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Interacting boson model plus broken-pair description of high-spin dipole bands

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The interacting boson model with broken pairs has been extended to include mixed proton-neutron configurations in the fermion model space. The extended version of the model has been used to describe high-spin bands in the transitional nucleus ¹³⁶Nd. Model calculations reproduce ten bands of positive and negative parity states, including the two dipole high-spin structures based on the $(\pi h_{11/2})^2 (v h_{11/2})^2$ configuration. [S0556-2813(98)02902-1]

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I. INTRODUCTION

Models that are based on the interacting boson approximation [1] provide a consistent description of nuclear structure phenomena in spherical, deformed, and transitional nuclei. Many extensions of the original interacting boson model (IBM-1) [2] have been studied. In particular, models have been constructed to describe the physics of high-spin states in nuclei ($10\hbar \leq J \leq 30\hbar$). In the formulation of these models one has to go beyond the boson approximation and include selected noncollective fermion degrees of freedom. By including part of the original shell-model fermion space through successive breaking of correlated *S* and *D* pairs, the IBM is extended to describe the structure of high-spin states. This extension of the model is especially relevant for transitional regions, where single-particle excitations and vibrational collectivity are dominant modes, and the traditional cranking approach to high-spin physics is not adequate.

The model that we present is an extension of our previous work on the physics of high-spin states in even-even and odd-even nuclei [3–5]. The model is based on the IBM-1 [1]; the boson space consists of *s* and *d* bosons, with no distinction between protons and neutrons. To generate high-spin states, the model allows one or two bosons to be destroyed and to form noncollective fermion pairs, represented by two- and four-quasiparticle states which recouple to the boson core. High-spin states are described in terms of broken pairs. The model space for an even-even nucleus with $2N$ valence nucleons is

$$\begin{aligned} &|N \text{ bosons}\rangle \oplus |(N-1) \text{ bosons} \otimes 1 \text{ broken pair}\rangle \\ &\oplus |(N-2) \text{ bosons} \\ &\otimes 2 \text{ broken pairs}\rangle \oplus \dots \end{aligned}$$

The model Hamiltonian has four terms: the IBM-1 boson Hamiltonian, the fermion Hamiltonian, the boson-fermion interaction, and a pair breaking interaction that mixes states with different number of fermions. In previous versions of the model for even-even nuclei, we could only separately consider neutron or proton excitations. Mixed configurations

were not included in the model space, i.e., fermions in broken pairs had to be identical nucleons. For odd-*A* nuclei, we have extended the IBFM [6] to describe one- and three-fermion structures [5]. In that model mixed proton-neutron configurations have been included in the model bases. The two fermions in a broken pair can be of the same type as the unpaired fermion, resulting in a space with three identical fermions. If the fermions in the broken pair are different from the unpaired fermion, the fermion basis contains two protons and one neutron or vice versa. We have applied the model to the description of high-spin states in even-even and odd-even nuclei in the Hg [3–5], Sr-Zr [7–10], and Nd-Sm [11–13] regions. In several even-even nuclei high-spin bands have been observed that could not be described by model calculations. They were interpreted as possible two proton–two neutron excitations, not included in our fermion model space. In the present work we extend the model to include, in addition to two broken pairs of identical nucleons, the configurations with two different broken pairs, one of protons and one of neutrons. We continue our investigation of high-spin bands in the Nd-Sm region, and apply the model to the structure of positive and negative parity bands in ¹³⁶Nd. This nucleus is especially interesting since, in addition to quadrupole bands, a number of high-spin dipole bands have been observed. The occurrence of regular $\Delta J=1$ sequences in nearly spherical or transitional nuclei is a relatively new phenomenon. It has attracted much interest, but this is the first time that dipole bands in even-even nuclei are described in the framework of a model that is based on the interacting boson approximation. This means that we do not have to assume a geometrical picture for these bands, and that they result from a consistent calculation of the complete excitation spectrum, which includes also the ground state band and two- and four-quasiparticle quadrupole bands. In Sec. II we present an outline of the model. The calculated excitation spectrum of ¹³⁶Nd is compared with recent experimental data in Sec. III.

II. THE INTERACTING BOSON MODEL PLUS BROKEN PAIRS

An even-even nucleus with $2N$ valence nucleons is described as a system of N interacting bosons. The bosons

represent collective fermion pair states (correlated S and D pairs) that approximate the valence nucleon pairs. In order to generate high angular momentum states, one or two bosons can be destroyed. They form noncollective pairs, represented by two- and four-quasiparticle states that couple to the boson core. Fermions in broken pairs should in principle occupy all the valence single-particle orbitals from which the bosons have been mapped. However, for the description of high-spin bands close to the yrast line, the most important are the unique parity high- j orbitals for which the Coriolis antipairing effect is much more pronounced. We define the model Hamiltonian

$$H = H_B + H_F + V_{BF} + V_{\text{mix}}, \quad (1)$$

where H_B is the boson Hamiltonian of IBM-1 [2], and H_F is the fermion Hamiltonian which contains single-fermion energies and fermion-fermion interactions. The interaction between the unpaired fermions and the boson core contains the dynamical, exchange and monopole interactions of the IBFM-1 [6]. In order to describe high-spin dipole bands in ^{137}Nd , in Ref. [11] we have modified the quadrupole-quadrupole dynamical interaction

$$V_{\text{dyn}} = \Gamma_0 \sum_{j_1 j_2} (u_{j_1} u_{j_2} - v_{j_1} v_{j_2}) \langle j_1 \| Y_2 \| j_2 \rangle \times ([a_{j_1}^\dagger \times \tilde{a}_{j_2}]^{(2)} \cdot Q^B), \quad (2)$$

The standard boson quadrupole operator Q^B

$$Q^B = [s^\dagger \times \tilde{d} + d^\dagger \times \tilde{s}]^{(2)} + \chi [d^\dagger \times \tilde{d}]^{(2)} \quad (3)$$

has been extended by the higher order term

$$\chi' \sum_{L_1 L_2} [[d^\dagger \times \tilde{d}]^{(L_1)} \times [d^\dagger \times \tilde{d}]^{(L_2)}]^{(2)}. \quad (4)$$

The boson-fermion interactions of IBFM were derived in the lowest seniority approximation [6]. In the extension of the model to high-spin states, most of the structures can be described by the simplest form of the boson fermion interaction. However one should also expect polarization effects, as for example those that result in dipole bands, for which the boson-fermion interaction has to be extended by higher order terms. The term (4) might arise as a nontrivial higher-order contribution in the construction of the quadrupole moment from the generators of the $U(6)$ boson algebra. For the exchange and monopole interactions we have retained their standard forms.

The terms H_B , H_F and V_{BF} conserve the number of bosons and the number of fermions separately. In our model only the total number of nucleons is conserved, bosons can be destroyed and fermion pairs created, and vice versa. In the same order of approximation as for V_{BF} , the pair breaking interaction V_{mix} which mixes states with different number of fermions, conserving the total nucleon number only, reads

$$V_{\text{mix}} = -U_0 \left\{ \sum_{j_1 j_2} u_{j_1} u_{j_2} (u_{j_1} v_{j_2} + u_{j_2} v_{j_1}) \times \langle j_1 \| Y_2 \| j_2 \rangle^2 \frac{1}{\sqrt{2j_2+1}} ([a_{j_2}^\dagger \times a_{j_2}^\dagger]^{(0)} \cdot s) + \text{H.c.} \right\} - U_2 \left\{ \sum_{j_1 j_2} (u_{j_1} v_{j_2} + u_{j_2} v_{j_1}) \langle j_1 \| Y_2 \| j_2 \rangle \times ([a_{j_1}^\dagger \times a_{j_2}^\dagger]^{(2)} \cdot \tilde{d}) + \text{H.c.} \right\}. \quad (5)$$

This is the lowest order contribution to a pair-breaking interaction. The first term represents the destruction of one s boson and the creation of a fermion pair, while in the second term a d boson is destroyed to create a pair of valence fermions.

If mixed proton-neutron configurations are included in the fermion model space, i.e., if broken pairs contain both protons and neutrons, the full model Hamiltonian reads

$$H = H_B + H_{\nu F} + H_{\pi F} + V_{\nu BF} + V_{\pi BF} + V_{\nu}^{\text{mix}} + V_{\pi}^{\text{mix}} + V_{\nu\pi}, \quad (6)$$

where the proton-neutron interaction term is defined

$$V_{\nu\pi} = \sum_{\nu\nu'} \sum_{\pi\pi'} (-)^J h_J(\nu\nu' \pi\pi') (u_{\nu} u_{\nu'} - v_{\nu} v_{\nu'}) \times (u_{\pi} u_{\pi'} - v_{\pi} v_{\pi'}) ([a_{\nu}^\dagger \times \tilde{a}_{\nu'}]^{(J)} \cdot [a_{\pi}^\dagger \times \tilde{a}_{\pi'}]^{(J)}). \quad (7)$$

The two-body reduced matrix elements of a residual proton-neutron interaction define the coefficients $h_J(\nu\nu' \pi\pi')$:

$$h_J(\nu\nu' \pi\pi') = (-)^{j_{\nu}+j_{\pi}} \sum_{J'} (-)^{J'} \sqrt{2J'+1} \times W(j_{\nu} j_{\pi} j_{\nu'} j_{\pi'}; J' J) \langle (j_{\nu} j_{\pi}) J' \| V(1,2) \| \rangle \times (j_{\nu'} j_{\pi'}) J'. \quad (8)$$

In the present work we use the surface δ -force (SDI) for the residual interaction between unpaired fermions

$$V_{\text{SDI}}(1,2) = 4\pi V_0 \delta(\vec{r}_1 - \vec{r}_2) \delta(r_1 - R_0). \quad (9)$$

III. THE NUCLEUS $^{136}_{60}\text{Nd}_{76}$

Nuclei in the $A = 130$ – 140 mass region are γ -soft and the polarizing effect of the aligned nucleons induces changes in the nuclear shape. In all even-even nuclei the alignment of both proton and neutron pairs in the $h_{11/2}$ orbital generates low-lying 10^+ states, in many cases isomeric. By coupling the proton and neutron pairs to collective core excitations, $\Delta J = 2$ decoupled bands are generated with the 10^+ as band-head states. Because of the different nature of the excitations (particles for proton, and holes for neutron configurations), the alignment of a pair of $h_{11/2}$ protons drives the nucleus towards a collective shape opposite to that induced by the alignment of the neutron pair. One therefore expects to ob-

serve different coexisting structures at similar excitation energies. Employing the IBM with broken pairs, we have described many low-spin and high-spin properties of γ -soft nuclei of this region [in the IBM language $O(6)$ nuclei]: ^{137}Nd [11], ^{138}Nd [12], and ^{139}Sm [13].

In the present work we use the IBM with proton and neutron broken pairs to describe the excitation spectrum of ^{136}Nd . In particular, we want to obtain a correct description of high-spin dipole bands which have been interpreted as two proton–two neutron structures. Experimental data on high-spin structures in ^{136}Nd have been recently reported in Refs. [14] and [15]. In addition to two-proton and two-neutron quadrupole bands, as well as highly-deformed bands, five dipole bands dominated by strong $M1$ transitions have been observed. Theoretical calculations based on the projected shell model suggested quasiparticle configurations involving two protons and two neutrons for four of these bands. The model interpretation for the fifth, highest lying band, was a configuration with four identical nucleons. The experimental level scheme of positive-parity states is displayed in Fig. 1. The labels of bands are from Refs. [14] and [15]; in addition to the ground state band and the quasi γ -band, bands 3, 5, 7 and 8 result from the alignment of two protons or two neutrons in the $h_{11/2}$ orbital, and the dipole bands 10 and 11 are based on four-quasiparticle configurations.

In ^{136}Nd there are 6 neutron valence *holes* and 10 proton valence *particles*. The resulting boson number is $N=8$. The set of parameters for the boson Hamiltonian is $\epsilon=0.36$, $C_0=0.16$, $C_2=-0.12$, $C_4=0.19$, $V_2=0.11$, and $V_0=-0.15$ (all values in MeV). The boson parameters have values similar to those used in the calculation of ^{137}Nd , ^{138}Nd , and ^{139}Sm .

In $A \approx 140$ nuclei the structure of positive parity high-spin states close to the yrast line is characterized by the alignment of both proton and neutron pairs in the $h_{11/2}$ orbital. For positive-parity states we have only included the proton and neutron $h_{11/2}$ orbitals in the fermion model space. Additional single-nucleon states make the bases of the two broken pairs prohibitively large. The single quasiparticle energies and occupation probabilities are obtained from a BCS calculation using Kisslinger-Sorensen [16] single-particle energies. For the proton $h_{11/2}$ orbital the occupation probability is $v_\pi^2(h_{11/2})=0.07$ and the single-quasiparticle energy is $E_\pi(h_{11/2})=1.74$ MeV. For neutrons $v_\nu^2(h_{11/2})=0.83$ and $E_\nu(h_{11/2})=1.13$ MeV. As we have shown in our previous calculations for ^{137}Nd , ^{138}Nd , and ^{139}Sm , the single-quasiparticle energy $E_\nu(h_{11/2}) \approx 1.1$ MeV is too low. In order to reproduce the excitation energy of the two lowest 10^+ states, we have renormalized the value of $E_\nu(h_{11/2})$ to 1.75 MeV, and of $E_\pi(h_{11/2})$ to 1.60 MeV. If broken pairs are allowed to couple to all states of the boson core, the model bases are still too large. In order to further reduce the size of the space with two-broken pairs, we prediagonalize the boson Hamiltonian, and then couple the fermion states to the lowest eigenvectors. The collective structure of two- and four-quasiparticle high-spin bands is similar to that of the ground state band, but not identical, due to the smaller number of bosons. The parameters of the fermion-boson interactions are determined from IBFM calculations of low-lying negative-parity states in ^{137}Nd and neighboring odd-proton

nuclei. The parameters of the neutron dynamical fermion-boson interaction are $\Gamma_0=0.3$ MeV, $\chi=-1$ and $\chi'=-0.2$, and for protons: $\Gamma_0=0.22$ MeV, $\chi=+1$ and $\chi'=+0.2$. The strength parameters of the exchange interactions are $\Lambda_0^\nu=1.0$ and $\Lambda_0^\pi=1.5$ for neutrons and protons, respectively. The strength parameter of the pair-breaking interaction is $U_0=0$ and $U_2=0.2$ MeV, both for protons and neutrons in broken pairs. The residual interaction between unpaired fermions is a surface δ -force with strength parameters $V_0(\nu\nu)=-0.1$ MeV, $V_0(\pi\pi)=-0.1$ MeV and $V_0(\nu\pi)=-0.9$ MeV for neutron-neutron, proton-proton and neutron-proton configurations, respectively.

In Fig. 2 we display the calculated spectrum of positive-parity states. Only few lowest states of each angular momentum are included. According to the structure of wave functions, states are classified in bands labeled in such a way that a direct comparison can be made with their experimental counterparts. The calculated positive-parity structures 3, 5, 7, 8, 10 and 11, as well as the ground state band and the quasi γ -band, have to be compared with the experimental bands of Fig. 1. The collective ground state band is the yrast band up to angular momentum $J=8^+$. The calculation reproduces the experimental positions of states of the ground state band, as well as the quasi γ -band: 2_2^+ , 3_1^+ , and 5_1^+ . The bands 3, 5, 7, and the sequence of levels 14^+ , 16^+ and 18^+ on the left of band 7, result from the alignment of a pair of protons in the $h_{11/2}$ orbital. The main components in the wave functions of states of bands 3 and 5 are: $|(\pi h_{11/2})^2 J_F=10, I_B: J=J_F + I_B(-1)\rangle$, where (-1) refers the odd-spin band, and $|I_B\rangle$ denotes the lowest collective state of the boson core. In band 7 the two protons are predominantly coupled to $J_F=8$, and for $J \geq 14$ there is also a large contribution from components with $J_F=10$. Band 8 corresponds to two aligned $h_{11/2}$ neutrons with $J_F=10$, coupled to the boson core. We notice that the model calculation reproduces in detail the quadrupole collective two-proton and two-neutron bands. Finally, the two dipole bands 10 and 11 correspond to four-quasiparticle states (two broken pairs, one proton and one neutron). The states $[(\pi h_{11/2})^2 J_F=10][(\nu h_{11/2})^2 J_F=10]$ are coupled to the ground state band of the core. Bands 10 and 11 correspond to the two lowest calculated states for each angular momentum of the configuration $(\pi h_{11/2})^2 (\nu h_{11/2})^2$. The calculated structures are in very good agreement with the experimental bands.

The occurrence of regular dipole bands ($\Delta J=1$) in nearly spherical and transitional nuclei presents an interesting phenomenon. In the semiclassical picture of the cranked shell model, $\Delta J=1$ high-spin bands have been described as TAC (tilted axis cranking) solutions [17,18]. In our model such $\Delta J=1$ structures result from the fermion-boson interaction of unpaired fermions with the core, as well as from the residual proton-neutron interaction between unpaired fermions in unique-parity high- j orbitals. In order to obtain the correct energy spacings for bands 10 and 11, it was necessary to include the additional term (4) in the boson quadrupole operator. We have also found that the proton-neutron δ -interaction plays a crucial role in the excitation spectrum of these bands. This is illustrated in Fig. 3, where we display three steps in the construction of band 11, and compare them with the experimental band. In the left column we plot re-

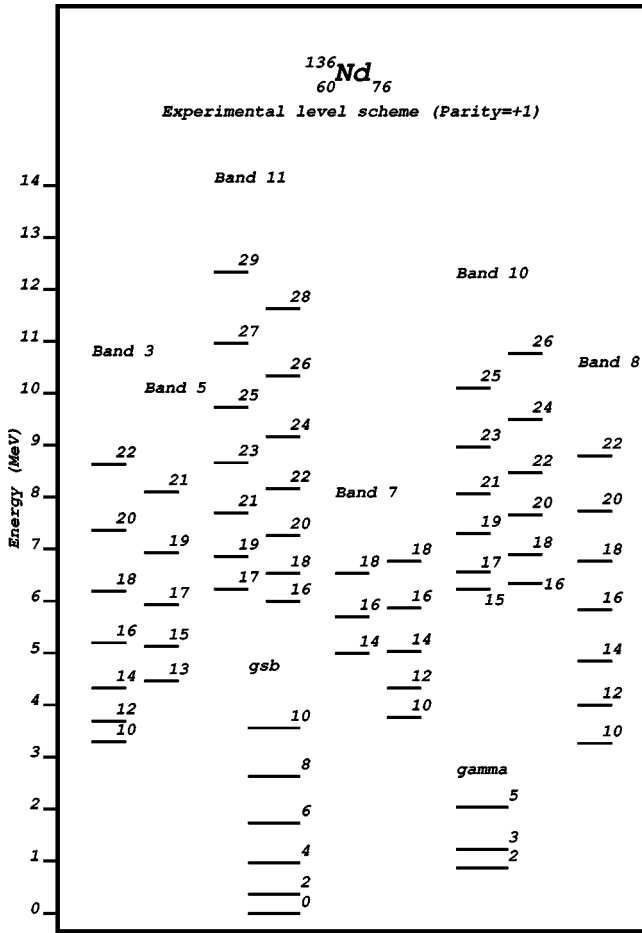


FIG. 1. Experimental excitation spectrum of positive-parity states in ^{136}Nd .

results of the model calculation obtained with parameters described above, except that the dynamical fermion-boson interactions for protons and neutrons do not contain the term (4), and the proton-neutron δ -interaction is not included. The structure essentially consists of two quadrupole bands, with almost degenerate doublets of odd and even spin states. This is a structure one normally expects to find in transitional γ -soft nuclei. The quasi-degeneracy of the doublets is removed in step two, where the term (4) has been included in the dynamical interactions for protons and neutrons. Finally, the correct excitation energies relative to the band-head 16^+ are obtained in step three with the inclusion of the proton-neutron residual interaction.

It should be noted that bands 10 and 11 have been recently described in the framework of the projected shell model [15]. This model uses a fixed deformation and provides a good description for the near yrast spectra of well deformed nuclei with stable shape. In order to obtain bands of dipole character, an axially symmetric shape with deformation $\epsilon_2=0.20$ had to be assumed, which in turn made energetically more favorable one of the neutrons to occupy the $\nu f_{7/2}$ orbital. There from the configuration $(\pi h_{11/2})^2 \nu f_{7/2} \nu h_{11/2}$ was assigned to bands 10 and 11. The dipole character of the bands results from the coupling of the neutron hole in $h_{11/2}$ and the neutron particle in $f_{7/2}$. This interpretation does not agree with the results of our calculation. When included in our model space, the structures based on

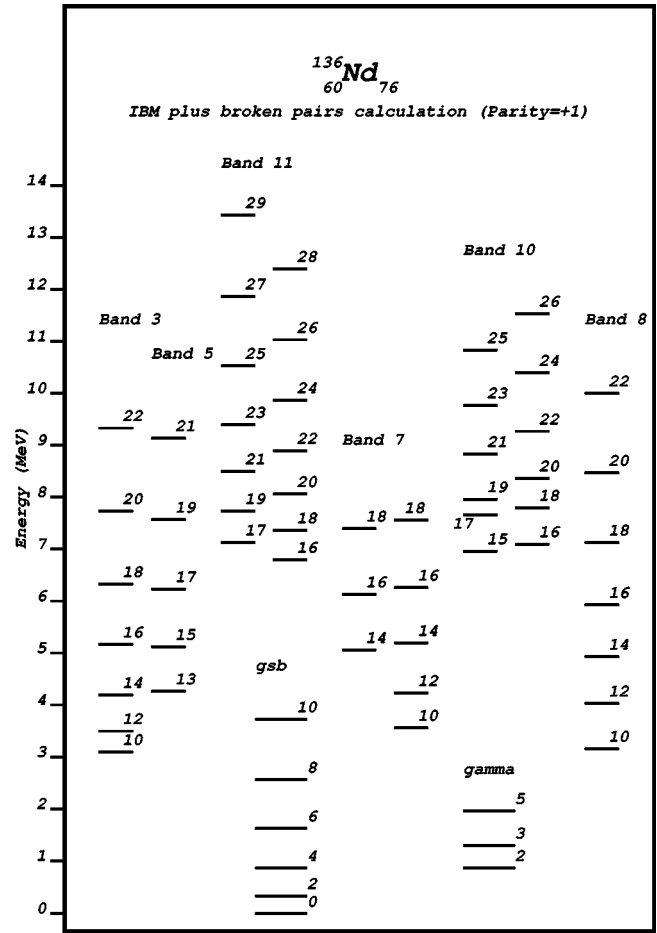


FIG. 2. Results of IBM plus broken pairs calculation for positive-parity bands in ^{136}Nd .

the $\nu f_{7/2}$ orbital are found high above the yrast. Since $(\pi h_{11/2})^2 J_F=10$ and $(\nu h_{11/2})^2 J_F=10$ are the lowest two-fermion bands, it is hard to see how could it be that four-fermion bands based on these configurations are not observed in the experiment. A larger deformation induced by polarization effects could place these configurations higher above the yrast. Such deformation however, should then result from the dynamics, and not as an additional parameter. We notice that a very similar dipole band, at approximately the same excitation energy, was found also in ^{138}Nd [12]. This nucleus is closer to the $N=82$ shell, and therefore deformations induced by polarization effects should be even smaller. On the other hand, using a unique set of parameters, we have been able to reproduce the complete experimental spectrum of positive parity states, from the ground state band to band 11, up to angular momentum $29 \hbar$ at more than 13 MeV excitation energy.

The structure of negative parity bands is displayed in Figs. 4 and 5. The labels are those of Ref. [15]. As in the case of ^{138}Nd [12], we assume that bands 1 and 2 correspond to a two-neutron configuration. For the calculation of negative parity bands the fermion part of the model space includes one and two neutron pairs. In addition to $h_{11/2}$, the neutron orbitals $s_{1/2}$ and $d_{3/2}$ are included in the model space. The quasiparticle energies and occupation probabilities: $E_\nu(d_{3/2})=0.91$ MeV, $v_\nu^2(d_{3/2})=0.32$ and $E_\nu(s_{1/2})=0.88$ MeV, $v_\nu^2(s_{1/2})=0.64$. The neutron parameters are the same

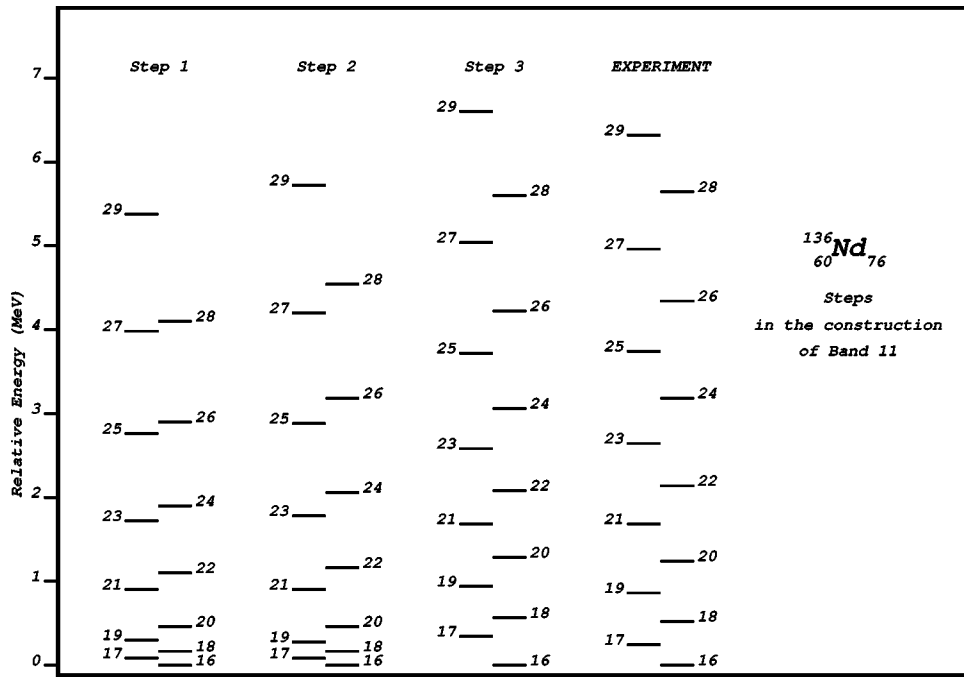


FIG. 3. Steps in the construction of band 11 compared with the experimental excitation spectrum. See text for details.

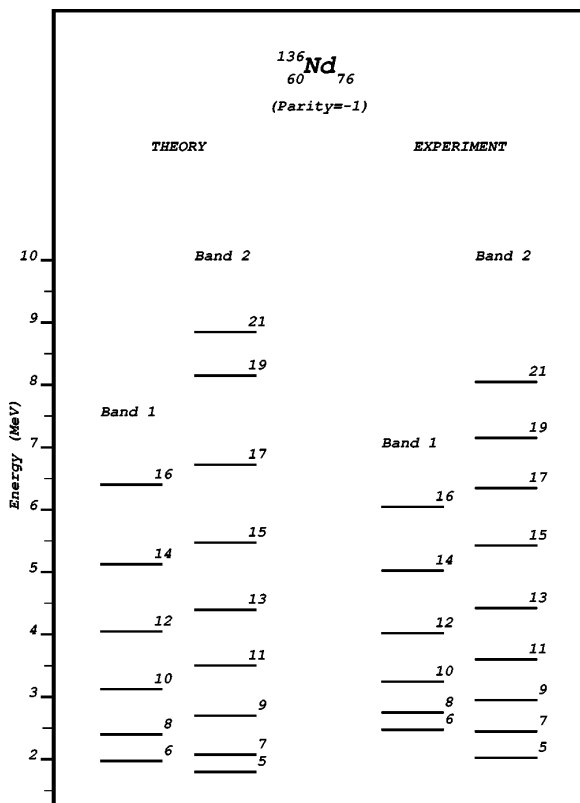


FIG. 4. Negative-parity bands 1 and 2 compared with results of the IBM plus broken pair calculation. Only states with possible experimental counterpart are shown.

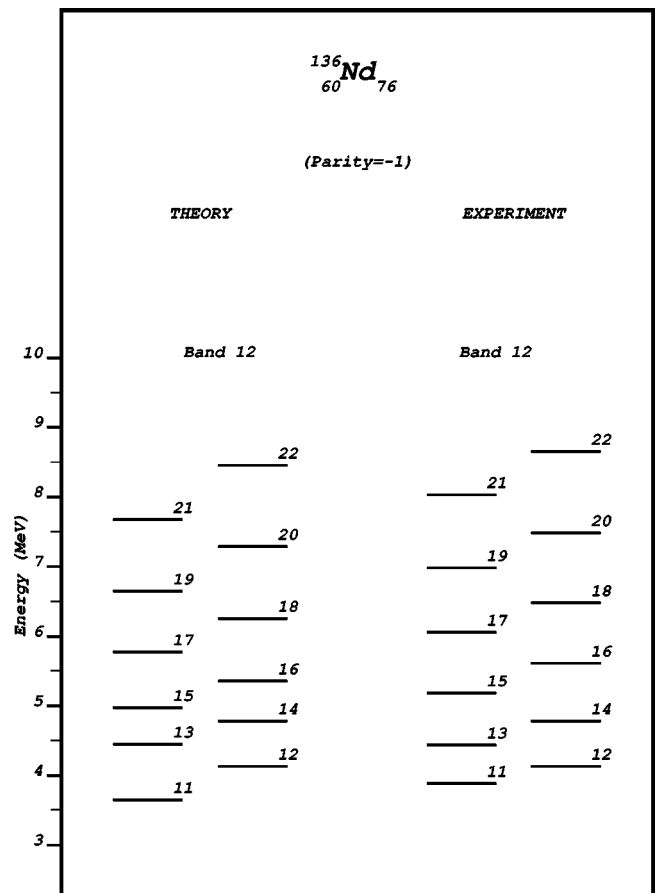


FIG. 5. Results of model calculation compared with the experimental negative-parity band 12.

as in the calculation of positive parity states, except that $\Lambda_0^v=0$ and $V_{\nu\nu}=-0.54$ MeV. These are effective interactions, and therefore the strength parameters depend on the fermion model space. However, the precise values are not that important for the structure of bands. In this particular case, the absence of the exchange interaction affects the moments of inertia of the bands, and the strength of the δ -interaction is determined from the energy splittings between even and odd spin states. The lowest calculated states are compared with the experimental bands 1 and 2 in Fig. 4. The theoretical bands are in good agreement with experimental data. The bands are based on the $(\nu d_{3/2}, \nu h_{11/2})$ configuration, coupled to the collective ground state band. In the lower part of the spectrum there are also significant contributions from components based on the configuration $(\nu s_{1/2}, \nu h_{11/2})$, but generally states based on this configuration are higher in energy. We have also investigated the possibility that the lowest negative parity bands are based on two-proton configurations in orbitals $h_{11/2}$, $d_{5/2}$ and $g_{7/2}$ as suggested in Ref. [14], but the calculation did not reproduce the experimental excitation energies. The lowest negative parity four-neutron states that result from our calculation are compared with band 12 in Fig. 5. The calculated band is based on the configuration $\nu d_{3/2}(\nu h_{11/2})^3$ coupled to the boson core, and reproduces the experimental spectrum. Of course, the possible assignment for band 12 could also be $(\nu d_{3/2}, \nu h_{11/2})(\pi h_{11/2})^2$, as suggested in Ref. [15]. However, we were not able to verify this assignment, since the fermion

model space for negative parity states with proton and neutron pairs is simply too large for any practical calculation. The experimental spectrum contains another negative parity structure (band 9 of Ref. [15]), based on a state 9^- at 3182 keV. The density of calculated negative parity states in this region is very high. States have strongly mixed wave functions, and we could not find a corresponding theoretical sequence.

In conclusion, we have used an extension of the interacting boson model with broken pairs to describe high-spin dipole bands in transitional nuclei. The model contains mixed proton-neutron configurations, i.e., proton and neutron broken pairs. Compared with models based on the cranking approximation, the present approach provides several advantages. High-spin structures can be described not only in well deformed, but also in transitional and spherical nuclei. A single set of parameters and a well defined Hamiltonian are used to calculate collective bands of the core, high-spin two- and four-quasiparticle bands. Polarization effects directly result from the model fermion-boson interactions. All calculations are performed in the laboratory frame, and therefore the results can be directly compared with experimental data. For the transitional nucleus ^{136}Nd , the model calculation reproduce the complete experimental excitation spectrum of positive and negative parity states. In particular, we have been able to obtain a correct description of the two high-spin $(\pi h_{11/2})^2$ $(\nu h_{11/2})^2$ dipole bands.

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- [1] F. Iachello and A. Arima, *The Interacting Boson Model* (Cambridge University Press, Cambridge, England, 1987).
- [2] A. Arima and F. Iachello, *Phys. Rev. Lett.* **35**, 10 (1975).
- [3] F. Iachello and D. Vretenar, *Phys. Rev. C* **43**, R945 (1991).
- [4] D. Vretenar, G. Bonsignori, and M. Savoia, *Phys. Rev. C* **47**, 2019 (1993).
- [5] D. Vretenar, G. Bonsignori, and M. Savoia, *Z. Phys. A* **351**, 289 (1995).
- [6] F. Iachello and O. Scholten, *Phys. Rev. Lett.* **43**, 679 (1979); F. Iachello and P. Van Isacker, *The Interacting Boson-Fermion Model* (Cambridge University Press, Cambridge, England, 1991).
- [7] P. Chowdhury, C. J. Lister, D. Vretenar, Ch. Winter, V. P. Janzen, H. R. Andrews, D. J. Blumenthal, B. Crowell, T. Drake, P. J. Ennis, A. Galindo-Uribarri, D. Horn, J. K. Johansson, A. Omar, S. Pilotte, D. Prévost, D. Radford, J. C. Waddington, and D. Ward, *Phys. Rev. Lett.* **67**, 2950 (1991).
- [8] C. J. Lister, P. Chowdhury, and D. Vretenar, *Nucl. Phys. A* **557**, 361c (1993).
- [9] A. A. Chishti, P. Chowdhury, D. J. Blumenthal, P. J. Ennis, C. J. Lister, Ch. Winter, D. Vretenar, G. Bonsignori, and M. Savoia, *Phys. Rev. C* **48**, 2607 (1993).
- [10] S. Cacciamani, G. Bonsignori, F. Iachello, and D. Vretenar, *Phys. Rev. C* **53**, 1618 (1996).
- [11] C. M. Petrache, R. Venturelli, D. Vretenar, D. Bazzacco, G. Bonsignori, S. Brant, S. Lunardi, N. H. Medina, M. A. Rizzutto, C. Rossi Alvarez, G. de Angelis, M. De Poli, and D. R. Napoli, *Nucl. Phys. A* **617**, 228 (1997).
- [12] G. de Angelis, M. A. Cardona, M. De Poli, S. Lunardi, D. Bazzacco, F. Brandolini, D. Vretenar, G. Bonsignori, M. Savoia, R. Wyss, F. Terrasi, and V. Roca, *Phys. Rev. C* **49**, 2990 (1994).
- [13] C. Rossi Alvarez, D. Vretenar, Zs. Podolyák, D. Bazzacco, G. Bonsignori, F. Brandolini, S. Brant, G. de Angelis, M. De Poli, M. Ionescu-Bujor, Y. Li, S. Lunardi, N. H. Medina, and C. M. Petrache, *Phys. Rev. C* **54**, 57 (1996).
- [14] C. M. Petrache, D. Bazzacco, S. Lunardi, C. Rossi Alvarez, R. Venturelli, D. Bucurescu, C. A. Ur, D. De Acuna, G. Maron, D. R. Napoli, N. H. Medina, J. R. B. Oliveira, and R. Wyss, *Phys. Lett. B* **373**, 275 (1996).
- [15] C. M. Petrache, Y. Sun, D. Bazzacco, S. Lunardi, C. Rossi Alvarez, R. Venturelli, D. De Acuna, G. Maron, M. N. Rao, Z. Podolyák, and J. R. B. Oliveira, *Phys. Rev. C* **53**, R2581 (1996).
- [16] L. S. Kisslinger and R. A. Sorensen, *Rev. Mod. Phys.* **35**, 853 (1963).
- [17] S. Frauendorf, *Nucl. Phys. A* **557**, 259c (1993).
- [18] S. Frauendorf and J. Reif, *Nucl. Phys. A* **621**, 736 (1997).