vec{\sigma}\vec{r}_0)\frac{\vec{\tau}}{2} \times \text{operators}\,. \tag{6.8}$$

By using Gauss' integral theorem one obtains after some integration

$$\vec{\nabla}_{\vec{x}}\vec{\rho}_{P,c}(\vec{x}) = -g\epsilon^{cab}\frac{m_0^2}{m_0^2}R^2 m_0 D_1^2 \frac{\tau^a}{2}\frac{\tau^b}{2}k_1(\mu_\pi R)\mu_\pi \frac{\mathrm{d}}{\mathrm{d}x}k_1(x)_{x=\mu\pi R}$$

$$\frac{4}{\pi}(\sigma_m u^+)(\sigma_n u^+)\sum_{lm} Y_l^{m*}(\Omega)\left[i_l(m_0R)-\gamma k_l(m_0R)\right]k_l(m_0R)\int \mathrm{d}\Omega' Y_l^m(\Omega')Y_l^m u(\Omega')Y_l^\nu(\Omega'). \tag{6.9}$$

Additional relations determining constants appearing in (6.3), (6.5) and (6.6) follow from the relation (6.2). One finds:

$$\frac{m_0}{g}\vec{r}_0\vec{\rho}_{r=R}^{\,a} = \epsilon^{abc}\frac{m_0^2}{m_A^2}D_1^2\frac{\tau^b}{2}\frac{\tau^c}{2}\mu_\pi\Big[\frac{1}{3}(\sigma_m u^+)(\sigma_m u)$$

$$-\big[m_0^2 k_1(m_0R)R^2 k_1(\mu_\pi R)\frac{\mathrm{d}}{\mathrm{d}x}k_1(x)_{x=\mu\pi R}(i_0(m_0R)-\gamma k_0(m_0R)))$$

$$+\frac{m_0^2}{3}(i_1(m_0R)+\gamma k_1(m_0R))\int_R^\infty \mathrm{d}r^2 r^a k_1(m_0r')k_1(\mu_\pi(r')\big[k_0(\mu_\pi r')+2k_2(\mu_\pi r')\big]\Big]$$

$$+m_0^2 Y_2^{\mu+\nu}(\Omega)\sqrt{\frac{8\pi}{15}}\langle 1\mu 1\nu|2\mu+\nu\rangle\sigma_\mu^+\sigma_\nu^+\big[R^2\left(\sqrt{\frac{2}{5}}jk_1(m_0R)\right.$$

$$\left.-\sqrt{\frac{3}{5}}kk_3(m_0R)\right)k_1(\mu_\pi R)\frac{\mathrm{d}}{\mathrm{d}x}k_1(x)_{x=\mu_\pi R}+m_0(i_1(m_1R)$$

$$+\gamma k_1(m_0R))\int_R^\infty \mathrm{d}r'\,r'^2 k_1(m_0r')k_1(\mu_\pi r')(k_0(\mu_\pi r')+k_2(\mu_\pi r'))$$

$$+m_0(i_3(m_0R)+\gamma k_3(m_0R))\int_R^\infty \mathrm{d}r'\,r'^2 k_3(m_0r')k_2(\mu_\pi r')\Big]\,. \tag{6.10a}$$

and

$$\left[\vec{\pi}\times\frac{\mathrm{i}}{j_\pi}\bar{\Psi}\gamma_5\vec{T}\Psi\right]^l = \left[\vec{\pi}\times\frac{m_0^2}{m_A^2}n^\nu\partial_\mu\vec{\pi}\right]^l$$

$$= \frac{m_0^2}{m_A^2}\epsilon^{abc}D_1^2k_1(\mu_\pi R)\mu_\pi\frac{\mathrm{d}}{\mathrm{d}x}k_1(x)_{x=\mu_p iR}\frac{(\tau^b)}{2}\frac{(\tau^c)}{2} \tag{6.10b}$$

$$\left[\frac{(\sigma_\mu^+)(\sigma_\nu)}{3}+(\sigma_\mu^+)(\sigma_\nu^+)\sqrt{\frac{8\pi}{15}}\langle 1\mu 1\nu|2\mu+\nu\rangle Y_2^{\mu+\nu}\right].$$

In order to have both (6.2) and (6.7) satisfied, where (6.2) is interpreted as:

$$\langle p|\left(\pi\times\frac{\mathrm{i}}{f_\pi}\bar{\Psi}\vec{T}\gamma_5\Psi-\frac{m_0^2}{g}\eta^\mu\vec{\rho}_\mu\right)\Big|_{r=R}|p\rangle=0\,, \tag{6.2b}$$

one has to fulfill the following conditions:

$$I^{ab}=-\sqrt{4\pi}\frac{m_0^2}{m_A^2}gR^2D_1^2\frac{\tau^a}{2}\frac{\tau^b}{2}k_1(\mu_\pi R)\mu_\pi\frac{\mathrm{d}}{\mathrm{d}x}k_1(x)_{x=\mu_p iR}$$

$$\frac{(\sigma_\mu^4)(\sigma_\mu}{3}\big[i_0(m_0R)-\gamma k_0(m_0R)\big]\,,$$

$$J^{ab}=-gR^2\frac{m_0^2}{m_A^2}D_1^2\frac{\tau^a}{2}\frac{\tau^b}{2}k_1(\mu_\pi R)\mu_\pi\frac{\mathrm{d}}{\mathrm{d}x}k_1(x)_{x=\mu_p iR}\sqrt{\frac{6\pi}{15}}$$

$$\langle 1\mu 1\nu|2\mu+\nu\rangle(\sigma_\mu^+)(\sigma_\nu^+)\big[i_2(m_0R)-\gamma k_2(m_0R)\big]\,,$$

$$-\sqrt{\frac{2}{5}}j+\sqrt{\frac{3}{5}}k=1\,,$$

$$k_1(\mu_\pi R)\frac{\mathrm{d}}{\mathrm{d}x}k_1(x)_{x=\mu_p iR}=m_0^2k_i(m_0R)R^2k_1(\mu_\pi R)\frac{\mathrm{d}}{\mathrm{d}x}k_1(x)\big[i_0(m_0R)$$

$$-\gamma k_0(m_0R)\big]+\frac{m_0^2}{3}\big[i_1(m_0R)+\gamma k_1(m_0R)\big]\int\limits_R^\infty \mathrm{d}r'\,r'^2k_1(m_0r')k_1(\mu_\pi r')$$

$$\big[k_0(\mu_\pi r')+2k_2(\mu_\pi r')\big]\,,$$

$$k_1(\mu_\pi R)\frac{\mathrm{d}}{\mathrm{d}x}k_1(x)_{x=\mu_\pi R}=-R^2m_0^2\left[\frac{2}{5}jk_1(m_0R)-\sqrt{\frac{3}{5}}kk_3(m_0R)\right]$$

$$k_1(\mu_\pi R)\frac{\mathrm{d}}{\mathrm{d}x}k_1(x)_{x=\mu_\pi R} - m_0\big[i_1(m_0 R) + \gamma k(m_0 R)\big]$$

$$\int\limits_R^\infty \mathrm{d}r'\, r'^2 k_1(m_0 r')k_1(\mu_\pi r')\big[k_0(\mu_\pi r') + 2k_2(\mu_\pi r')\big]$$

$$-m_0\big[i_1(m_0 r') + \gamma k_3(m_0 r')\big]\int\limits_R^\infty \mathrm{d}r'\, r'^2 k_3(m_0 r')k_1(\mu_\pi r')k_2(\mu_\pi r')\,. \tag{6.11}$$

They determine I^{ab}, J^{ab}, j, k and γ so that the vector mesonic phase

$$\vec{\rho} = \vec{\rho}^{\,HC} + \vec{\rho}^{\,PC}\,, \tag{6.12}$$

is completely specified. It is important to note that operator $\vec{\rho}$ which describes vector mesonic phase contains quark creation operators. That means that the current-current term,

$$\langle H'|\vec{\rho}^{\,a}\cdot\vec{\rho}^{\,b}|H\rangle$$

which would appear in the calculation of the nonleptonic decays[5)], in the valence quark approximation vanishes.

7. *Hadron masses and model parameters*

A mass of a baryon (hadron) in VCM consists of the same contributions as in the CM model[5)]. It is given by

$$M(R) = E_V + E_Q + E_0 + E_M = \frac{4\pi}{3}BR^2 + \frac{N_0}{R}(a_0 + b_0\ln R)$$

$$+\frac{8a_c}{3R}a_{00}0.175 - \frac{Z_0}{R} + \frac{\omega_0^2\widetilde{\Sigma}}{192\pi f_\pi^2(\omega_0-1)}\frac{m_A^2}{m_0^2} \tag{7.1}$$

$$\left[\frac{1+\mu_\pi R}{\big(1+\mu_\pi R+\mu_\pi^2R^2/2\big)R^3} + \frac{\mu_\pi^2\hat{g}f_\pi^2}{2m_0^2}\,\frac{1+\mu_\pi R/2}{R\big(1+\mu_\pi R+\mu_\pi^2R^2/2\big)^2}\right].$$

Here, E_V is the bag volume term, E_Q is the quark phase energy, E_C corresponds to the effective colour gluon exchange, E_0 is the zero-point energy and E_M is the energy of the mesonic phase. All such terms are well known[1,5)]. Only the E_Q and E_M require some additional comments. The value of ω (4.3), which follows from the boundary condition (4.4), and which is a function of the bag radius R, can be fitted by the logarithmic expression:

$$\omega(R) = a_0 + b_0 \ln R\,. \tag{7.2}$$

The mesonic phase energy E_M contains the well known expression for the pionic phase energy

$$E_\pi = \hat{E}_\pi \frac{1+\mu_\pi R}{R^3\left(1+\mu_\pi R+\mu_\pi^2 R^2/2\right)} \qquad \hat{E}_\pi = \frac{\omega_0^2}{192 f_\pi(\omega_0-1)^2}\bar{\Sigma}\,. \tag{7.3}$$

Here it is assumed that in the leading approximation of E_π depends on the MIT-bag value $\omega_0 = 2.0428$. The same assumption was used to calculate the contribution E_V comming from the axial-vector mesonic phase.

The vector mesonic phase does not contribute, as the expression for E_M has to be understood as an average value

$$E_M \to \langle B|E_M|B\rangle\,. \tag{7.4}$$

The terms comming from vector phase contain (6.10) eight quark operators, so that they do not contribute to (7.1) in the valence quark approximation used for hadron states.

By taking into account (5.11), the expression for E_V can be simplified and explicitly evaluated. This result, together with (7.4), leads to the term E_M in (7.1).

The factor a_{00} in (7.1) depends on the hadron state. Some useful values are shown in Table 2.

TABLE 2.

H	N	Δ	ρ	ω	π
a_{00}	-3	3	2	2	-6

Parameter which specifies the gluon contribution to mass.

The hadron masses are fitted taking into account that the equilibrium of the system containing bag is obtained, i.e.:

$$\left.\frac{\partial M}{\partial R}\right|_{R=R_0} = 0\,. \tag{7.5}$$

In Table 3 four possible fits (Fit. 1-4) are listed, and compared with the CM model results and with experimental hadron masses. The m_N and m_Δ are more or less imputs. The ρ-meson mass can serve as a test of the fitting accuracy. In that case the largest discrepancy is 7.4%. Unfortunately one also obtains $(m_\omega/m_\rho)_{th} < 1$ while the experimental results indicate > 1. Needles to say, as in all models with bag and valence quarks, pion mass is much too large, as such models do not properly represent the Goldstone boson character of the pion.

First column (Chir) gives one possible CM fit. The found VCM fits are in columns 2-5 (Fit. 1-4). The last column lists hadron masses in GeV, while radii R_α are in GeV^{-1}.

In the following the model parameters from Table 3 will be used to make predictions for static and semi-static quantities such as the axial vector coupling constant g_A, the charge radius $\langle r \rangle$ and the proton magnetic moment μ_p.

TABLE 3.

	Chir	Fit. 1	Fit. 2	Fit. 3	Fit. 4	Exptl.
$B^{\frac{1}{4}}$	0.125	0.199	0.14	0.139	0.14	
Z_0	0.495	0.585	1.15	1.04	1.3	
a_c	0.4	0.767	0.495	0.48	0.505	
$\hat{g}$		4.23	2.9	2	4	
m_N	0.938	0.938	0.938	0.938	0.938	0.938
m_Δ		1.232	1.230	1.230	1.222	1.232
m_ρ			0.819	0.827	0.802	0.770
m_ω			0.809	0.813	0.797	0.783
R_N	5.79	5.730	5.210	5.193	5.293	
R_Δ		6.776	5.326	5.336	5.369	
R_ρ			4.844	4.874	4.854	
R_ω			4.771	4.778	4.813	

Hadron masses and fitting parameters.

8. *The axial vector coupling constant*

The constant g_A is defined by the matrix element of the axial vector current $\vec{J}^A_\mu$ (3.3) between nucleon states

$$g_A = \lim_{\vec{R}\to 0} \int \mathrm{d}^3 r \, \mathrm{e}^{\mathrm{i}\vec{k}\vec{r}} \langle p | \vec{J}^{A+}(r) | n \rangle \, . \tag{8.1}$$

One obtains:

$$g_A = \frac{5}{9} \frac{j_0^2(\omega) + j_1^2(\omega)}{j_0^2(\omega) + j_1^2(\omega) - 2 j_0(\omega) j_1(\omega)/\omega}$$

$$+\frac{5}{9}\frac{j_0(\omega)j_1(\omega)+j_1(\omega)j_0(\omega)}{j_0^2(\omega)+j_1^2(\omega)-2j_0(\omega)j_1(\omega)/\omega}\;\frac{1+\mu_\pi R/2}{2\big(1+\mu_\pi R+\mu_\pi^2R^2/2\big)^2}\,. \tag{8.2}$$

This is identical in the form with CM value, what is not surprising. It is due to the identity (5.11). However the expressions for $M(R)$ in VCM differ from CM ones so that one obtains different sets of parameters, as shown in Table 3.

The theoretical values of g_A are listed in Table 4. They are always about 50% larger than the experimental value $g_A = 1.25 \pm 0.001$. Comparison with the first column in Table 4 shows that VCM model does no better than the old CM model.

TABLE 4.

	Chir.	Fit. 1	Fit. 2	Fit. 3	Fit. 4
R	5.645	5.730	5.210	5.193	5.293
ω	1.4483	1.4633	1.3638	1.3602	1.3810
g_A	1.8628	1.8563	1.8950	1.8963	1.8890

Coupling constant g_A for various model parameters.

9. *The proton magnetic moment*

Here the definition

$$\mu = \langle p|\int \mathrm{d}^3r\frac{1}{2}\vec{r}\times(\vec{J}^{\nu(I=0)}+(\vec{J}^{\nu(I=1)}|p\rangle \tag{9.1}$$

gives in VCM a different theoretical expression than in CM one. In VCM one has for the isoscalar the same expression as in CM, i.e.

$$\vec{J}^{V(I=0)} = \frac{1}{2}\bar{\psi}\vec{\alpha}\psi\,. \tag{9.2}$$

However the isovector current is in VCM given by (3.4), while in CM one has

$$\vec{J}^{V(I=1)}_{CM} = \bar{\psi}\gamma_\mu T^i\psi\Theta_\nu + \epsilon^{ijk}\pi^j\bar{\nabla}_\mu\pi^k(1-\Theta)\,. \tag{9.3}$$

With (3.4) and (9.2) one obtains

$$\mu_p(VCM) = \mu_p(Q) + \mu_p(M)\,,$$

$$\mu_p(Q) = \frac{2}{3}\frac{R}{\omega^4}\frac{1}{j_0^2(\omega)+j_1^2(\omega)-2j_0(\omega)j_1(\omega)/\omega}$$

$$\left[\frac{3\omega}{4} - \frac{3}{4}\sin(\omega)\cos(\omega) - \frac{\omega}{2}\sin^2(\omega)\right] . \tag{9.4}$$

$$\mu_p(M) = 2m_p\langle p|(-)\int \mathrm{d}^3x\vec{r}\times m_0^2\int \mathrm{d}^3xG(\vec{x},\vec{x}')\vec{\pi}(\vec{x})\times\vec{\nabla}_{\vec{x}'}\vec{\pi}(\vec{x})\frac{m_0}{m_A^2}|p\rangle .$$

In CM with (9.2) and (9.3) one finds

$$\mu(CM) = \mu_p(Q) + \mu_p^{CM}(M) ,$$

$$\mu_p^{CM}(M) = \mu_p(Q) = \frac{2R}{3\omega^4}\frac{1}{j_0^2(\omega)+j_1^2(\omega)-2j_0(\omega)j_1(\omega)/\omega}$$

$$\left[\frac{3\omega}{4} - \frac{3}{4}\sin(\omega)\cos(\omega) - \frac{\omega}{2}\sin^2(\omega)\right] \tag{9.5}$$

$$+\frac{R}{12\pi}\left(\frac{\mu_\pi}{f_\pi}\right)^2\frac{1+0.5\mu_\pi R}{\left[1+\mu_\pi R+(\mu_\pi^2R^2/2\right]^2}$$

$$\frac{11}{3}\frac{1}{\mu_\pi^2R^2}\frac{j_0(\omega)j_1(\omega)}{\left[j_0^2(\omega)+j_1^2(\omega)-2j_0(\omega)j_1(\omega)/\omega\right]^2} .$$

Both expressions (9.4) and 9.5) are explicit functions of ω and R. They can be compared by performing calculations for some arbitrary parameters. One finds, for example:

$$R = 5.7295\ \mathrm{GeV}^{-1},\quad \omega = 1.4633,\quad \mu(VCM) = 3.03,\quad \mu(CM) = 3.11 . \tag{9.6}$$

Obviously both expressions lead to very similar numerical results, the difference being less than 3%.

However fitting parameters, which include R and determine ω, are model dependent. Thus using values displayed in Table 5 one obtains a spectrum of μ values. Some of them are quite close to the experimental value $\mu_p = 2.79$[2)].

TABLE 5.

	Chir.	Fit. 1	Fit. 2	Fit. 3	Fit. 4
R	5.645	5.730	5.210	5.193	5.293
ω	1.4483	1.4633	1.3638	1.3602	1.3810
μ_p	3.1058	3.1131	2.8168	2.7430	2.8297

Proton magnetic moment μ_p.

10. *The proton charge radius*

The charge density $\rho = J_0^V$ which appears in the formula

$$\langle r^2 \rangle = \int d^3r\, r^2 \rho(r) \tag{10.1}$$

contains only the quark phase contribution. The vector mesonic phase contribution to (3.4) has only spatial components (6.3) and (6.5). Thus, as was the case with g_A, CM and VCM lead to the same expression

$$\langle r^2 \rangle = \frac{R_0^2}{2\omega^5} \frac{1}{j_0^2(\omega) + j_1^2(\omega) - 2j_0(\omega)j_1(\omega)/\omega} \left[\frac{2\omega^3}{3} + \omega + \omega\cos(\omega) - \sin(2\omega)\right] . \tag{10.2}$$

According to that expression the theoretical proton charge radius depends on R and ω which depend (See Table 3) on the model and on the particular fitting parameters. The corresponding numerical results are sumarized in Table 6.

TABLE 6.

Chir.	Fit. 1	Fit. 2	Fit. 3	Fit. 4
0.839	0.851	0.777	0.774	0.778
2.5%	1.1%	9.7%	10%	8.4%

Proton charge radius $\langle r^2 \rangle$ in fm.

The last row gives deviations from the experimental value $\langle r^2 \rangle_{exp} = 0.86 \pm 0.01$ fm. It is encouraging that the deviations from the experimental findings are never larger than 10%.

11. *Overview*

Enlargement of CM which is described here does not lead to any surprises. However this should be qualified by the reminder that VCM has been treated only approximately. A highly nonlinear system of equations has been linearized and solved sequentially. Furthermore one should not forget that all unknown functions are operators. They are approximately described by the simplest quark configurations, which would appear if those general operators are expanded in pieces containing large and large numbers of quark and gluon operators. This is a standard perturbation calculation in the Heisenberg picture[10)].

The inclusion of vector and axial vector phases did not cure the major ill of CM. The axial vector coupling constant g_A remains also much too large in VCM. One ventures to guess that might be connected with approximate linearization of the theory, by which the chiral invariance is severely broken, even for massless VCM.

Space in VCM, and CM as well, is devided in two pieces: inside bag and outside. As vector (axial-vector) fields do not penetrate inside of the bag, there is no gauge invariance[8)] for $r > R$.

As already has been discussed PCAC and CVC condition force VCM results close to the old CM ones. Thus the explicitly apparent VMD for the region $r > R$ has mostly a formal character. This is explicitly, within the approximations used demonstrated for the axial-vector "field" (5.11). That phase can be replaced by the derivative of the meson phase thus making the theoretical expressions for g_A equal in both CM and VCM. As discussed above, this does not lead to equal numerical results. The theoretical expressions for hadron masses are not equal and this leads to unequal model parameters (i.e.: ω, R. etc.) values.

VCM can produce the predictions of μ_p and of $\langle r^2 \rangle$ which are within 10% of experimental values. Seemingly this is somewhat better than CM, but objectively speaking, no model version has a clear cut advantage. In VCM it seems impossible to reproduce both quantities equally well. Almost perfect theoretical prediction of μ_p leads to not so good $\langle r^2 \rangle$ and vice versa. This is always achieved with various VCM parameter sets which all do predict hadron masses with about the same accuracy.

One might say that VMD, formally expressed in formulae (3.3) and (3.4), is openly displayed in VCM and "hidden" in CM. Statement holds for linearised theories, solved in the leading order in the perturbation calculation. In that approximation both CM and VCM lead to almost equal prediction for static (masses, g_A) and semistatic (μ_p, $\langle r^2 \rangle$) quantities. All this makes sense in the context of the model structure in which vector "fields" are not directly coupled to quarks. They are driven by vector-"meson" interaction terms. One can hope that detailed and explicit calculation which is presented here sheds some light on the inner workings of others, similarly constructed, models[2,3)].

Appendix

In the evaluation of mesonic phase one has used spherical Bessel functions

$$
\begin{aligned}
j_n(x) &= \sqrt{\frac{\pi}{2x}}\, J_{n+1/2}(x)\,, \\
n_n(x) &= (-1)^{n+1}\sqrt{\frac{\pi}{2x}}\, J_{n-1/2}(x)\,, \\
h_n^1(x) &= j_n(x) + \mathrm{i} n_n(x)\,, \\
h_n^2(x) &= j_n(x) - \mathrm{i} n_n(x)\,,
\end{aligned}
\tag{A1}
$$

and modified Bessel functions

$$
i_n(x) = \mathrm{i}^{-n} j_n(\mathrm{i}x), \qquad k_n(x) = -\mathrm{i}^n h_n^1(\mathrm{i}x)\,. \tag{A2}
$$

The vector fields were expressed through irreducible tensors

$$
T_{L\lambda}^M = \sum_\mu \langle 1-\mu\ \lambda\ M+\mu | LM\rangle Y_\lambda^{M+\mu} \vec{e}_1^{\,\mu} \tag{A3}
$$

Here $\vec{e}_1^{\,\mu}$ is a spherical component of a unit vector.

Some useful relations for tensors (A3) are[11]:

$$\vec{\nabla}k_1(mr)T^M_{L\lambda} = -\sqrt{\frac{\lambda+1}{2\lambda+3}}\delta_{L,\lambda+1}Y^M_{\lambda+1}k_{\lambda+1}(mr)$$

$$+\sqrt{\frac{\lambda}{2\lambda-1}}\delta_{L,\lambda-1}Y^M_{\lambda-1}mk_{\lambda-1}(mr)$$

$$= -\left[\sqrt{\frac{\lambda+1}{2\lambda+1}}\delta L-1,\lambda - \sqrt{\frac{L}{2L+1}}\delta\lambda, L-1\right] mY^M_L k_L(mr)\,, \qquad (A4)$$

and

$$T^0_{01} = -\frac{\vec{r}_0}{\sqrt{4\pi}}\,,$$

$$\sum_\nu T^\nu_{12}\sigma^+_\nu = \frac{\vec{\sigma}}{\sqrt{4\pi}}\,, \qquad (A5)$$

$$\sum_\nu T^\nu_{12}\sigma^+_\nu = -\frac{3}{\sqrt{8\pi}}\left[\vec{r}_0(\vec{r}_0\vec{\sigma}) - \frac{\vec{\sigma}}{3}\right]\,.$$

A general form of a Green function is

$$G(\vec{x},\vec{x}') = m\sum_{lm} Y^m_l(\Theta,\Phi)Y^{m8}_l(\Theta',\Phi')\big[i_l(mr_<) + \gamma_1 k_1(mr_<)\big]k_l(mr_>)\,. \qquad (A6)$$

Here m, which multiplies the r.h.s., is the "mass" of a vector "field". Actually m^{-1} is a characteristic range for a given vector (axial-vector) phase.

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VEKTORSKA MEZONSKA FAZA I KIRALNI VREĆASTI MODEL

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Mezonski sektor u standardnom kiralnom vrećastom modelu je povećan uključivanjem vektorskih i aksialno-vektorskih komponenata. Novi model otvoreno pokazuje polje-struje identiteta. Njegova pretkazivanja su bliža starijem modelu. To je, čini se, posljedica kiralne nepromjenljivosti te PCAC i CVC uvjeta. Proračunati su: čestične mase, aksialnovektorska vezna konstanta, protonski magnetski moment i nabojni polumjer.