Role of collectivity in the structure of 120,122,124Sb nuclei

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Energy spectra, spectroscopic factors and electromagnetic moments of the low-lying states of $^{120-124}$Sb have been calculated in the framework of the interacting boson-fermion-fermion model, and compared with the experimental values. It was found that the collectivity has a significant effect on the energy splitting of the multiplets. The dynamical boson-fermion interaction leads to a smoothing of the energy splitting, while the exchange interaction tends to cause a fourth order polynomial splitting in the E^* versus $J(J+1)$ plot. The smoothing of the energy splitting can be interpreted in a quasiparticle picture as the lengthening of the range of the p-n effective interaction. The collective and single particle effects have comparable contributions in the electromagnetic moments, and the single particle part of the transfer operator determines the spectroscopic factors.

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I. INTRODUCTION

The nuclei in the single closed-shell region were traditionally treated as typical examples for the quasiparticle shell model. In the last few years the collective description of the odd-odd In nuclei [1, 2], as well as of lighter odd-odd Sb nuclei [3–5] proved to be successful, indicating the necessity of considering also the collective degree of freedom in the interpretation of the structure of Sb nuclei. It has been pointed out earlier also by Walters [6] that the various underlying core structures may affect different properties of proton-neutron multiplets.

In order to investigate the role of collectivity in the behavior of proton-neutron multiplets of the odd-odd Sb nuclei we calculated the energy spectra, electromagnetic moments, and spectroscopic factors of the low-lying states of $^{120-124}$Sb, which have been studied already with different reactions [7–17]. As a result of these works many low-energy levels are known in the $^{120-124}$Sb nuclei, and also the spins, parities, and spectroscopic factors have been determined for many states.

The earlier theoretical interpretations of the experimental level scheme of ^{122,124}Sb, given by Van Gunsteren and Rabenstein [12], Alexeev *et al.* [13, 17], and Artamonov and Isakov [18] were based on different quasiparticle models. The spectroscopic factors were calculated by Van Gunsteren for ^{122}Sb [12].

The present approach differs from the previous ones as in our case the proton-particle neutron-quasiparticle multiplets are coupled to the bosonic excitations of the core, and it is investigated how the collective excitations disturb the energies, electromagnetic moments, and particle transfer cross sections of $^{120-124}$Sb states, as well as how the core-(quasi)particle interactions can be simulated with an appropriate proton-neutron effective interaction.

II. INTERACTING BOSON-FERMION-FERMION MODEL DESCRIPTION

A. Hamiltonian

The Hamiltonian of the interacting boson-fermion-fermion (IBFFM) model [19] is

$$H_{\mathrm{IBFFM}} = H_{\mathrm{IBFM}}(\pi) + H_{\mathrm{IBFM}}(\nu) - H_{\mathrm{IBM}} + H_{\mathrm{eff}},$$

where $H_{\mathrm{IBFM}}(\pi)$ and $H_{\mathrm{IBFM}}(\nu)$ denote the IBFM Hamiltonians for the neighboring odd-even nuclei with an odd proton and odd neutron, respectively [21, 20]. H_{IBM} denotes the IBM Hamiltonian [22, 23] for the even-even core nucleus. H_{eff} denotes the effective proton-neutron interaction.

In this work the core Hamiltonian was approximated with its SU(5) limit, which is reasonable for spherical nuclei in this energy region. A spin dependent delta interaction with an additional spin polarization term was taken as the effective proton-neutron interaction:

$$H_{\mathrm{eff}} = V_0\delta(\mathbf{r}_\pi - \mathbf{r}_\nu)(1 + \alpha\sigma_\pi\sigma_\nu) + V_{\mathrm{sp}}[\sigma_\pi\sigma_\nu]_0.$$

The IBFFM Hamiltonian was diagonalized in the proton-neutron-boson basis:

$$|(j_\pi, j_\nu)J, n_d R; I\rangle,$$

where j_π and j_ν stand for the proton and neutron angular momenta coupled to J, n_d is the number of d bosons, R is their total angular momentum, and I is the spin of the state. The computer code IBFFM, used for the calculations, was written by Brant, Paar, and Vretenar [24].

B. Parameters

The single parameter of the core, the *d-boson energy* (ε_d) was taken as the energy of the 2_1^+ states of the neighboring even-even tin isotopes: 1.23, 1.14, and 1.13 MeV for $^{120-124}$Sb, respectively.

Since in the SU(5) limit we can use a reduced boson number, as a first approximation, we restricted the maximal number of the d bosons to 2, which largely simplified the calculations. The present calculations show, however, that the total contribution of the two d-boson components were weak in the wave functions of the states investigated, suggesting that there is no need for further d bosons. To check this statement we allowed for a third d boson, and found its effects negligible. So we kept the two-d-boson approximation.

The *model space* consisted of the $s_{1/2}$, $d_{3/2}$, $d_{5/2}$, $g_{7/2}$, and $h_{11/2}$ states both for the protons and neutrons. The single proton was assumed to be a particle, while the neutrons were taken as quasiparticles. The *occupation probabilities* for the neutrons were calculated in BCS approximation with $\Delta = 1.3$ MeV pairing gap. The single particle energies were calculated in modified harmonic oscillator potential with $\mu = 0.40$ and $\kappa = 0.066$, extrapolated from the values given by Maldeghem *et al.* [1]. The single particle energies obtained this way are consistent with the single particle energy systematics of Seo [25], and the V^2 values deduced are consistent with the experimental systematics. The occupation probabilities used in the IBFFM calculations are given in Table I.

The single proton and the quasineutron energies as well as the *strength parameters* of the nucleon-core interaction were fitted to the level energies and electromagnetic moments of the neighboring odd ^{119,121,123,125}Sb and ^{119,121,123}Sn isotopes by IBFM calculations. The dynamical and monopole proton interaction strengths were Γ_π=0.65 and A_π=0.1 MeV, respectively, and the exchange interaction strength was neglected, as the boson consists of neutron excitations. The neutron dynamical, monopole, and exchange interaction strengths were Γ_ν=0.6, A_ν=0.1 and Λ_ν=1.3 MeV, respectively.

TABLE I. Quasiparticle parameters of the IBFFM calculations.

Mass number		120	122	124
E_π (MeV)	$s_{1/2}$	1.20	1.60	2.00
	$d_{3/2}$	1.20	1.50	1.80
	$d_{5/2}$	0.06	0.12	0.30
	$g_{7/2}$	0.00	0.00	0.00
	$h_{11/2}$	1.40	1.50	1.60
E_ν (MeV)	$s_{1/2}$	0.00	0.00	0.06
	$d_{3/2}$	0.26	0.10	0.00
	$d_{5/2}$	1.20	1.40	1.40
	$g_{7/2}$	0.60	0.80	1.00
	$h_{11/2}$	0.44	0.28	0.04
V_ν^2	$s_{1/2}$	0.454	0.553	0.643
	$d_{3/2}$	0.354	0.439	0.538
	$d_{5/2}$	0.925	0.938	0.948
	$g_{7/2}$	0.865	0.893	0.913
	$h_{11/2}$	0.354	0.448	0.547

The energy difference of the $g_{7/2}$ and $d_{5/2}$ proton states were close to the values corresponding to the heavier odd Sb neighbor of the odd-odd nuclei investigated, in agreement with the fact that the nearest doubly closed-shell nucleus is the ^{132}Sn.

We had to renormalize slightly the relative energy of the $s_{1/2}$, $d_{3/2}$, and $h_{11/2}$ quasineutron states to get better agreement in the odd-odd nuclei. The new values obtained this way somewhat differ from the BCS expectations.

In this connection, it is needed to comment on the relation between the BCS approximation and the IBFFM calculations. The BCS approximation is designed for treating the pairing force employing the particle-hole channel: each quasiparticle state j is a linear combination of a hole state j^{-1} (with the probability amplitude U_j) and of a particle state j (with the probability amplitude V_j). Thus, strictly speaking, the BCS approximation is appropriate for the use in the quadrupole-phonon [22] representation of IBM, where the internal boson structure is expressed in the particle-hole channel. However, in the standard IBM representation of Arima and Iachello [23], the internal boson structure is expressed in the particle-particle channel. In this representation, therefore, the occupation probabilities and particle energies can differ from the values obtained by solving the BCS equations.

The proton single particle and neutron quasiparticle energies applied in the IBFFM calculations are summarized in Table I.

The short range *proton-neutron effective interaction* strengths $V_0 = -500$ MeV fm^3 and $\alpha = 0.15$ are deduced from the doubly closed-shell nuclei [26]. The radial matrix elements were calculated using harmonic oscillator wave functions with oscillator parameter $b = 2.29$ fm. The spin polarization interaction with a strength of $V_{\rm sp}$=0.06 MeV was used. The strength of the effective spin-spin interaction, which simulates the admixing of $M1$ giant resonance to the low-lying states, was estimated from the effective spin gyromagnetic ratios applying the relations given by Bohr and Mottelson [27].

The applied *effective charges and gyromagnetic ratios* were the commonly used, standard values in the mass region [28]: $e_p = 1.5e$, $e_n = 0.5e$, $e_{\rm vib} = 1.6e$, g_{lp} =1, g_{ln} =0, $g_{sp} = 0.6 g_{sp}^{\rm free}$, $g_{sn} = 0.5 g_{sn}^{\rm free}$, $g_R = Z/A$.

The standard form of the transfer operator, as given in IBFM [29], was applied in the IBFFM calculations with the additional $\Delta N_d = 2$ term defined in Ref. [30]. Its weight $\gamma = 0.2$ is a reasonable value for this kind of calculation.

C. Results

The *energy spectra* of ^{122,124}Sb calculated in the IBFFM are compared with the experimental data in Figs. 1 and 2, respectively. Since the IBFFM parameters for ^{120}Sb are nearly the same as in Ref. [5], we refer to that paper for the IBFFM versus experimental energy comparison. The rms deviation of the calculated levels is less than 100 keV. The trend of the change of the en-

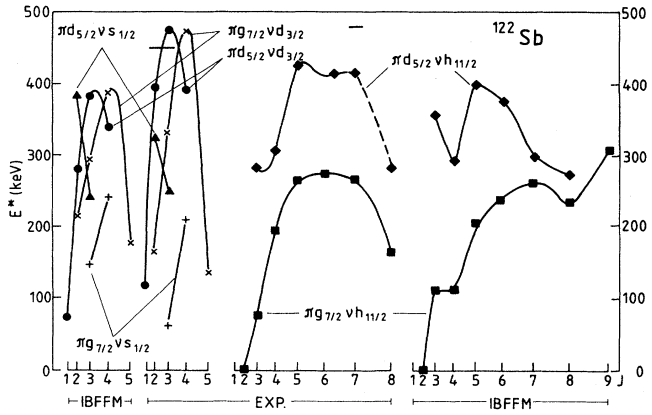

FIG. 1. IBFFM and experimental energy spectra of low-lying ^{122}Sb states. The abcissa is scaled according to $J(J+1)$, where J is the spin of the state. Levels belonging to the same p-n multiplets are connected. The horizontal lines indicate unassigned experimental states. The experimental data are taken from Ref. [13].

ergy of the multiplet members, which will be analyzed in the next section, is also well reproduced. The reasonable description of the energy correlation of the states within a p-n multiplet is important, since the parameters of the IBFFM determining the multiplet splitting were not fitted to the odd-odd nuclei. The connection of the experimental and theoretical states is based on the comparison of experimental and theoretical results on one nucleon transfer reactions and on electromagnetic moments, as well as on the analysis of the decay properties of the states [13, 17]. The assignment of the positive parity ^{124}Sb states is given in the next subsection.

The main components of the *wave functions* of the states for which the electromagnetic properties were calculated are given in Table II. Although the vector space used is quite large, many of the states investigated are based on one proton-neutron multiplet. The 2^+ and 3^+ states as well as some of the negative parity states of

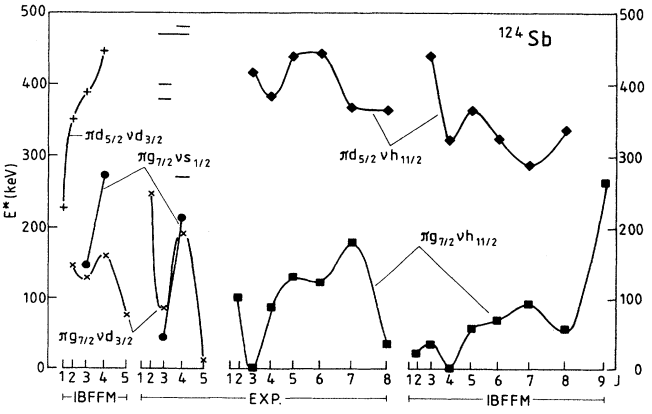

FIG. 2. Comparison of the calculated in IBFFM and experimental energy spectra of the low-lying ^{124}Sb states. The experimental data are taken from Ref. [17].

^{120}Sb are clearly mixed, while both in ^{122}Sb and ^{124}Sb the positive parity states are relatively pure. In the case of 7^- and 8^- ^{122}Sb states the mixing reaches 20%. These data are in a good agreement with the results deduced from the phenomenological analysis of the branching ratios [17]. Emigh *et al.* [9], from the number of states observed in their transfer reaction study, concluded that more states are mixed than we gave, but in our approximation due to the explicit treatment of collectivity a lot of higher-lying, mainly one-d-boson states contain also some single nucleon component, making them observable in transfer reactions.

For the pure states the strongest component in the wave function is the proton-neutron multiplet state with about 70% weight, the summed weight of the one-d-boson components is about 25%. Only 5% is left for the two-d-boson components, which justifies our two-boson approach.

The calculated and measured *spectroscopic factors* are compared in Tables III and IV for ^{120,122}Sb and the calculated values for ^{124}Sb are given in Table V. The spectroscopic factors not given are less than 0.03. In the case of the pickup reactions, the spectroscopic factors defined in Ref. [32] are not given in Tables III and IV, instead they are divided by $2(j_{\text{target}}+1)$ according to the convention

TABLE II. Wave functions of some low-lying states of $^{120-124}$Sb states. For the given J^π states the $|(j_\pi, j_\nu)I; NR\rangle$ wave function components and the corresponding amplitudes are given. Only amplitudes larger than 0.2 are shown.

Nucleus	E^*	J^π	Component	Amplitude
^{120}Sb	0 keV	1^+	$\|(5/2,3/2)1;00\rangle$	0.827
			$\|(5/2,3/2)2;12\rangle$	0.266
			$\|(5/2,1/2)1;00\rangle$	−0.225
	78 keV	3^+	$\|(5/2,1/2)3;00\rangle$	−0.791
			$\|(7/2,1/2)3;00\rangle$	0.349
			$\|(5/2,1/2)3;12\rangle$	0.315
		8^-	$\|(5/2,11/2)8;00\rangle$	0.823
			$\|(5/2,11/2)8;12\rangle$	−0.341
^{122}Sb	0 keV	2^-	$\|(7/2,11/2)2;00\rangle$	0.844
			$\|(7/2,11/2)4;12\rangle$	−0.349
			$\|(7/2,11/2)3;12\rangle$	−0.292
	61 keV	3^+	$\|(7/2,1/2)3;00\rangle$	−0.844
			$\|(7/2,1/2)3;12\rangle$	0.391
	137 keV	5^+	$\|(7/2,3/2)5;00\rangle$	−0.829
			$\|(7/2,3/2)5;12\rangle$	0.463
^{124}Sb	0 keV	3^-	$\|(7/2,11/2)3;00\rangle$	0.765
			$\|(7/2,11/2)5;12\rangle$	−0.444
			$\|(7/2,11/2)3;12\rangle$	0.335
	41 keV	3^+	$\|(7/2,1/2)3;00\rangle$	−0.808
			$\|(7/2,1/2)3;12\rangle$	0.365
			$\|(7/2,3/2)3;00\rangle$	−0.302
	125 keV	6^-	$\|(7/2,11/2)6;00\rangle$	−0.814
			$\|(7/2,11/2)4;12\rangle$	0.294
			$\|(7/2,11/2)8;12\rangle$	0.270
			$\|(7/2,11/2)7;12\rangle$	−0.217
			$\|(7/2,11/2)5;12\rangle$	0.214

TABLE III. Comparison of measured and calculated in IBFFM spectroscopic factors for the ^{121}Sb$(p,d)^{120}$Sb one-neutron transfer reaction. The first two columns contain the energy, spin, and parity of the final states involved in the reaction taken from Ref. [8].

Energy (keV)	J^π	ℓ_n	$S_{(p,d)}$ [a]	$S_{\rm IBFFM}$ [b]
0	1^+	2	0.11	0.15
78	3^+	0	0.19	0.12
149	3^+	0	0.05	0.03
166	3^-	5	0.63	0.44
193	2^+	0	0.11	0.08
		2	0.07	0.05
233	2^+	0	0.06	0.03
		2	0.12	0.05
334	$4^{(+)}$	2	0.10	0.19
343	$4^{(-)}$	5	0.54	0.39
387	$(3-5)$	5	0.44	0.61
390	$(2,3)^+$	2	0.06	0.07
438	(2)	0	0.01	0.02
		2	0.02	0.13

[a]Reference [9].
[b]$S/(2j_{\rm target}+1)$.

of the experimental papers [9, 13].

The calculated and measured spectroscopic factors agree in most cases within a factor of 3. The ^{120}Sb spectroscopic factors fit as well as the gamma branching ratios in Ref. [5], as the IBFFM parametrization is nearly the same.

In the case of ^{122}Sb the large difference between the calculated and measured spectroscopic factors for the 283 and 311 keV levels may have experimental origin, because the experimental spectroscopic factors are well above the sum rule limit. Even if we assume that the 283 keV peak is a doublet, containing also the 8^- member of the $\pi d_{5/2}\nu\tilde{h}_{11/2}$ multiplet predicted at about this energy, the experimental spectroscopic factor remains a bit large. The deviation of the calculated and measured spectroscopic factors for the 3^+_1 and 3^+_3 states suggest that the calculated wave function differs somewhat from the real one.

The spectroscopic factor calculations of Van Gunsteren and Rabenstein [12] for ^{122}Sb show much stronger mixing of the negative parity states than our calculations and the experiment. The spectroscopic factors of the positive parity states given in Ref. [12] are very different from the experimental values mainly due to a too large

TABLE IV. Comparison of measured and calculated in IBFFM spectroscopic factors for the one-neutron transfer reactions leading to ^{122}Sb. The first two columns contain the energy, spin, and parity of the final states involved in the reaction taken from Ref. [13].

Energy (keV)	J^π	ℓ_n	$S_{(p,d)}$ [a]	ℓ_n	$S_{(d,t)}$ [b]	$S_{\rm IBFFM}$ [d]	ℓ_n	$S_{(d,p)}$ [c]	$S_{\rm IBFFM}$
0	2^-	5	0.20		0.46	0.46			
61	3^+	0	0.23	(0)	0.45	0.13	(2)	0.05	0.00
78	3^-	5	0.46			0.52			
121	1^+						(2)	0.30	0.47
137	5^+	2	0.38	(2)	0.37	0.20			
164	8^-	5	0.50			0.53			
167	2^+	2	0.10			0.14	(2)	0.09	0.09
193	4^-	5	0.41			0.58			
210	4^+	0	0.30	(0)	0.34	0.11			
255	3^+						0	0.34	0.44
264	5^-					0.66			
265	7^-	5	0.62			0.57			
272	6^-					0.68			
283	3^-						(5)	2.40	0.40
311	4^-						(5)	0.80	0.43
323	2^+						0	0.30	0.40
334	3^+	2	0.07		0.09	0.22	2	0.16	0.01
394	4^+						2	0.45	0.46
397	2^+	2	0.03		0.10		2	0.45	0.38
414	6^-								0.55
$\approx$ 420	7^-	5	0.03			0.16	(5)	1.70	0.43
425	5^-								0.49
481	4^+	2	0.19	(2)	0.30	0.22			
484	3^+						2	0.51	0.60

[a]Reference [9].
[b]Reference [7].
[c]Reference [13].
[d]$S/(2j_{\rm target}+1)$.

TABLE V. Calculated in IBFFM spectroscopic factors for the ^{123}Sb$(d,p)^{124}$Sb one-neutron transfer reaction. The first two columns contain the energy, spin, and parity of the final states involved in the reaction taken from Ref. [17].

Energy (keV)	J^π	ℓ_n	$S_{\rm IBFFM}$
0	3^-	5	0.35
11	5^+	2	0.44
37	8^-	5	0.47
41	3^+	0	0.39
81	3^+	2	0.48
88	4^-	5	0.41
104	2^-	5	0.30
125	6^-	5	0.52
132	5^-	5	0.46
180	7^-	5	0.52
196	4^+	2	0.49
218	4^+	0	0.36
248	$2^+(3^+,4^+)$	2	0.38

configuration mixing.

In the case of ^{124}Sb the (d,p) strength is concentrated below 300 keV, and there is a general agreement between the calculated and experimental data.

The presence of collective excitations leads to the quenching of the quasiparticle strength, so it affects the spectroscopic factors through the structure of the wave functions. The collective part of the transfer operator had a different effect on different states and different orbital momentum transfers, but in general it lead to a 5–50 % change of the spectroscopic factors for the states with reasonable strength.

The results of the *electromagnetic moment* calculations and the corresponding experimental data are summarized in Table VI. The agreement between the experimental and theoretical magnetic moments is good for the positive parity states and acceptable for the negative parity ones. The electric quadrupole moments show a similar trend, but the quadrupole moments of the negative parity states are much smaller than the measured values. A closer agreement could be acheived if the $\nu h_{11/2}$ states were occupied to a larger extent, but the splitting of the $\pi d_{5/2}\nu h_{11/2}$ multiplet suggests just the opposite. Some admixture of the $11/2^-$ component from the $d_{5/2}\oplus 3^-$ octupole vibrational multiplet to the $h_{11/2}$ neutron state may also cause some deviation in the electromagnetic moments of the above states. The IBFFM calculations show that the contribution of the collective $M1$ operator to the magnetic moments is negligible, while in the quadrupole moments the off-diagonal collective contributions are dominant. The contribution of the proton single particles to the quadrupole momentum is about half of the collective component, whereas the contribution of the neutron quasiparticles is unimportant.

D. Assignments of ^{124}Sb states

The magnetic moment of the 41 keV 3_1^+ state of ^{124}Sb ($2.970\mu_N$) is closer to the value expected from the $\pi g_{7/2}\nu\tilde{s}_{1/2}$ multiplet ($3.01\mu_N$) than to the value obtained for the $\pi g_{7/2}\nu\tilde{d}_{3/2}$ multiplet ($2.26\mu_N$), so we identified the 41 keV 3_1^+ state of ^{124}Sb as a member of the $\pi g_{7/2}\nu\tilde{s}_{1/2}$ multiplet, and the 3_2^+ state at 81 keV as a member of the $\pi g_{7/2}\nu\tilde{d}_{3/2}$ multiplet.

The 196 keV 4^+ state decays strongly to the 41 keV 3^+ $\pi g_{7/2}\nu\tilde{s}_{1/2}$ state, indicating that these two states form the $\pi g_{7/2}\nu\tilde{s}_{1/2}$ doublet. The branching ratio of gamma rays from this state could be reproduced within a factor of 3 in the IBFFM calculations. The 215 keV 4^+ and 248 keV $2^+,(3^+,4^+)$ states decay by strong gamma transitions to the 81 keV 3^+ $\pi g_{7/2}\nu\tilde{d}_{3/2}$ multiplet state, suggesting that these states also belong to the same multiplet. The expected $M1$ transition from the 4^+ state to the 5^+ at 11 keV is only a factor of 2 weaker than expected from the IBFFM calculations.

Although these assignments contradict the assignment of Alexeev *et al.* [17], they remain in agreement with the results of the single nucleon transfer experiment [7] used as a basis of the assignment by Alexeev *et al.* [17]. The lowest-lying (d,p) peak (-20 keV in Ref. [7]) has ℓ_n=2+5 character in agreement with our assignment, the peak 55 keV higher with ℓ_n=0+2 is an *sd* mixture in our calculation, followed by a group of $\nu h_{11/2}$ levels, which may correspond to the peak lying 111 keV higher than the

TABLE VI. Magnetic dipole (μ in μ_N) and electric quadrupole (Q in e b) moments of some $^{120-124}$Sb states.

Nucleus	E^*	J^π	$\mu_{\rm expt}$ [a]	$\mu_{\rm IBFFM}$	$Q_{\rm expt}$ [a]	$Q_{\rm IBFFM}$
^{120}Sb	0 keV	1^+	$\pm$2.34(22)	+2.25		−0.10
	78 keV	3^+	+2.584(6)	+2.67	$\pm$0.41(4)	−0.47
		8^-	$\pm$2.34(4)	+2.45		−0.51
^{122}Sb	0 keV	2^-	−1.905(20)	−2.33	+0.85(11)	−0.08
	61 keV	3^+	+2.983(12)	+3.07	$\pm$0.41(4)	−0.48
	137 keV	5^+	+3.05(10)	+3.07		−0.62
^{124}Sb	0 keV	3^-	$\pm$1.20(2)	−1.23	+1.87(38)	+0.35
	41 keV	3^+	+2.970(33)	+3.01		−0.46
	125 keV	6^-	+0.384(12)	+0.36		+0.16

[a]The experimental data were taken from Ref. [31].

ground state, with $\ell_n = 5$. The peak at 190 keV relative to the ground state peak, taking into account the ≈ 40 keV resolution, may correspond to the averaged 196 and 215 keV levels, with correct $\ell_n = 0 + 2$ values.

III. ROLE OF COLLECTIVITY

A. Systematics of the multiplet splitting

The splitting of the $\pi d_{5/2}\nu\tilde{s}_{1/2}$ and $\pi g_{7/2}\nu\tilde{s}_{1/2}$ multiplets does not depend on the mass number (the occupation probability of the $\nu\tilde{s}_{1/2}$ state). This is because the splitting of the doublets is determined by the spin dependent part of the effective interaction, which is not dependent on the occupation probabilities, according to the quasiparticle models. For the two multiplets investigated, about half of the splitting is caused by the short range interaction, and the other half is caused by the spin polarization interaction. The explicit treatment of the collective quadrupole degree of freedom plays a negligible role in the splitting of the doublets.

The splitting of the $\pi d_{5/2}\nu\tilde{d}_{3/2}$ and $\pi g_{7/2}\nu\tilde{d}_{3/2}$ multiplets is determined by the dipole and quadrupole components of the effective interaction, so the E^* versus $J(J+1)$ plot of these multiplets has a parabolic shape for any residual interaction. The parabolic rule [33] predicts the vertex of the parabolas at $J = 3.0$ and $J = 3.9$, respectively, in agreement with the experiment for the case of ^{120,122}Sb. The parabolas of ^{124}Sb are strongly distorted, they are analyzed in Sec. III D.

Since we are in the SU(5) limit of the IBFFM, the contribution of the collective excitations to the multiplet splitting can be approximated in leading order with a quadrupole-quadrupole interaction:

$$V_{QQ} = 10\Gamma_\pi\Gamma_\nu\frac{N_d}{\varepsilon_d}Y_2(\pi)Y_2(\nu).$$

The splitting, caused by this interaction, is nearly the same as the splitting caused by the delta interaction for the $\pi d_{5/2}\nu\tilde{d}_{3/2}$ and $\pi g_{7/2}\nu\tilde{d}_{3/2}$ multiplets.

The $\pi d_{5/2}\nu\tilde{h}_{11/2}$ and $\pi g_{7/2}\nu\tilde{h}_{11/2}$ multiplets are much longer than the previous ones, so they are more sensitive to the details of the interactions. Most of the members of these multiplets are found experimentally, making possible the checking of different assumptions used in the description of odd-odd Sb nuclei. Van Gunsteren and Rabenstein [12] used a short range Schiffer interaction, Artamonov and Isakov [18] and Alexeev *et al.* [13,17] applied a long range Gaussian interaction, while the IBFFM corresponds in leading order to a delta plus quadrupole-quadrupole interaction in quasiparticle approximation. The pure short range interaction has a very strong staggering compared to the experiment, while the long range interaction of Alexeev *et al.* [17] produces as good a description as the IBFFM, since the staggering is smoothed out in both cases.

B. Smoothing of the multiplet splitting

In IBFFM some smoothing of the multiplet splitting in the E^* versus $J(J+1)$ plot is caused by the presence of the polarization interaction [34], which represents a smooth parabola in this plot. The simple summation of the polarization and the delta interactions, anyhow, cannot cancel the staggering feature of the short range interaction [35].

The reason of the additional smoothing is in the difference between the IBFFM and quasiparticle wave functions. While the IBFFM wave functions contain admixing from one-, two-, etc., d-boson components, in the quasiparticle model the multiplets are pure in this sense. Using a one-d-boson approximation the

$$|j_\pi j_\nu, I\rangle \longrightarrow a|j_\pi j_\nu, I00; I\rangle + \sum_{i=-2}^{2} b_i|j_\pi j_\nu, (I+i)12; I\rangle$$

substitution takes place. The b_i amplitudes vanish if the $|j_\pi - j_\nu| \le I + i \le j_\pi + j_\nu$ relation does not hold. In a similar way the matrix elements of the delta interaction are also replaced:

$$\langle j_\pi j_\nu, I|\delta|j_\pi j_\nu, I\rangle \longrightarrow a^2\langle j_\pi j_\nu, I00; I|\delta|j_\pi j_\nu, I00; I\rangle + \sum_{i=-2}^{2} b_i^2\langle j_\pi j_\nu, (I+i)12; I|\delta|j_\pi j_\nu, (I+i)12; I\rangle$$

$$= a^2\langle j_\pi j_\nu, I|\delta|j_\pi j_\nu, I\rangle + \sum_{i=2}^{2} b_i^2\langle j_\pi j_\nu, (I+i)|\delta|j_\pi j_\nu, (I+i)\rangle.$$

That is, instead of the simple matrix element, the energy of the multiplet state is a weighted average of the matrix elements of the neighboring multiplet states. The weights are determined by the dynamical interaction strength. In this way, the explicit treatment of the d-boson components leads to an averaging procedure, which is quite similar to a five-point smoothing method.

The effect of the smoothing is shown in Fig. 3. In the calculations $\Gamma_\nu = 0$ was applied in order to avoid the disturbance of the polarization interaction. It is seen that the multiplet gets smoother and smoother, and the large 1^+-2^+ distance, characteristic for the delta interaction, is gradually decreasing. It is to be mentioned that the splitting gets also weaker and weaker, while the average interaction strength remains practically the same.

C. Renormalization of p-n interaction

The smoothing procedure can also be analyzed by assuming the calculated IBFFM multiplets to be quasiparticle ones, and determining the multipole components of the corresponding effective interaction. The multipole coefficients of the interaction (α_k) were defined as [36]

$$\alpha_k = [(2k+1)(2j_\pi+1)(2j_\nu+1)]^{-1/2} \sum_{J=|j_\pi-j_\nu|}^{j_\pi+j_\nu} (-1)^{j_\pi+j_\nu+J}(2J+1)W(j_\pi j_\nu j_\pi j_\nu : Jk)E_J,$$

and the relative quadrupole coefficient (β_2) as

$$\beta_2 = \alpha_2/\alpha_0.$$

Here W is the standard Racah coefficient and E_J is the energy of the multiplet member with spin J. The zero point of the calculated energy scale is chosen so that the average energy of the multiplet should be equal to the monopole component of the delta interaction.

The calculated even rank relative multipole coefficients of the IBFFM are compared with the multipole coefficients of the delta, Schiffer [37], and Alexeev [17] interactions in Fig. 4.

The IBFFM values represent an average over several multiplets. All the multipole coefficients depend on the parameters of the model. The model parameters, used for the calculations shown in Fig. 4 were $\Gamma_\pi = 1.0$ MeV and $\Gamma_\nu = 0.8$ MeV, $V_\pi^2 = V_\nu^2 = 0$.

It is seen in Fig. 4 that the delta and Schiffer interactions have quite similar multipole structure, as both give a good description of the multiplet splitting in the doubly closed-shell regions. The IBFFM and the Alexeev interaction, describing the single closed-shell nuclei, give much larger quadrupole coefficients, while the higher rank coefficients are clearly smaller. This is the typical situation for the long range interactions.

The lengthening of the range of the effective interaction, which was artificially introduced in the quasiparticle model, could be reproduced by the use of a zero range effective interaction plus the explicit treatment of the collective quadrupole degree of freedom in IBFFM. According to the above result, in order to take into account the presence of the collective quadrupole excitations in quasiparticle models not only the quadrupole coefficient must be renormalized, but all of the multipole coefficients of the proton-neutron interaction, which can be done by lengthening the range of the effective interaction. For the case of the above boson-fermion dynamical interaction strengths, the 2 fm Gaussian interaction of Alexeev *et al.* [17] is a good estimation.

D. Deviation from the parabolic splitting

The correspondence between IBFFM and its quasiparticle approximation remains valid as long as only the dynamical interaction influences the splitting of the multiplets. The effect of the exchange interaction may cause some deviation from the above picture. The deviation is most significant when the neutron occupation probability (V_ν^2) is around 0.5. In this case in quasiparticle approximation the multiplet splitting is governed by the spin dependent interactions. As all the even multipole members of the effective interaction are blocked by the $U_\nu^2 - V_\nu^2$ quasiparticle factor, even the core polarization interaction has no effect.

In IBFFM, the proton-boson dynamical interaction acts without any distortion, and leads to a smoothing of the spin dependent part of the delta interaction in agreement with the lengthening of the range of the effective proton-neutron interaction. But in the IBFFM the neu-

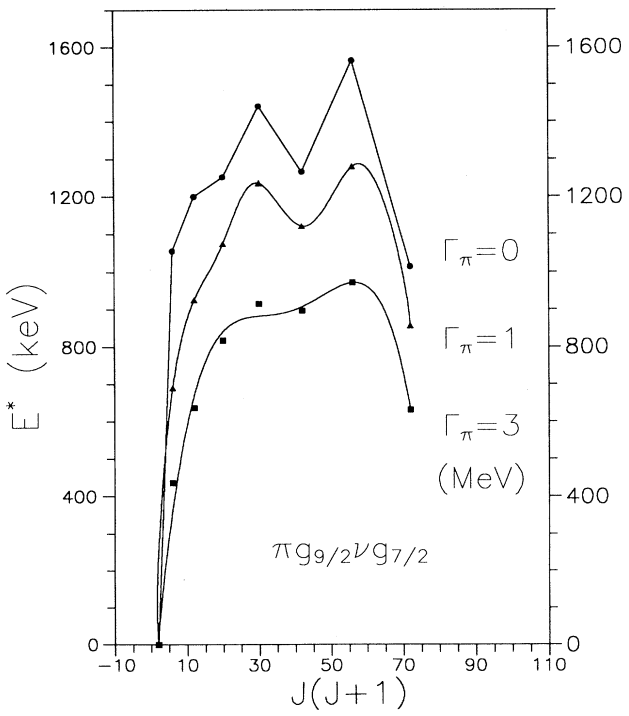

FIG. 3. Effect of the dynamical interaction on the splitting of the $\pi g_{9/2}\nu g_{7/2}$ multiplet.

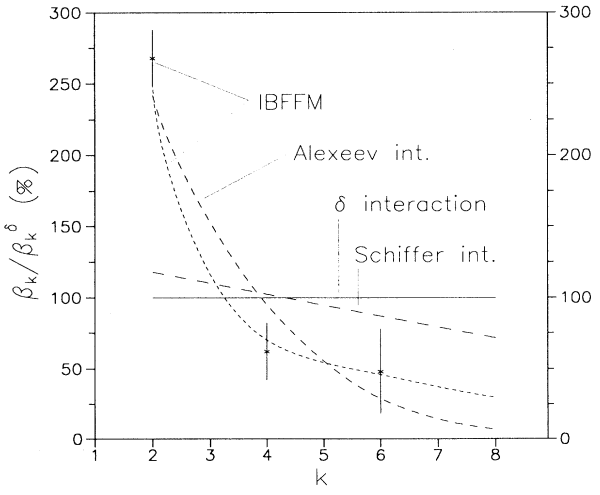

FIG. 4. Comparison of the even multipole coefficients of the delta, Schiffer [37], and Alexeev [17] effective proton-neutron interactions with the multipole coefficients of the interaction, simulating the IBFFM. The vertical bars indicate the uncertainty in the IBFFM multipole coefficients.

tron exchange interaction also acts. Although it works only for the d-boson states, the proton dynamical interaction also admixes d-boson components to the wave function of the proton-neutron multiplets, and makes effective the exchange interaction for multipletlike states, too [35]. The proton-hole-boson-dynamical and the neutron-quasiparticle-boson-exchange interactions together produce an additional splitting of the multiplets. The magnitude of this splitting, in a similar way to the polarization interaction, is proportional (in the weak coupling limit) to the product of boson-fermion interaction strengths Γ_π and Λ_ν.

The generated splitting of the multiplets depend both on j_π and j_ν, but not on the orbital angular momentum values. It is the same for particlelike or holelike proton states. For doublets no splitting is caused, the shape of quadruplets' splitting in the E^* versus $J(J+1)$ plot has an N-like character (see, e.g., the $\pi g_{7/2}\nu\tilde{d}_{3/2}$ multiplet of ^{124}Sb), while the longer multiplets split W-like, which is a fourth order polynomial in the plot. The fourth order nature of the splitting is in connection with the fact that the matrix element of the exchange interaction in IBFFM can be given as

$$\langle (j_\pi j_\nu)I,R;|H_{\rm exc}|(j_\pi j_\nu)I,R;J\rangle$$

$$=\sum_{\ell=0}^{4} C_\ell W(I,R,I,R;J,\ell) \approx \sum_{\ell=0}^{4} C_\ell^1 P_\ell(\cos\Theta),$$

where W is the Racah coefficient, J is the spin of the state, I is the summed angular momentum of the proton and neutron, and R is the spin of the d bosons. C_ℓ is a complicated expression, independent of J. Taking the classical limit of the Racah coefficients a sum of zeroth, second, and fourth order Legendre polynomials can be obtained, where Θ is the classical angle between I and R, and $\cos\Theta$ is

$$\cos\Theta = \frac{J(J+1)-I(I+1)-R(R+1)}{2\sqrt{I(I+1)R(R+1)}}$$

leading to a fourth order polynomial as a function of $J(J+1)$.

The splitting of some multiplets is shown in Fig. 5. The calculations were performed with $\Gamma_\pi = 1$ MeV, $\Gamma_\nu = 0$, $\Lambda_\nu = 2$ MeV, and $V_\nu^2 = 0.5$.

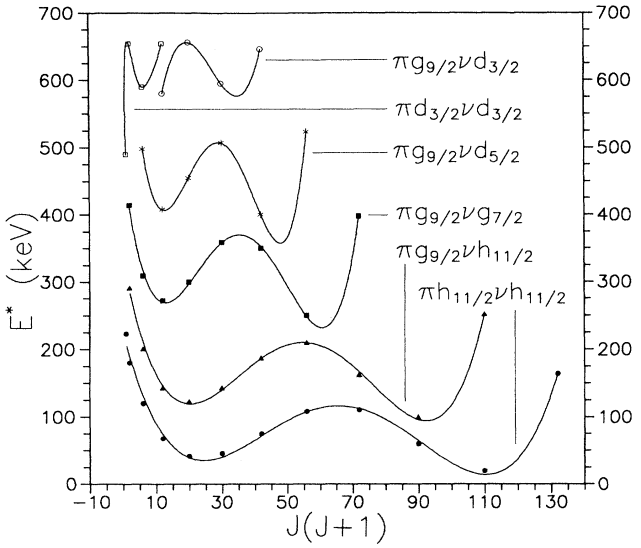

FIG. 5. Splitting of several half-filled multiplets due to the combined effect of the proton-boson dynamical and neutron-boson exchange interactions. The solid line connecting the points of the multiplets is a fourth order polynomial.

IV. CONCLUSIONS

In the framework of the IBFFM we were able to understand the general features and systematics of the energy splitting of proton-neutron multiplets of the $^{120-124}$Sb nuclei. Although the particle-core interactions are relatively weak in these nuclei, their contribution to the splitting of the multiplets is significant. The need for explicit treatment of the collectivity, especially in the case of nearly half-filled orbits, was pointed out. We have shown that in quasiparticle models one has to renormalize the effective interaction by lengthening its range to take in the collective degree of freedom.

The splitting of proton-neutron multiplets is very sensitive to the details of the interactions, so its investigation can be a useful tool for detecting phenomena manifesting as weak effects.

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