

# Rearrangement of valence neutrons by proton excitation in odd-odd Sb nuclei

---

**Dombradi, Zs.; Danko, I.; Brant, Slobodan; Paar, Vladimir**

*Source / Izvornik:* **Physical Review C - Nuclear Physics, 1996, 53, 1244 - 1248**

**Journal article, Published version**

**Rad u časopisu, Objavljena verzija rada (izdavačev PDF)**

<https://doi.org/10.1103/PhysRevC.53.1244>

*Permanent link / Trajna poveznica:* <https://urn.nsk.hr/urn:nbn:hr:217:706218>

*Rights / Prava:* [In copyright](#)/[Zaštićeno autorskim pravom.](#)

*Download date / Datum preuzimanja:* **2024-07-15**



*Repository / Repozitorij:*

[Repository of the Faculty of Science - University of Zagreb](#)



## Rearrangement of valence neutrons by proton excitation in odd-odd Sb nuclei

Zs. Dombrádi and I. Dankó

*Institute of Nuclear Research of the Hungarian Academy of Sciences, H-4001, Debrecen, P.O. Box 51, Hungary*

S. Brant and V. Paar

*Department of Physics, Faculty of Science, University of Zagreb, 41000 Zagreb, Croatia*

(Received 31 May 1995)

The distribution of valence neutrons of odd-odd  $^{120,122,124}\text{Sb}$  nuclei was investigated in the framework of the interacting boson-fermion-fermion model. It was found that the occupation probability of the  $\nu h_{11/2}$  orbit increased by  $0.084 \pm 0.037$  when exciting the odd proton from the  $d_{5/2}$  to the  $g_{7/2}$  state. The change was interpreted as a result of the proton-neutron interaction.

PACS number(s): 21.60.Fw, 21.30.Fe, 21.60.Cs, 27.60.+j

### I. INTRODUCTION

Adding several nucleons to a spherical nucleus, it gets deformed, as the sphericity-favoring pairing correlations between the pairs of protons and neutrons are gradually exceeded by spatial correlations between the proton-neutron pairs. From a shell model point of view, the isoscalar ( $T=0$ ) component of the proton-neutron effective interaction is responsible for the shape change [1] through mixing of the shell model states. The energy of the proton-neutron interaction reaches its maximum value when proton and neutron radial wave functions are strongly overlapping ( $N_p=N_n$  and  $\ell_p \approx \ell_n$ ), because of the short range of the  $p$ - $n$  interaction. Recent Hartree-Fock-Bogolyubov calculations [2,3] justify the role of the proton-neutron interaction and put emphasis on its quadrupole-quadrupole component. Federman and Pittel have proposed an impressive picture for interpretation of the onset of deformation [4]. In their approach, when, e.g., protons start to occupy an orbit, they pull neutrons into spatially strongly overlapping orbits, via the  $p$ - $n$  interaction. To reach the maximal overlap the neutrons move as close to the equatorial plane of the proton orbit as possible. This kind of polarization mechanism results in formation of a deformed shape.

According to the above mechanism, when adding protons to a spherical nucleus, the distribution of neutrons over different orbits must be changed. The neutrons should be moved into those orbits which have the largest spatial overlap with the filled proton orbits; thus, the occupation probabilities of the strongly overlapping orbits have to be increased, which is expected to be a measurable effect. Although the proposed mechanism for development of deformation is a very plastic one, no direct observation of a change of the occupation probabilities is known up to now.

To search for the Federman-Pittel effect, it is most straightforward to study those spherical nuclei which remain spherical even after adding one or two valence nucleons to it. This condition ensures that the concepts of the occupation probabilities as well as of the states themselves keep their original meaning. The singly closed shell nuclei have enough valence particles which can be polarized and remain spherical after adding a pair of nucleons to the closed shell; thus,

they seem to be optimal for such an investigation. We have chosen the Sn isotopes, having a long chain of isotopes, providing the possibility of systematic investigations.

Adding a single proton to the Sn nuclei can lead to the rearrangement of the valence neutrons, and in this way to a change of the occupation probabilities of the neutron orbits. The occupation probabilities can be measured by single-nucleon transfer reactions. The average reliability of the occupation probabilities deduced from a distorted-wave Born approximation (DWBA) analysis is  $\sim 20\%$  for states strongly excited in transfer reaction [5,6]. The precision can be further enhanced by taking into account the appropriate corrections to the DWBA method, and taking care in choosing the optical model parameters and radial form factors [5–7]. Analyzing a whole set of reactions, and interrelating the different spectroscopic factors, the uncertainties can be pushed down to  $\sim 5\%$ , as in the case of the  $f_{7/2}$  orbit in the Ca region [8].

Single-nucleon transfer reactions were performed both for Sn and Sb nuclei [9,10], but, unfortunately, they were not aimed at searching for a fine effect, but to get information on the structure of the nuclei investigated. In the case of odd Sb isotopes the final nuclei are odd-odd ones, and probably due to the high level density of these nuclei even at low energies, relatively large amounts of single-particle transfer strengths were missing from the sum rule limit even in high quality measurements, which resulted in a situation where practically all  $V^2$  values are smaller in Sb than in Sn. Adding two protons to the Sn nuclei, the even Te isotopes are obtained, for which experimental occupation probabilities are also available. For the overlapping region of nuclei with  $N=70$ –74, the measured occupation probabilities in Te [11] are quite close to those in Sn. Taking into account the scattering of the data and the systematic uncertainty of the deduction of occupation probabilities from the results of single-nucleon transfer reactions, it can be concluded that the differences in the occupation probabilities of the neutron states,  $\Delta V^2$ , are less than 0.15.

To push lower the limit on the change of neutron occupation probabilities because of adding a proton to the nucleus, more precision measurements of the spectroscopic factors are required, or an other quantity sensitive to occupation probabilities can be used.

## II. METHOD

It is known that the shape of splitting of the proton-neutron multiplets in the  $E^*$  versus  $J(J+1)$  plot is very sensitive to the occupation probabilities [12]. The matrix elements of the effective interaction are proportional to  $(U^2 - V^2)$  in the quasiparticle model, where  $U^2 = 1 - V^2$ . In a more realistic case, where the presence of the core nucleons is also considered, the shape of splitting of the multiplets changes from an open down parabolic form through a  $W$ -like fourth order splitting to an open up parabolic shape [13]. The splitting is very sensitive to the occupation probabilities in the  $V^2 = 0.2-0.8$  range.

From the experimental splittings of the multiplets the values of the occupation probabilities can be deduced by use of a particle-vibration coupling model. Applying the interacting boson-fermion-fermion formalism, which takes into account in a consequent way the most important particle-vibration and particle-particle interactions as well as the anharmonicities of the core, the splittings of the proton-neutron multiplets of In and Sb nuclei [13–15] could be successfully described.

The Hamiltonian of the interacting boson-fermion-fermion model (IBFFM) [16] is

$$H_{\text{IBFFM}} = H_{\text{IBFM}}(\pi) + H_{\text{IBFM}}(\nu) - H_{\text{IBM}} + H_{\text{eff}},$$

where  $H_{\text{IBFM}}(\pi)$  and  $H_{\text{IBFM}}(\nu)$  denote the IBFM hamiltonians for the neighboring odd-even nuclei with an odd proton and odd neutron, respectively [17].  $H_{\text{IBM}}$  denotes the IBM Hamiltonian [18] for the even-even core nucleus.  $H_{\text{eff}}$  denotes the effective proton-neutron interaction.

In this work, as in Refs. [13–15], the core Hamiltonian was simplified to its vibrational limit, all the particle-vibration interactions, namely, the dynamical interaction having a quadrupole-quadrupole nature, as well as the exchange and monopole interactions were kept both for protons and neutrons; furthermore, a spin-dependent delta interaction with an additional spin polarization term was taken as the effective proton-neutron interaction:

$$H_{\text{eff}} = V_0 \delta(\mathbf{r}_\pi - \mathbf{r}_\nu) (1 + \alpha \sigma_\pi \sigma_\nu).$$

The IBFFM Hamiltonian was diagonalized in the proton-neutron-boson basis:

$$|(j_\pi, j_\nu) J, n_d R; I\rangle,$$

where  $j_\pi$  and  $j_\nu$  stand for the proton and neutron angular momenta coupled to  $J$ ,  $n_d$  is the number of  $d$  bosons,  $R$  is their total angular momentum, and  $I$  is the spin of the state. The computer code IBFFM, used for the calculations, was written by Brant, Paar, and Vretenar [19].

In addition to the occupation probabilities, all the coupling strengths in the model, and even the core parameters, can affect the shape of the proton-neutron multiplet splitting. All of these parameters are bound to some physical region of possible values, but none of them can be determined as sharp quantities. Taking into account the uncertainty of these parameters, the occupation probability, which can be deduced from an IBFFM analysis, has much larger uncertainty than those determined from the spectroscopic factors. That is why

an IBFFM analysis cannot compete with the transfer reaction measurements in determination of the *absolute* occupation probabilities.

We can get information on the polarization effect performing a *relative* occupation probability determination by investigation of the change of neutron occupation probability, not by adding a proton to the system, but by changing the proton state in the *same* nucleus. In this case the correlations determining the neutron-core interactions remain the same, as the neutron remains in the same state. Similarly, the core structures, in terms of the phonon energy and the anharmonicities, are expected to be the same, as they do not depend on the state of the odd proton. The proton-core coupling will, nevertheless, change, but if the proton is a single particle, the proton core interaction can be well approximated with a quadrupole-quadrupole interaction, the state dependence of which is well understood [20]. The state dependence of the proton-neutron interaction can also be handled in a simple manner, approximating the effective interaction with a  $\delta$  force, which is accepted as a good approximation [12,21]. Although the absolute value of the occupation probability of the neutron state deduced from the multiplet splitting will even now be quite uncertain ( $\Delta V^2 \sim 0.15$ ), the difference in the occupation probabilities deduced for different proton states will already be stable against reasonable changes of the model parameters. In this way we can get rid of the uncertainties of the absolute method and reduce the error of the difference by a factor of 3–4 compared to the error of the difference between the  $V^2$  values determined from IBFFM. This may be enough to go beyond the limits set by the presently available transfer reaction data in this region.

## III. IBFFM ANALYSIS OF THE SPLITTINGS

For the investigation of the change of occupation probability of the  $h_{11/2}$  neutron state depending whether the proton is in  $d_{5/2}$  or  $g_{7/2}$  state the  $\pi d_{5/2} \nu \tilde{h}_{11/2}$  and  $\pi g_{7/2} \nu \tilde{h}_{11/2}$  multiplets of the  $^{124-120}\text{Sb}$  nuclei could be applied. The following states were accepted as the experimental basis of the comparison: The 103, 0, 88, 132, 125, 180, and 37 keV levels of  $^{124}\text{Sb}$  were assigned to the  $2^- - 8^-$  states of the  $\pi g_{7/2} \nu \tilde{h}_{11/2}$  multiplet and the 419, 384, 439, 446, 372, and 368 keV states to the  $3^- - 8^-$  states of the  $\pi d_{5/2} \nu \tilde{h}_{11/2}$  multiplet [22]. In  $^{122}\text{Sb}$  the 0, 78, 193, 264, 272, 265, and 164 keV levels were accepted as the  $2^- - 8^-$  members of the  $\pi g_{7/2} \nu \tilde{h}_{11/2}$  multiplet and the 283, 311, 425, 414,  $\sim 420$ , and 280 keV states as the  $3^- - 8^-$  members of the  $\pi d_{5/2} \nu \tilde{h}_{11/2}$  multiplet, respectively [23]. In  $^{120}\text{Sb}$  the 8, 260, 262, and 341 keV levels were taken as the  $2^- - 4^-$  members of the  $\pi g_{7/2} \nu \tilde{h}_{11/2}$  multiplet and the 166, 343, 387, and  $\sim 165$  keV states were assumed to be the  $3^-$ ,  $4^-$ ,  $5^-$ , and  $8^-$  members of the  $\pi d_{5/2} \nu \tilde{h}_{11/2}$  multiplet [14].

The wave functions of the above proton-neutron multiplet states were analyzed on the basis of spectroscopic factors and electromagnetic properties of the states using IBFFM [13,14]. It was found that most states are pure proton-neutron-boson multiplet states, but there is some mixing between the negative parity states. The mixing is strongest in the case of  $7^-$  and  $8^-$   $^{122}\text{Sb}$  states, where it reaches even 20%, which leads to a  $\sim 20$  keV change in the energy of these states.

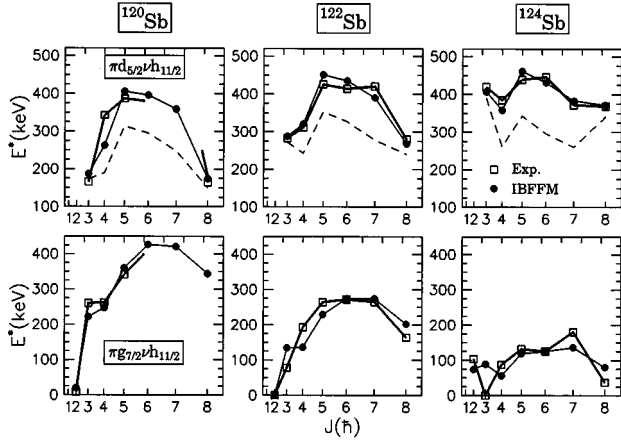


FIG. 1. Experimental ( $\square$ ) and calculated with the IBFFM ( $\bullet$ ) energy splittings of the  $\pi d_{5/2}\nu\tilde{h}_{11/2}$  and  $\pi g_{7/2}\nu\tilde{h}_{11/2}$  multiplets of  $^{120,122,124}\text{Sb}$ . In the case of  $\pi d_{5/2}\nu\tilde{h}_{11/2}$  multiplets also the theoretical splittings calculated with the assumption of best fits for the  $\pi g_{7/2}\nu\tilde{h}_{11/2}$  multiplet  $V^2$  values are shown as dashed lines. The abscissa is scaled according to  $J(J+1)$ , where  $J$  is the spin of the state.

In order to determine the occupation probability of the  $h_{11/2}$  neutron state we calculated the energy splitting of the  $\pi d_{5/2}\nu\tilde{h}_{11/2}$  and  $\pi g_{7/2}\nu\tilde{h}_{11/2}$  multiplets in IBFFM as a function of the occupation probabilities, and compared the calculated values to the experimental data.

The parameters of the model were as follows: The core was approximated with a harmonic vibrator with  $\hbar\omega=1.2$  MeV, the average energy of the  $2_1^+$  state in the neighboring even-even Sn nuclei. The standard values used earlier to describe the structure of  $^{114-124}\text{Sb}$  nuclei [13–15] were taken for the particle-vibration coupling strengths. The dynamical and monopole proton interaction strengths were  $\Gamma_\pi=0.65$  and  $A_\pi=0.1$  MeV, and the exchange interaction strength was neglected, as the bosons consist of neutron excitations. The neutron dynamical, monopole, and exchange interaction strengths were  $\Gamma_\nu=0.6$ ,  $A_\nu=0.1$ , and  $\Lambda_\nu=1.3$  MeV, respectively. The short range proton-neutron effective interaction strengths  $V_0=-500$  MeV  $\text{fm}^3$  and  $\alpha=0.15$  are characteristic for the doubly closed shell nuclei [24]. The radial matrix elements were calculated using harmonic oscillator wave functions with oscillator parameter  $b=2.27$  fm.

To allow for different neutron occupation probabilities in the case of different multiplets, only one neutron and one proton state was allowed for in the calculations; that is, we did not include the mixing of the multiplets in the present description. To consider the neglected mixings and correlations we assumed a 20 keV systematic uncertainty in the calculated energies.

#### IV. RESULTS AND DISCUSSION

The experimental energies of the members of the  $\pi d_{5/2}\nu\tilde{h}_{11/2}$  and  $\pi g_{7/2}\nu\tilde{h}_{11/2}$  multiplets are compared in Fig. 1 with the theoretical curves obtained as best fits. The IBFFM curves fit very closely to the experimental points. Their rms deviations are in the 20–50 keV range. For the sake of comparison, in the upper part of the figure also the

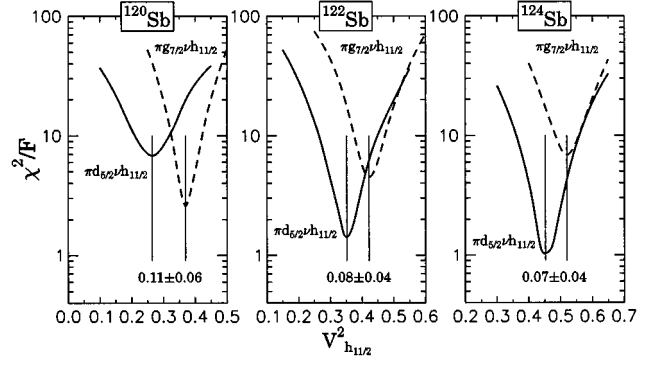


FIG. 2. Quality of the IBFFM description of the splittings of  $\pi d_{5/2}\nu\tilde{h}_{11/2}$  and  $\pi g_{7/2}\nu\tilde{h}_{11/2}$  multiplets of  $^{120,122,124}\text{Sb}$  as a function of the occupation probability of the  $\nu\tilde{h}_{11/2}$  state. The numbers at the bottom of figures show the differences of the occupation probabilities deduced at the best fits. The uncertainties of the differences were determined at  $\chi^2_{\text{min}}+1$ .

splittings of the  $\pi d_{5/2}\nu\tilde{h}_{11/2}$  multiplet calculated with the same  $h_{11/2}$  occupation probability as the  $\pi g_{7/2}\nu\tilde{h}_{11/2}$  multiplet are shown as dashed lines. It is interesting to see that these curves resemble much more closely the splitting of the same multiplet in the nucleus having two more neutrons.

The quality of the IBFFM descriptions was characterized by the  $\chi^2/F$  values. The change of  $\chi^2/F$  as a function of  $V^2$  is shown in Fig. 2. It is seen that the positions of the minima for the two multiplets are systematically at different positions, the minima of the  $\pi g_{7/2}\nu\tilde{h}_{11/2}$  multiplets are always at higher occupation probability, and even the values of the differences determined at the optimal occupation probabilities are more or less the same. The differences are less than 0.1, and have their significance levels between  $1\sigma$  and  $2\sigma$  values. Taking advantage of the systematic study and the averaging over the three nuclei the significance level could be pushed above the  $2\sigma$  limit. On average, the occupation probability of the neutron  $h_{11/2}$  orbit is  $0.084\pm 0.037$  larger, when the proton is in the  $g_{7/2}$  state compared to the occupation probability, obtained when the proton is in the  $d_{5/2}$  state. It suggests that the proton in the  $g_{7/2}$  state forces a pair of neutrons to spend  $\sim 35\%$  more time in the  $h_{11/2}$  orbit. Although this difference is very small, it corresponds to  $\sim 20\%$  of the occupation probability, and so it is expected to be visible in high precision transfer reaction experiments, too.

A similar analysis to the  $\nu\tilde{h}_{11/2}$  case could be performed in  $^{120}\text{Sb}$  for the  $d_{3/2}$  neutron state. In this nucleus all the members of the  $\pi d_{5/2}\nu\tilde{d}_{3/2}$  and  $\pi g_{7/2}\nu\tilde{d}_{3/2}$  multiplets are identified [14]. The  $\chi^2$  analysis of these multiplets shows that the occupation probability of the  $d_{3/2}$  state is larger by  $0.09\pm 0.05$  when the proton is in  $d_{5/2}$  state [25].

The above results indicate that there is some positive correlation between the increase of the occupation probability of certain neutron orbitals and the overlap of the proton and these neutron states, as was proposed by Federman and Pittel [4]. As in our analysis only the state of the single proton was changed, and the effect observed can be traced back to the state dependence of the  $p$ - $n$  interaction. Two different components of the  $p$ - $n$  interaction are relevant for the interpreta-

tion of the above effect: the monopole one, which shifts the positions of the single-particle orbits, and the quadrupole one, which is responsible for the polarization [26].

The monopole effect means that when a proton is added to a Sn nucleus it makes the neutrons more bound as a result of the attractive proton-neutron interaction. The single-particle energy, which includes the proton-neutron self-energy term, can be given [27] as

$$\bar{E}_n = E_n + V_{pn}^0,$$

where  $V_{pn}^0$  is the matrix element of the average (monopole) proton-neutron interaction. Depending on the radial behavior of the effective proton-neutron interaction, the different orbits can gain energy to different extents. A volume delta interaction produces the strongest state dependence, while a surface delta interaction, with no radial dependence, leads to the same energy shift for all valence orbits.

A changing of the state of the single proton leads to the change of the proton-neutron matrix element  $V_{pn}^0$ , and in this way to some change in the neutron single-particle energies. The magnitude of this monopole effect can be estimated using the same proton-neutron interaction as for the calculation of the splitting of the multiplets, and approximating again the single-particle wave functions with oscillator wave functions. Because of the larger radial overlap with the  $\pi g_{7/2}$  orbit, the single-particle energy of the  $h_{11/2}$  neutron state will be  $\sim 140$  keV lower, if the proton is in  $g_{7/2}$  state instead of being in the  $d_{5/2}$  one, while the  $d_{3/2}$  and  $s_{1/2}$  states gain 275 and 125 keV more energy, respectively, when the proton is in the  $d_{5/2}$  state.

The change in the single-particle energies has its effect on the occupation probabilities, too. In the case of  $^{120-124}\text{Sb}$  the

$h_{11/2}$  state lies close to the Fermi surface, that is, in that region where the occupation probability changes nearly linearly with the single-particle energy. The 140 keV change in the single-particle energy of the  $\nu h_{11/2}$  state corresponds to an increase of 0.041 in its occupation probability. The quasilinear regime grants that the uncertainty of the change of the occupation probability, because of using different initial energies from different systematics, remains small,  $\sim 0.004$ . The average  $p$ - $n$  interaction strength has also a  $\sim 10\%$  uncertainty; thus, all together the monopole effect caused a  $\Delta V^2 = 0.041 \pm 0.008$  increase of the occupation probability of the  $\nu h_{11/2}$  orbit. For the quadrupole effect,  $\Delta V^2 = 0.043 \pm 0.045$  remains. Its order of magnitude is in the range calculated by Federman and Pittel for the heavy Mo isotopes ( $\Delta V^2 < 0.05$ ) [4], but, unfortunately, it is completely covered by the uncertainty of the present analysis.

## V. CONCLUSIONS

In the present paper we proposed a novel method for the deduction of a change of occupation probabilities of one type of nucleon, because of a change of the state of the other type of nucleon. Using this method, it was possible to show that the occupation probability of the strongly overlapping orbitals slightly increased. This increase is just at the detection limit, but it shows some experimental evidence in relatively clean conditions for the polarization mechanism proposed by Federman and Pittel. A more significant effect is expected in a softer nucleus. To search for such a more impressive effect an investigation of  $^{124,126}\text{I}$  nuclei, which have two more protons than  $^{122,124}\text{Sb}$  of the present study, was started [28].

This work was supported in part by the Hungarian Foundation for Scientific Research, OTKA, Contract No. 3004.

- 
- [1] A. DeShalit and M. Goldhaber, Phys. Rev. **92**, 1211 (1953); I. Talmi, Rev. Mod. Phys. **34**, 704 (1962).
- [2] J. Dobaczewski, W. Nazarewicz, J. Skalski, and T. Werner, Phys. Rev. Lett. **60**, 2257 (1988); J. Dobaczewski, in *Contemporary Topics in Nuclear Structure Physics*, edited by R. F. Casten, A. Frank, M. Moshinsky, and S. Pittel (World Scientific, Singapore, 1988), p. 227.
- [3] S. Aberg, H. Flocard, and W. Nazarewicz, Annu. Rev. Nucl. Part. Sci. **40**, 439 (1990).
- [4] F. Federman and S. Pittel, Phys. Rev. C **20**, 820 (1978); E. D. Kirchuk, P. Federman, and S. Pittel, *ibid.* **47**, 567 (1993).
- [5] N. Austern, *Direct Nuclear Reaction Theories* (Wiley, New York, 1970), Chaps. 5 and 8.
- [6] P. E. Hodgson, *Nuclear Reactions and Nuclear Structure* (Clarendon, Oxford, 1971), Chap. 15.
- [7] G. R. Satchler, *Direct Nuclear Reactions* (Oxford University Press, Oxford, 1983), Chaps. 16 and 17.
- [8] C. F. Clement and S. M. Perez, Nucl. Phys. **A284**, 469 (1977).
- [9] E. J. Schneid, A. Parakash, and B. L. Cohen, Phys. Rev. **156**, 1316 (1967); P. L. Carlson and L. C. Melntire, Jr., Nucl. Phys. **A198**, 289 (1979), and references therein; D. G. Fleming, Can. J. Phys. **60**, 428 (1982).
- [10] R. A. Emigh, C. A. Fields, M. L. Gartner, L. E. Samuelson, and P. A. Smith, Z. Phys. A **308**, 165 (1982); S. A. Hjorth, Ark. Fys. **33**, 183 (1966).
- [11] R. K. Jolly, Phys. Rev. **136** B683 (1964); J. R. Lien, J. S. Vaagen, and A. Grane, Nucl. Phys. **A253**, 165 (1975), and references therein; M. A. G. Fernandez and M. N. Rao, J. Phys. G **3**, 1397 (1977).
- [12] J. P. Schiffer and W. W. True, Rev. Mod. Phys. **48**, 191 (1976).
- [13] Zs. Dombrádi, S. Brant, and V. Paar, Phys. Rev. C **47**, 1539 (1993).
- [14] T. Fényes and Zs. Dombrádi, Phys. Lett. B **275**, 7 (1992).
- [15] Z. Gácsi, Zs. Dombrádi, T. Fényes, S. Brant, and V. Paar, Phys. Rev. C **44**, 642 (1991); J. Gulyás, T. Fényes, M. F. F. M. Hassan, and Zs. Dombrádi, *ibid.* **46**, 1218 (1992); Zs. Dombrádi, T. Fényes, S. Brant, and V. Paar, in *Capture Gamma-Ray Spectroscopy*, edited by R. W. Hoff, AIP Conf. Proc. No. 238 (AIP, New York, 1991), p. 425, and references therein.
- [16] V. Paar, in *In-Beam Nuclear Spectroscopy*, edited by Zs. Dombrádi and T. Fényes (Akadémiai Kiadó, Budapest, 1984), p. 675.
- [17] F. Iachello and O. Scholten, Phys. Rev. Lett. **43**, 679 (1979).
- [18] A. Arima and F. Iachello, Phys. Rev. Lett. **35**, 1069 (1975).
- [19] S. Brant, V. Paar, and D. Vretenar, computer code IBFFM/OTQM, Institut für Kernphysik, KFA Jülich, 1985 (unpublished).

- [20] A. Bohr and B. R. Mottelson, *Nuclear Structure* (Benjamin, New York, 1975), Vol. 2, Chap. 6.
- [21] L. Zamick, in *Nuclear Structure and Nuclear Spectroscopy*, edited by H. P. Block and A. E. L. Dieperink (Free University, Amsterdam, 1974), p. 24.
- [22] V. L. Alexeev *et al.*, Nucl. Phys. **A345**, 93 (1980).
- [23] V. L. Alexeev *et al.*, Nucl. Phys. **A297**, 373 (1978).
- [24] Zs. Dombrádi, Zs. Podolyák, S. Brant, and V. Paar, Phys. Scr. T **56**, 239 (1995).
- [25] Zs. Dombrádi, T. Fényes, Z. Gácsi, J. Gulyás, S. Brant, and V. Paar, in *Perspectives for the Interacting Boson Model*, edited by R. F. Casten, A. Vitturi, A. B. Balantekin, B. R. Barrett, J. N. Ginocchio, G. Manio, and T. Otsuka (World Scientific, Singapore, 1994), p. 469.
- [26] K. Heyde, P. Van Isacker, R. F. Casten, and J. L. Wood, Phys. Lett. **155B**, 303 (1985).
- [27] J. N. L. Akkermans and K. Allart, Z. Phys. A **304**, 254 (1982); J. N. L. Akkermans, Ph.D. thesis, Vrije Universiteit, Amsterdam, 1981.
- [28] M. Fayez Hassan, J. Gulyás, I. Dankó, and Zs. Dombrádi, ATOMKI Annual Report 1994, Institute for Nuclear Research, Debrecen, 1995, p. 15.