Interacting boson-fermion model calculation of the level scheme and electromagnetic properties of ^{99}Zr

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A theoretical study of the ^{99}Zr nucleus is presented. Energy levels, wave functions, and electromagnetic rates were calculated in the framework of the interacting boson-fermion model and are compared to the available data for low-lying states. In particular, we discuss the sensitivity of the g factor of the $3/2_1^+$ state to the quenching of the spin gyromagnetic ratio and to the structure of the respective wave function. [S0556-2813(98)07208-2]

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In Ref. [1] the low-lying spherical states and electromagnetic properties of the $N=59$ nucleus $^{97}_{38}\mathrm{Sr}_{59}$ were described in the framework of the interacting boson-fermion model (IBFM). The purpose of this Brief Report is to present a similar calculation for $^{99}_{40}\mathrm{Zr}_{59}$, compare it with available experimental data, and in particular with the recently reported result for the g factor of the $3/2_1^+$ state [2].

It is well known that the theoretical approach to nuclear structure in the framework of the interacting boson model (IBM) [3], the interacting boson-fermion model (IBFM) [4], and the interacting boson-fermion-fermion model (IBFFM) [5] is capable of treating even-even, odd-even, and odd-odd transitional nuclei, respectively. In this framework the structure of some nuclei in the $A=100$ region was investigated in recent years [1,6–8]. In particular, two $N=59$ isotones have been studied so far: $^{97}_{38}\mathrm{Sr}_{59}$ [1] and $^{98}_{39}\mathrm{Y}_{59}$ [7]. In Ref. [1] it was pointed out that the structure of $^{97}_{38}\mathrm{Sr}_{59}$ and $^{99}_{40}\mathrm{Zr}_{59}$ is similar, with the coexistence of spherical and deformed states: the lowest-lying triplet of states $1/2_1^+,3/2_1^+,7/2_1^+$ is of spherical character, while above 0.5 MeV there appear states of deformed nature. These deformed states were associated with the [422] 3/2 and [541] 3/2 Nilsson orbitals. In ^{97}Sr the lowest member of the [541] 3/2 band was assigned to the $3/2^-$ level at 644.7 keV and the lowest member of the [422] 3/2 band to the $3/2^+$ level at 585.1 keV. In ^{99}Zr the possible candidates for these two band heads are the $3/2^-$ level at 613.96 keV and the $3/2^+$ level at 724.30 keV [8].

One of the main difficulties encountered in IBFM calculations in cases where sufficient experimental data is not available is the relatively large number of parameters. We approached this problem by considering the similarity of the low-lying states in ^{99}Zr and ^{97}Sr. Of all the interaction strengths only the boson-fermion dynamical interaction was adjusted for ^{99}Zr. The other boson and quasiparticle parameters are taken the same as in the previous IBFM calculation for ^{97}Sr [1] with one modification: in Ref. [1] it was assumed that the $\nu d_{5/2}$ configuration was completely occupied and thus it was omitted from the calculation, while in the present calculation for ^{99}Zr the $\nu\tilde{d}_{5/2}$ quasiparticle was included, with $\epsilon(\tilde{d}_{5/2})=1.3$ MeV, $v^2(\tilde{d}_{5/2})=0.86$. The values of the boson-fermion interaction strengths A_0 and Λ_0 were taken from the previous calculation for ^{97}Sr [1]. The dynamical boson-fermion interaction strength Γ_0 was changed from the value $\Gamma_0=0.5$ MeV for ^{97}Sr to $\Gamma_0=0.8$ MeV for ^{99}Zr. This increase of Γ_0 causes a lowering of the first two excited states. The value of the quadrupole strength parameter χ is taken to be zero in the calculation for ^{99}Zr, in accordance with the calculation of the energy spectrum.

In Fig. 1 the calculated IBFM states of spherical type in ^{99}Zr are presented in comparison to the available experimental data. As in the case of the $N=59$ isotone ^{97}Sr, the wave function of the $1/2^+$ ground state of ^{99}Zr is dominated by the $\tilde{s}_{1/2}$ quasiparticle, while the $3/2^+$ first excited state is of a more complex character. Components of the latter wave function larger than 1% are given in Table I, expressed in the boson-fermion coupled basis:

$$|3/2_1^+\rangle=\sum_{jn_dvI}\xi^{3/2}_{jn_dvI}|j,n_dvI;\tfrac{3}{2}\rangle. \tag{1}$$

In the basis state $|j,n_dvI;\frac{3}{2}\rangle$ the quasiparticle j and the n_d d-boson state $|n_dvI\rangle$ of angular momentum I are coupled to the total angular momentum $\frac{3}{2}$. Here, $|n_dvI\rangle$ denotes the IBM basis state $|n_dvI,n_s=N-n_d;I\rangle$, where n_d d bosons and n_s s bosons are coupled to the total angular momentum I. The quantity v denotes an additional quantum number, if needed, which distinguishes the d-boson states having the same values of quantum numbers n_dI. As seen from Table I, the wave function of the $3/2_1^+$ state does not have a single dominant component. The two largest components in the $|3/2_1^+\rangle$ wave function are comparable: the quasiparticle state $|\tilde{d}_{3/2},00;3/2\rangle$ (27.8%) and the one-d-boson multiplet state based on the $\tilde{g}_{7/2}$ quasiparticle $|\tilde{g}_{7/2},12;3/2\rangle$ (27.4%). The fact that two components account for more than 50% of the wave function is significant: it means that, within the limitations of the present model, the state has a predominantly spherical character—as opposed to deformed states, whose wave functions are expected to contain many different configurations, with no particular preference for any of them. A possible way to check this statement is by comparing experimental values of observables of the $3/2_1^+$ state to calculations

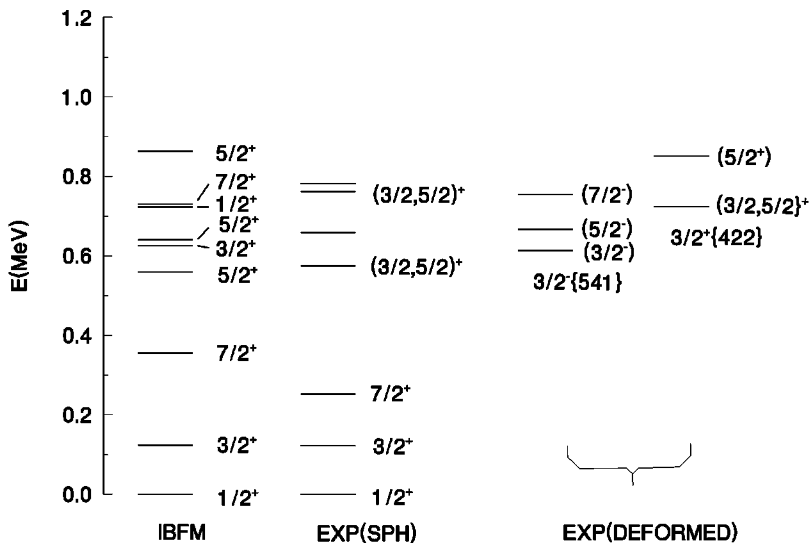

FIG. 1. IBFM positive-parity levels calculated for ^{99}Zr (spectrum denoted by IBFM) in comparison to available positive-parity states of spherical nature [spectrum denoted by EXP(SPH)]. For completeness the available experimental levels of possible rotational nature [labeled EXP(DEFORMED)] are also presented. Possible members of the rotational band $3/2^-[541]$ are assigned according to Ref. [7]. The assignment of the 658.06 keV, 761.5 keV, and 782.2 keV levels as spherical states, and the 724.30 keV and 851.89 keV levels as members of the $3/2^+$ [422] band is only tentative.

using the above wave function. We focus here on electromagnetic properties, and in particular on the magnetic moment of this state, for which an experimental value was recently reported [2].

Employing the IBFM wave functions, the electromagnetic properties of the four lowest positive-parity states in ^{99}Zr were calculated. The effective charges and gyromagnetic ratios, which are input parameters in the IBFM calculation, have been chosen as follows. The effective electric charges were taken as $e^{\mathrm{sp}}=0.5e, e^{\mathrm{vib}}=0.5e$ and the following gyromagnetic ratios were used (in μ_N):

$$g_R=\frac{Z}{A}=0.404, \quad g_l=g_l^{\mathrm{free}}=0, \quad g_s=0.4g_s^{\mathrm{free}}=-1.53,$$

$$g_T=\frac{1}{35}g_s^{\mathrm{free}}\langle r^2\rangle=-2.02. \tag{2}$$

The values of parameters e^{sp}, g_R and g_l are standard, while the values of parameters e^{vib}, g_s and g_T were adjusted to the overall agreement with available data on electromagnetic properties of four lowest-lying positive parity levels of ^{99}Zr, assuming that the $(3/2,5/2)^+$ level at 575.4 keV is $5/2^+$. (For definition of parameters in electromagnetic operators see Ref. [1].) The adjusted value for e^{vib} is larger than the value used in the previous calculation for ^{97}Sr [1].

As is well known [9–11], the quenching of g_s arises as a polarization effect associated with the presence of unsaturated spins in the closed shells, which can be partially aligned by the interactions with the spin of the extra nucleon. A rough overall estimate for this quenching given in Ref. [11] is $g_s \approx \frac{1}{2}\ g_s^{\mathrm{free}}$. In previous boson-fermion calculations the quenching of g_s was adjusted to the available experimental values of magnetic dipole moments and transitions. In most cases, such values lie in the range $g_s/g_s^{\mathrm{free}} \approx 0.4$–$0.7$. Thus, the present value for quenching lies at the lower end of the

TABLE I. IBFM wave function of the $3/2_1^+$ state in ^{99}Zr.

Component	Amplitude
$\lvert s_{1/2},12;3/2\rangle$	0.23
$\lvert g_{7/2},12;3/2\rangle$	−0.52
$\lvert g_{7/2},22;3/2\rangle$	0.32
$\lvert g_{7/2},24;3/2\rangle$	0.21
$\lvert g_{7/2},32;3/2\rangle$	0.12
$\lvert g_{7/2},34;3/2\rangle$	−0.11
$\lvert d_{3/2},00;3/2\rangle$	−0.53
$\lvert d_{3/2},12;3/2\rangle$	0.32
$\lvert d_{3/2},20;3/2\rangle$	0.21
$\lvert d_{3/2},22;3/2\rangle$	−0.16
$\lvert d_{3/2},30;3/2\rangle$	0.12

TABLE II. Comparison of experimental and calculated electromagnetic properties of four low-lying states in ^{99}Zr. Effective charges and gyromagnetic factors are adjusted to an overall agreement with all available data and assuming that the $(3/2,5/2)^+$ level at 575.4 keV is $5/2_1^+$.

	IBFM	Expt.
$B(E2;3/2_1^+\rightarrow 1/2_1^+)(e^2\mathrm{b}^2)$	0.0034	
$B(M1;3/2_1^+\rightarrow 1/2_1^+)(\mu_N^2)$	0.0046	0.0179(18)
$B(E2;7/2_1^+\rightarrow 3/2_1^+)(e^2\mathrm{b}^2)$	0.0079	0.0036(1)
$\mu(1/2_1^+)(\mu_N)$	−0.44	
$\mu(3/2_1^+)(\mu_N)$	+0.51	+0.42(6)
$Q(3/2_1^+)(e\mathrm{b})$	−0.12	
$\frac{I_\gamma(5/2_1^+\rightarrow 7/2_1^+)}{I_\gamma(5/2_1^+\rightarrow 1/2_1^+)}$	0.07	not observed
$\frac{I_\gamma(5/2_1^+\rightarrow 3/2_1^+)}{I_\gamma(5/2_1^+\rightarrow 1/2_1^+)}$	0.55	0.35

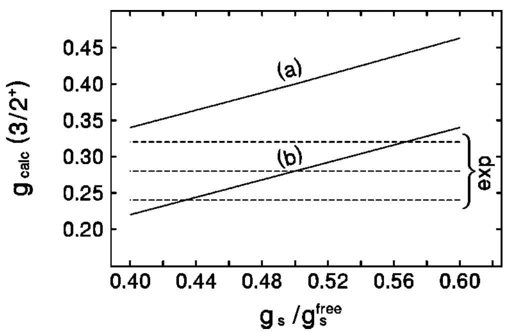

FIG. 2. Calculated g factor of the $3/2_1^+$ state versus the quenching factor $g_s/g_s^{\rm free}$. The solid curves present results for: (a) $g_T = -2.02$; (b) $g_T=0$. The dashed lines mark the experimental g factor with its error bars.

overall interval. Furthermore, we note that this value of the quenching factor $g_s/g_s^{\rm free}$ is similar to the one used in previous calculations for several nuclei in the $A=100$ region, like ^{103}Ru and ^{105}Pd [6].

The value chosen here for the tensor interaction strength g_T is somewhat smaller than the value used in the previous calculation for ^{97}Sr [1]. We also note that the value $g_T \approx +0.5 g_s^{\rm free}$ is rather close to an overall estimate of $g_T \approx +0.4 g_s^{\rm free}$ from Ref. [11].

In Table II, the calculated electromagnetic properties corresponding to the above parametrization are compared to the available data under the previously mentioned assumption for the spin and parity of the 575.4 keV level. The agreement between the calculations and experiment for the few observables for which we have data is within a factor of 2–3, which can be considered as reasonable and in fact typical for IBFM calculations of electromagnetic observables in medium and heavy nuclei. Now we discuss in some detail the structure of the $3/2_1^+$ wave function and the value of its magnetic moment. The experimental value of the g factor is $g(3/2_1^+) = +0.28(4)\mu_N$ [2]. For the parametrization in Eq. (2) we obtain in IBFM the value $g(3/2_1^+) = +0.34$. On the other hand, without the contribution from the tensor term in the $M1$ operator the calculated result is $g(3/2_1^+) = +0.22$. Moreover, the calculated value of $g(3/2_1^+)$ is sizably dependent on the value of the quenching factor $g_s/g_s^{\rm free}$. This is shown in Fig. 2 by the solid line labeled (a), by keeping the values of the other parameters (g_R, g_l, g_T) fixed as given by the parametrization in Eq. (2). The solid line labeled (b) presents the value of $g(3/2_1^+)$ versus $g_s/g_s^{\rm free}$, without inclusion of the tensor term. From Fig. 2 it is clear that at present, one cannot unambiguously determine the importance of the tensor term in the $M1$ operator, although it is clear that for $g_T=0$ a better agreement with the experimental value is obtained for the entire range of $g_s/g_s^{\rm free}$ values.

TABLE III. Contributions of various components of the wave function to the g factor of the $3/2_1^+$ state of ^{99}Zr.

j	n_d	I	j'	n_d'	I'	Contribution to $g(3/2^+)$
5/2	2	4	5/2	2	4	0.011
7/2	1	2	7/2	1	2	0.027
7/2	2	2	7/2	2	2	0.010
7/2	2	4	7/2	2	4	0.018
7/2	3	4	7/2	3	4	0.006
1/2	1	2	1/2	1	2	0.041
1/2	1	2	3/2	1	2	0.011
3/2	0	0	3/2	0	0	0.130
3/2	1	2	1/2	1	2	0.011
3/2	1	2	3/2	1	2	0.043
3/2	2	0	3/2	2	0	0.020
3/2	2	2	3/2	2	2	0.010
3/2	3	0	3/2	3	0	0.007

In Table III we present the contributions to the calculated $g(3/2_1^+)$ from various components of the wave function. We see that about 70% of the experimental value of the g factor is due to three matrix elements, thus supporting the above contention that the $3/2_1^+$ state has a predominantly spherical character.

In conclusion, the IBFM provides a reasonable description of low-lying states of the transitional ^{99}Zr nucleus. A detailed comparison of the calculated value of the magnetic moment of the first excited state with a recently reported experimental value supports the expectation that this state has spherical character, in accordance with the conclusions of previous works regarding shape coexistence in this nucleus.

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[1] G. Lhersonneau, B. Pfeiffer, K.-L. Kratz, H. Ohm, K. Sistemich, S. Brant, and V. Paar, Z. Phys. A **337**, 149 (1990).
[2] A. Wolf, R. L. Gill, Z. Berant, and D. S. Brenner, Phys. Rev. C **51**, 2381 (1995).
[3] A. Arima and F. Iachello, Phys. Rev. Lett. **35**, 1069 (1975).
[4] F. Iachello and O. Scholten, Phys. Rev. Lett. **43**, 679 (1979).
[5] V. Paar, in *Capture Gamma-Ray Spectroscopy and Related Topics*, edited by S. Raman, AIP Conf. Proc. No. 125 (AIP, New York, 1984), p. 70; S. Brant, V. Paar, and D. Vretenar, Z. Phys. A **319**, 351 (1984).
[6] S. Brant *et al.*, in *Proceedings of the International Workshop on Nuclear Structure of the Zirconium Region*, edited by J. Eberth, R.A. Meyer, and K. Sistemich (Springer-Verlag, Berlin, 1988), p. 199.
[7] S. Brant, V. Paar, G. Lhersonneau, O. W. B. Schult, H. Seyfarth, and K. Sistemich, Z. Phys. A **334**, 517 (1989).
[8] S. Brant, K. Sistemich, V. Paar, and G. Lhersonneau, Z. Phys. A **330**, 365 (1988).
[9] A. Arima and H. Horie, Prog. Theor. Phys. **12**, 623 (1954).
[10] R. J. Blin-Stoyle and M. A. Perkes, Proc. R. Soc. London, Ser. A **67**, 885 (1954).
[11] A. Bohr and B. R. Mottelson, *Nuclear Structure* (Benjamin, New York, 1969), Vol. I.