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Generalized Bethe formula for the total level density

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It is pointed out that the generalized Bethe formula for the total level density encompasses the standard Bethe formula and the constant temperature formula as limiting cases. It is shown that the generalized Bethe formula provides an excellent fit to the results of combinatorial calculations in the region of doubly closed shell nucleus ^{208}Pb where the standard Bethe formula fails. The new parameter ξ associated with the generalized Bethe formula shows a systematic behavior with distance from double shell closure. [S0556-2813(97)50604-2]

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For many years experimental observations of nuclear level densities have been analyzed employing the level density formula of the basic type introduced by Bethe [1]. Bethe derived an analytical expression for the total level density in an equidistant model where single particle levels are equally spaced and nondegenerate. This model represents only a zeroth-order approximation of a Fermi gas [2,3]. Rosenzweig and Kahn have shown [4] that the effect of schematic periodic shell structure can be approximately accounted for by introducing an energy backshift in the Bethe formula. On this basis an approach was adopted, referred to as the backshifted Fermi gas model [5–7], where both the energy backshift E_1 and the level density parameter a in the Bethe formula are adjusted to experiment. The corresponding formula is of the form

$$\rho(E)=\frac{\exp[2\sqrt{a(E-E_1)}]}{12\sqrt{2}\sigma a^{1/4}(E-E_1)^{5/4}}, \tag{1}$$

where the spin cutoff parameter σ is given by [8]

$$\sigma^2=0.0888A^{2/3}\sqrt{a(E-E_1)}. \tag{2}$$

This level density formula is referred to as the standard Bethe formula.

On the other hand, the applicability of the level density forms different from the Bethe form was found in the nuclei near singly or doubly closed shells [9–12]. In particular, in these nuclei below an energy of about 10 MeV the total level densities have been better represented by the constant temperature formula [2,8]

$$\rho(E)=\frac{1}{T}\exp\left(\frac{E-E_0}{T}\right) \tag{3}$$

than by the Bethe formula. The appearance of the level density forms different from the Bethe formula was related to the existence of energy gaps between the major shells in single particle level scheme [9]. At higher energies, however, in these nuclei too the level densities exhibit a concave shape [9]. Thermodynamic calculations [13,14] based upon the realistic shell model single particle spectra have also led to the conclusion that the behavior of the total level densities near closed shells cannot be reproduced by Bethe formula.

In this paper, we point out that in the region of doubly closed shell nucleus ^{208}Pb the generalized Bethe formula gives an excellent fit to the total level densities obtained by combinatorial calculations for noninteracting particles in realistic single particle levels.

In his landmark paper [1], in addition to the investigation of the level density for noninteracting particles in an equidistant model, Bethe has discussed a more general assumption that the excitation energy depends on the temperature τ according to the power law

$$E=\frac{(a\tau)^n}{a}, \tag{4}$$

where a and n are parameters. Equation (4) does not depend on the particular model and represents the low-temperature behavior of any system in quite a general way. Here we apply this long-buried idea of Bethe. As was pointed out by Bethe, in the case of model of noninteracting particles the exponent in Eq. (4) is $n=2$, while in the case of particles with large interaction the exponent is $n=\frac{7}{3}$ at low and $n=4$ at high excitation energies [1]. It should be noted that in the case of noninteracting fermions, $n=2$ holds if we can expand energy as a function of temperature in a power series around $\tau=0$ MeV by expanding the function g (density of single particle states) around Fermi energy.

In evaluating the expression for the total level density, we employ the standard saddle point method as in Ref. [8] and keep the general low-temperature power law (4) as a solution of the saddle point equations. It means that we do not assume any particular model except the fact that the nucleus is composed of two kinds of particles, neutrons, and protons. As was already pointed out by Bethe, the Eq. (4) leads to the expression for entropy

$$S(E)=\frac{(aE)^\xi}{\xi}, \quad \xi=\frac{n-1}{n} \tag{5}$$

not only for the canonical, but for the grand canonical ensemble as well (the treatment of the nucleus that we employ here) provided the number of particles is constant. Using Eqs. (4) and (5), and the standard saddle point result for the total state density [1,8]

$$\omega(E)=\frac{\exp[S(N,Z,E)]}{\sqrt{(2\pi)^3\|D\|}}, \tag{6a}$$

$$\|D\|\simeq\tau^4\frac{dE}{d\tau}\frac{dN}{d\mu_n}\frac{dZ}{d\mu_p}\simeq\tau^4\frac{dE}{d\tau}\left(\frac{3a}{\pi^2}\right)^2 \tag{6b}$$

one obtains in a straightforward way a new formula for the total level density

$$\rho(E)=\frac{\sqrt{1-\xi}}{12}\cdot\frac{\exp\{[a(E-E_1)]^{\xi}/\xi\}}{\sigma\ \ (E-E_1)[a(E-E_1)]^{1-3\xi/2}}, \tag{7}$$

where the spin cutoff parameter σ is given by

$$\sigma^2=\frac{1}{2\pi}\left(\frac{\omega}{\rho}\right)^2\simeq 0.0888A^{2/3}[a(E-E_1)]^{1-\xi}. \tag{8}$$

Here, the energy backshift E_1 is introduced phenomenologically to account for shell effects and pairing. The backshift as a shell-effect was obtained analytically for the simple case of equidistant single particle level scheme in [4]. The spin cutoff parameter is used and derived in analogy to the standard Bethe formula [(1) and (2)] as in Ref. [8]. The expression (7) will be referred to as the generalized Bethe formula and applied to the model of noninteracting fermions.

Let us first discuss the dependence of Eq. (7) on the parameter ξ. For $\xi=0.5$ the generalized Bethe formula (7) reduces to the standard Bethe formula (1). In this limit the parameter a corresponds to the well-known level density parameter. On the other hand, in the limit $\xi\rightarrow 1$ the generalized Bethe formula asymptotes to the form of the constant temperature formula (3), but it is only the exponential part of the constant temperature expression that is exactly recovered. For the intermediate values $0.5<\xi<1$ the generalized Bethe formula describes a transitional pattern between the standard Bethe formula (1) and the constant temperature formula (3). The maximum value of $n=4$ discussed by Bethe corresponds to such an intermediate case. On the other hand, for $\xi<0.5$ the generalized Bethe formula describes a pattern which is concave down more than given by the standard Bethe formula (1), i.e., the total level density exhibits a slower increase with energy (a ''sub-Bethe'' form); such a pattern can be approximately simulated with the standard Bethe formula by using an energy dependent level density parameter a which decreases with increasing energy [15–24].

Let us investigate the fit of Eq. (7) to the total level densities obtained by combinatorial calculations for the nuclei in the region near the doubly closed shell nucleus ^{208}Pb, using realistic single particle levels. We treat the parameters a, the energy backshift E_1, and the new parameter ξ in Eq. (7) as free parameters which are fitted to the total level densities obtained by combinatorial calculations.

The combinatorial calculations are performed using the Gaussian polynomial generating function method (GPM) [25,26]. Single particle levels are taken from Ref. [27]. In Fig. 1 we present, as an illustration, the combinatorial total level densities for noninteracting particles for ^{208}Pb, ^{198}Au, and ^{190}Pt (closed circles). Both the generalized Bethe formula (7) and the standard Bethe formula (1) are fitted to the

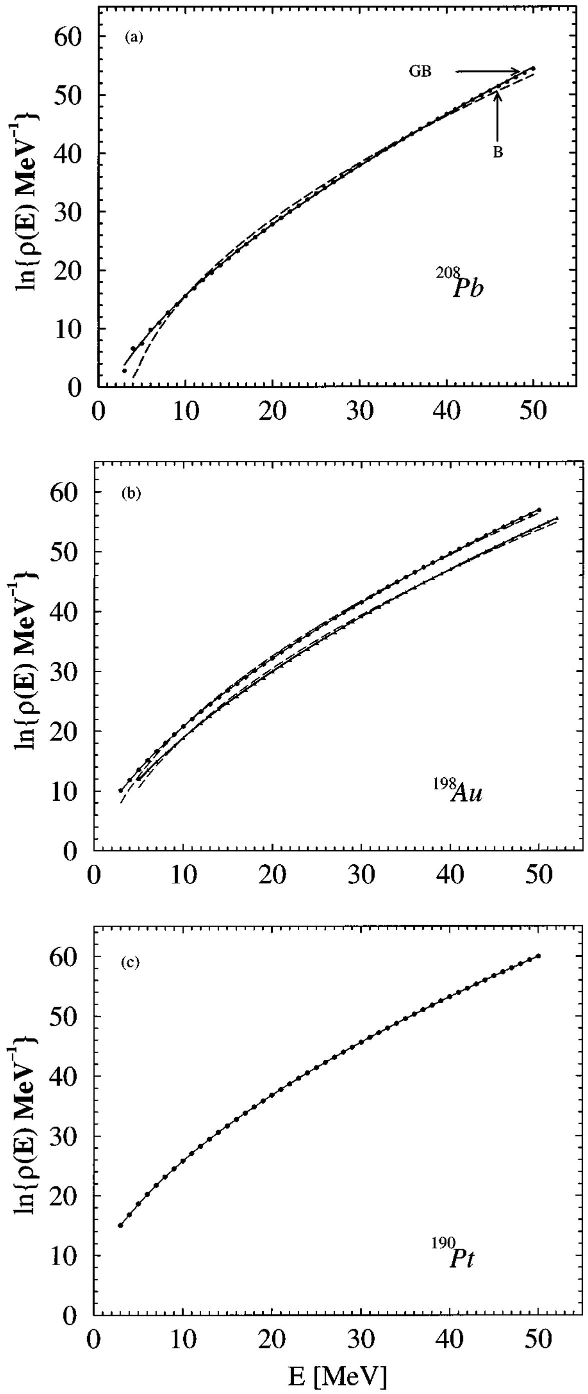

FIG. 1. Fit of the generalized Bethe formula (7) to the combinatorial total level densities for (a) ^{208}Pb, (b) ^{198}Au, and (c) ^{190}Pt in comparison to the fit of the standard Bethe formula (1). Results of the GPM combinatorial calculations are presented for noninteracting particles in the realistic single particle levels [27] (closed circles) and for the inclusion of pairing (closed triangles). The fit of the generalized Bethe formula to the GPM results is shown by solid lines (label GB) and the fit of the standard Bethe formula by dashed lines (label B). The GPM level densities below 5 MeV are shown, but they are not included in the fitting.

combinatorial level densities (solid and dashed lines, respectively). The parameters ξ, a, E_1, and the variance s^2 of the best fit for several nuclei in this region are listed in Table I. It is seen that near the doubly closed shell nucleus ^{208}Pb the

TABLE I. ξ, a, E_1, and variance s^2 at the best fit of the generalized Bethe formula (7) (GB) to the GPM combinatorial results for some nuclei in the ^{208}Pb region in comparison to the best fit of the standard Bethe formula (1) (B). The value $\xi=0.5$ corresponds to the standard Bethe formula (1). The combinatorial level densities were calculated exactly using the GPM method [25,26]. The normalization of variance s^2 for fit of the generalized Bethe formula (7) to the combinatorial level density is given by $1/(N-n_p)\,\Sigma_{i=1}^{N}[y_i-f(x_i)]^2$, where y_i is equal to $\ln(\rho_{\rm GPM})$, N is the number of points, and n_p is the number of parameters.

Nucleus	Formula	ξ	a (MeV^{-1})	E_1 (MeV)	s^2
^{208}Pb	GB	0.615 ± 0.002	8.0	1.07	0.008
	B	0.5	22.5	4.4	0.4
^{206}Tl	GB	0.610 ± 0.001	8.0	0.0	0.004
	B	0.5	21.5	2.7	0.4
^{210}At	GB	0.575 ± 0.0008	11.42	−1.58	0.001
	B	0.5	23.32	0.61	0.1
^{202}Pt	GB	0.604 ± 0.001	8.5	−0.78	0.003
	B	0.5	21.5	1.95	0.3
^{198}Au	GB	0.559 ± 0.0007	12.73	−1.11	0.0007
	B	0.5	22.95	0.82	0.09
^{198}Hf	GB	0.555 ± 0.001	13.1	−1.50	0.001
	B	0.5	22.50	0.19	0.07
^{220}Pb	GB	0.522 ± 0.001	20.5	−0.05	0.003
	B	0.5	25.97	0.56	0.02
^{190}Pt	GB	0.495 ± 0.0004	25.4	−2.23	0.0002
	B	0.5	25.3	−2.26	0.0006

generalized Bethe formula (7) gives much better fit than the standard Bethe formula (1) (in the former case the values of variance s^2 are smaller by two orders of magnitude). As the number of valence shell particles or holes increases, the quality of fit for the standard Bethe formula tends to improve. The density of levels at low energy is rather small (≈ 50 levels below 5 MeV in ^{208}Pb) and thus the pronounced level density fluctuations appear and therefore the low-lying part of the energy spectrum is excluded from the fit.

We note that the value $\xi=0.615$ associated with the doubly closed shell nucleus ^{208}Pb corresponds to $n=2.6$ in the power law (4), which is slightly above the lower value ($n=\frac{7}{3}$) associated with the model of strongly interacting particles. Near ^{208}Pb the dependence of the calculated ξ on the number of valence shell particles or holes, $N_v=|Z-82|+|N-126|$, can be roughly described by

$$\xi=\xi_0(1-\kappa N_v), \tag{9}$$

where ξ_0 is the maximum value of ξ, corresponding to the doubly closed shell nucleus, $\xi_0=\xi(^{208}\text{Pb})=0.615$, and $\kappa=0.01$. On the other hand, in accordance with expectations the parameter a has a minimum value at doubly closed shell nucleus ^{208}Pb and gradually increases for the nuclei with open shells. The behavior of the energy shift E_1 is less regular, but generally it is positive around the doubly closed nucleus and negative for the nuclei towards the middle of the shells.

In addition to the investigation of the region of doubly closed shell nucleus ^{208}Pb, we have performed some preliminary calculations to test the generalized Bethe formula approach for a broader range of nuclei. Similar type of behavior was also found for the region of $N=82$ nuclei, and in particular for the neighborhood of ^{132}Sn doubly closed nucleus (with maximum $\xi=0.620$ and minimum $a=5.16$ MeV^{-1}).

Finally, we have made preliminary investigations of the influence of pairing interaction. As a general pattern, inclusion of pairing in the GPM combinatorial calculations leads to decrease of the total level density, but the energy dependence remains similar as in the case of noninteracting nucleons [26]. In Fig. (1) this is demonstrated for ^{198}Au (closed triangles), where the pairing strength in GPM is fitted to the neutron resonance. In this case, with pairing included, we get $\xi=0.574$, $a=10.46$ MeV^{-1}, $E_1=-0.92$ MeV, and $s^2=0.0013$, in comparison to $\xi=0.559$, $a=12.73$ MeV^{-1}, $E_1=-1.11$ MeV, and $s^2=0.0007$ for the case of noninteracting particles.

In conclusion, we point out that the generalized Bethe formula (7) contains as limiting cases the standard Bethe formula (1) and the exponential form of the constant temperature formula (3), and it encompasses two transitional regimes, the one lying between the pattern of the standard Bethe formula and the constant temperature formula, and the other characterized by slower increase of level density with energy then given by the standard Bethe formula (''sub-Bethe'' pattern). We have shown that the generalized Bethe formula gives an excellent fit to the combinatorial total level densities calculated for nuclei in the region of doubly closed shell nucleus ^{208}Pb, where the standard Bethe formula fails. Going away from doubly closed shells the new parameter ξ gradually decreases from its maximum value $\xi=0.615$, which corresponds to an intermediate situation between the constant temperature and the standard Bethe formula, towards $\xi=0.5$ which corresponds to the standard Bethe formula.

Our results would suggest that this new unified parametrization of the total level density, using Eq. (7), may be useful in regions of the closed shell nuclei and in the ''sub-Bethe'' regions (where the increase of level density with energy is slower than predicted by the standard Bethe formula). Second, because the Eq. (7) does not depend on a particular model it will be interesting, in this framework, to deduce theoretically an information on the power law (4).

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