The extended chiral quark model in a Tamm-Dancoff inspired approximation

Horvat, Dubravko; Horvatić, Davor; Podobnik, Boris; Tadić, Dubravko

Permanent link / Trajna poveznica: https://urn.nsk.hr/urn:nbn:hr:217:318959

Download date / Datum preuzimanja: **2025-03-24**

THE EXTENDED CHIRAL QUARK MODEL IN A TAMM-DANCOFF INSPIRED APPROXIMATION

DUBRAVKO HORVAT[a], DAVOR HORVATIĆ[b], BORIS PODOBNIK[b] and
DUBRAVKO TADIĆ[b]

[a] *Department of Physics, Faculty of Electrical Engineering, University of Zagreb,
HR-10000 Zagreb, Croatia*
E-mail: dubravko.horvat@fer.hr

[b] *Department of Physics, Faculty of Natural Sciences, University of Zagreb,
HR-10000 Zagreb, Croatia*
E-mail addresses: davorh@phy.hr   bp@phy.hr   tadic@phy.hr

A procedure inspired by the Tamm-Dancoff method is applied to the chiral quark model which has been extended to include additional degrees of freedom: a pseudoscalar isoscalar field as well as a triplet of scalar isovector fields. The simpler, generic σ-model has been used before as a test for the Tamm-Dancoff inspired approximation (TDIA). The extended chiral quark model is employed here to investigate possible novel effects of the additional degrees of freedom as well as to point out the necessesity to introduce a SU(3) flavour. Model predictions for the axial-vector coupling constant and for the nucleon magnetic moment obtained in TDIA are compared with experimental values.

1. Introduction

The Tamm-Dancoff method (TDM) [1, 2] has been intensively investigated during the 1950s [3]. It has been revived recently [4–8]. In some calculations TDM can be a much better approximation than perturbation theory [9].

As chiral bag models [10, 11] are simple effective theories of quark bound states,

hadrons, it is not unreasonable to hope that TDM might be useful in that case, too.

The Tamm-Dancoff inspired approximation (TDIA) [12] will be used here. It will be developed for an extended chiral sigma model which besides usual modes [7, 8] contains additional degrees of freedom: a pseudoscalar isoscalar field and a triplet of scalar isovector fields. The U(2) symmetry of the simple model [7, 8] has thus been enlarged to the present U(2)×U(2) [13]. That allows closer comparison with the SU(3) symmetry based linear sigma model [14, 15]. In such model one has to introduce 9 scalar and 9 pseudoscalar degrees of freedom, thus creating an U(3)×U(3) symmetry [16–19]. While in the SU(3) based case one has 18 mesonic degrees, in our simpler case one deals with 8 degrees only. The simple σ-model [12] used 4 mesonic degrees. Our enlarged U(2)×U(2) model can serve as a transition to the full SU(3) treatment showing the (non)importance of the scalar deegres of freedom.

It will be shown to what extent the new mesonic degrees modify previously found results [12].

2. Tamm-Dancoff inspired approximation

Full description of TDIA can be found in Ref. [12]. Here we will give only some details pertinent for the U(2)×U(2) model. Working in the Heisenberg picture [20], all field operators are expanded in the free fields [12]. Eventually one ends with an infinite set of coupled differential equations instead of integral ones, which appear in TDM [3, 4]. These differential equations are closely related to the familiar chiral quark model equations.

The Lagrangian containing the extended linear U(2)×U(2) sigma model embedded in the bag environment is:

$$\mathcal{L} = \mathcal{L}_\psi \Theta + \mathcal{L}_{\text{int}} \delta_S + [\mathcal{L}_\chi + U(\chi)]\overline{\Theta} \,, \tag{2.1}$$

where

$$\mathcal{L}_\psi = \frac{\mathrm{i}}{2}(\bar{\psi}\gamma^\mu\partial_\mu\psi - \partial_\mu\bar{\psi}\gamma^\mu\psi) \,,$$

$$\mathcal{L}_{int} = \frac{g}{2}\bar{\psi}(\sigma + \mathrm{i}\vec{\tau}\vec{\pi}\gamma_5)\psi - \frac{\mathrm{i}g'}{2}\bar{\psi}\gamma_5(\eta + \mathrm{i}\vec{\tau}\vec{s}\gamma_5)\psi \,,$$

$$\mathcal{L}_\chi = \frac{1}{2}\left(\partial^\mu\sigma\partial_\mu\sigma + \partial^\mu\vec{\pi}\partial_\mu\vec{\pi} + \partial^\mu\eta\partial_\mu\eta + \partial^\mu\vec{s}\partial_\mu\vec{s}\right) \,, \tag{2.2}$$

$$U(\chi) = \frac{\lambda^2}{4}\left[\sigma^2 + \vec{\pi}^2 + \eta^2 + \vec{s}^2 - \nu^2\right]^2 \,,$$

$$+\frac{\lambda^2\mu^2}{2}\left[\eta^2 + \vec{s}^2\right] + f_\pi m_\pi^2 \sigma \,,$$

and $f_\pi = 0.093\,\text{GeV}$. The $\Theta-$function signalizes that $\mathcal{L}_\psi$ is different from zero inside the bag ($r < R_{bag}$). The surface $\delta-$function, δ_S, gives the surface quark – meson

interaction, and $\overline{\Theta}$ ensures that the potential U and the $(\sigma, \vec{\pi}, \eta, \vec{s})$ kinetic-energy terms exist (only) outside the bag. In the *spherical* bag, Θ and $\overline{\Theta}$ become $\theta(R_{bag}-r)$ and $\theta(r - R_{bag})$, respectively. In the product symmetry U(2)×U(2), the coupling constants g and g' can have different values. In actual calculations we were forced to break this symmetry thus introducing an assortment of g's whose values are connected trough boundary conditions (see Eq. (2.13) below). The self-interaction potential U contains the symmetry-breaking (SB) term $c\sigma(x) \equiv -f_\pi m_\pi^2 \sigma(x)$. The other parameters are fixed by particle mass spectrum, i.e.

$$
\begin{aligned}
m_\sigma^2 &= -\lambda^2\nu^2 + 3\lambda^2 f_\pi^2 \,, \\
m_\pi^2 &= -\lambda^2\nu^2 + \lambda^2 f_\pi^2 \,, \\
m_\eta^2 &= m_s^2 = \lambda^2\mu^2 - \lambda^2\nu^2 + \lambda^2 f_\pi^2 \,,
\end{aligned}
\tag{2.3}
$$

by the PCAC and by the usual potential minimum conditions [14,15].

The effective empirical quantum field theory (2.1) describes quark dynamics as an approximant for the underlying, fundamental and exact QCD.

The field operators ψ, $\vec{\pi}$, σ, η and $\vec{s}$ are expanded in terms of the free field creation (annihilation) operators. For the quark field, for example, one introduces

$$
\begin{aligned}
\psi_f^c(x) = {}&\phi_m^f \mathrm{b}_{m,f}^c + \tilde{\phi}_m^f \mathrm{d}_{m,f}^{c\dagger} \\
&+ \chi_{m_1 m_2 m_3}^{fgh}(x) \mathrm{b}_{m_1,f}^c \mathrm{d}_{m_2,g}^{e\dagger} \mathrm{b}_{m_3,h}^e + \dots
\end{aligned}
\tag{2.4}
$$

Many complex operator combinations are possible, besides the one shown. However, in our approximation we use only first two terms. Here c is a quark colour and f is a quark flavour, whereas m is the spin projection. $\mathrm{b}_{m,f}^c$ and $\mathrm{d}_{m,f}^c$ are the quark and antiquark annihilation operators, respectively. This infinite expansion is truncated leading to a physically motivated finite basis, which defines the Tamm-Dancoff inspired approximation (TDIA).

The truncation of the ψ field (2.4) as well as the corresponding Ansätze for the $\vec{\pi}$, σ, η and $\vec{s}$ fields lead to the following system of the coupled nonlinear Euler-Lagrange equations

$$
\begin{aligned}
rli\gamma_\mu\partial^\mu\psi &= 0 \quad (r < R_{\text{Bag}}), \\
\partial_\mu\partial^\mu\sigma - \lambda^2\sigma\left(\sigma^2 + \vec{\pi}^2 + \eta^2 + \vec{s}^2 - \nu^2\right) - f_\pi m_\pi^2 &= 0 \quad (r > R_{\text{Bag}}), \\
\partial_\mu\partial^\mu\pi^a - \lambda^2\pi^a\left(\sigma^2 + \vec{\pi}^2 + \eta^2 + \vec{s}^2 - \nu^2\right) &= 0 \quad (r > R_{\text{Bag}}), \\
\partial_\mu\partial^\mu\eta - \lambda^2\eta\left(\sigma^2 + \vec{\pi}^2 + \eta^2 + \vec{s}^2 - \nu^2 + \mu^2\right) &= 0 \quad (r > R_{\text{Bag}}),
\end{aligned}
$$

$$rl\partial_\mu\partial^\mu s^a - \lambda^2 s^a \left(\sigma^2 + \vec{\pi}^2 + \eta^2 + \vec{s}^2 - \nu^2 + \mu^2\right) = 0 \quad (r > R_{\text{Bag}}),$$

$$in_\mu\gamma^\mu\psi + g_\sigma\sigma\psi + g_\pi i\vec{\tau}\vec{\pi}\gamma_5\psi - g_\eta i\gamma_5\eta\psi + g_s\vec{\tau}\vec{s}\psi = 0 \quad (r = R_{\text{Bag}}), \qquad (2.5)$$

$$\left(\partial^\mu\sigma\right)\hat{n}_\mu = -\frac{g_\sigma}{2}\overline{\psi}\psi \quad (r = R_{\text{Bag}}),$$

$$\left(\partial^\mu\pi^a\right)\hat{n}_\mu = -\frac{g_\pi}{2}\overline{\psi}i\vec{\tau}\gamma_5\psi \quad (r = R_{\text{Bag}}),$$

$$\left(\partial^\mu\eta\right)\hat{n}_\mu = \frac{g_\eta}{2}\overline{\psi}i\gamma_5\psi \quad (r = R_{\text{Bag}}),$$

$$\left(\partial^\mu s^a\right)\hat{n}_\mu = -\frac{g_s}{2}\overline{\psi}\vec{\tau}\psi \quad (r = R_{\text{Bag}}).$$

In the TDIA sense, one keeps just the terms in the field expansion which are needed to obtain a nontrivial coupled system of differential equations.

The leading approximation follows if in Eqs. (2.5) one keeps just two first terms in the expansion (2.4).

The result is then sandwiched between the initial quark or antiquark states

$$\langle f| = \langle \bar{q}^a_{p,r}| = \langle 0|d^a_{p,r} \qquad |i\rangle = |0\rangle, \quad \text{or}$$
$$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (2.6)$$
$$\langle f| = \langle 0| \qquad\qquad\qquad |i\rangle = |q^a_{p,r}\rangle = b^{a\dagger}_{p,r}|0\rangle.$$

and the vacuum, leading to the terms such as

$$\langle 0|\gamma_\mu\partial^\mu\psi|q^c_{f,m}\rangle = \gamma_\mu\partial^\mu\phi^f_m(x). \qquad (2.7)$$

The Dirac equation for the free quark inside the bag [21], according to (2.4), leads to the following approximate TDIA Ansatz for the quark field

$$\psi^c_f(x) = \frac{N}{\sqrt{4\pi}}\begin{pmatrix} j_0 \\ i(\vec{\sigma}\hat{r})j_1 \end{pmatrix}\chi^f_m b^c_{m,f} + \frac{N}{\sqrt{4\pi}}\begin{pmatrix} (\vec{\sigma}\hat{r})j_1 \\ ij_0 \end{pmatrix}\chi^f_m d^{c\dagger}_{m,f}. \qquad (2.8)$$

Here the quantities $j_{0,1}(r)$ are spherical Bessel functions of the order $(0,1)$ and χ^f_m is the quark isospinor ($\tilde{\chi}^f$)-spinor (χ_m) product.

The boundary conditions (2.5) can be satisfied with the first two terms in the expansion (2.4) if the corresponding $\vec{\pi}$, σ, η and $\vec{s}$ field expansion contains terms such as

$$b^{c\dagger}_{m,f}b^c_{m',f'}. \qquad (2.9)$$

All terms in (2.5), either the bispinor ones ($\overline{\psi}\Gamma\psi$) or the meson ones ($\vec{\pi}$, σ) must contain the same number and the same kind of the creation (annihilation) quark operators. Thus the TDIA Ansätze given in terms of the *chiral*-quark operators are [12]:

$$\sigma(r) = \sigma_s(r) \cdot (b^{c\dagger}_{m,f} b^c_{m,f} + d^{c\dagger}_{m,f} d^c_{m,f}) - f_\pi \,,$$

$$\pi^a(r) = \pi_s(r)(b^{c\dagger}_{m,f} d^{c\dagger}_{m',f'} + d^c_{m,f} b^c_{m',f'}) \cdot [\chi^{\dagger}_{m,f} \tau^a \chi_{m',f'}]$$

$$+ \pi_p(r)(b^{c\dagger}_{m,f} b^c_{m',f'} - d^{c\dagger}_{m',f'} d^c_{m,f}) \cdot [\chi^{\dagger}_{m,f}(\vec{\sigma}\hat{r})\tau^a \chi_{m',f'}] \,,$$

$$\eta(r) = \eta_s(r)(b^{c\dagger}_{m,f} d^{c\dagger}_{m',f'} + d^c_{m,f} b^c_{m',f'})$$

$$+ \eta_p(r)(b^{c\dagger}_{m,f} b^c_{m',f'} - d^{c\dagger}_{m',f'} d^c_{m,f}) \cdot [\chi^{\dagger}_{m,f}(\vec{\sigma}\hat{r})\chi_{m',f'}] \,,$$

$$s^a(r) = s_s(r) \cdot (b^{c\dagger}_{m,f} b^c_{m,f} + d^{c\dagger}_{m,f} d^c_{m,f}) \cdot [\chi^{\dagger}_{m,f} \tau^a \chi_{m',f'}] \,.$$

(2.10)

The Ansätze obey spin-isospin and Lorentz properties of corresponding particles. They were inspired by the valence quark content of mesons and they correctly match the quark field approximation (2.8).

At this level of TDIA expansion only the quark operators are important. The boson operators can be introduced later on or one can assume that the theory (2.1, 2.2) contains the fermions only. Then the terms like σ^2, $\vec{\pi}^2$ etc. describe various nonlinear interactions among fermions (quarks) which have to be coupled in scalar (pseudoscalar) combinations. Such models (theories) [11] would be effective nonrenormalizable field theories.

In the following the terms *meson*, *pion*, *sigma*, *eta* or *s* are used in that generalized sense refering to expressions like (2.10).

The expansions (2.10) for bosonic quantities appear quite naturally. They have been encountered in the past applications of the Tamm-Dancoff procedure, as for example in the formula (4.6) of Ref. [8]. As the operators b and d have the opposite parity [22], the TDIA conserves parity throughout.

The boundary conditions in (2.4) are now specified using the Ansätze (2.8) and (2.10). When they are sandwiched between the final and initial states as given in (2.6), one finds

$$\frac{\partial}{\partial r}\sigma_s(r)\Big|_{r=R} = -\frac{g_\sigma N^2}{8\pi}\left[j_0^2(\omega) - j_1^2(\omega)\right],$$

$$\frac{\partial}{\partial r}\pi_s(r)\Big|_{r=R} = +\frac{g_{\pi_s} N^2}{8\pi}\left[j_0^2(\omega) + j_1^2(\omega)\right],$$

$$\frac{\partial}{\partial r}\pi_p(r)\Big|_{r=R} = +\frac{g_{\pi_p} N^2}{4\pi}j_0(\omega)j_1(\omega),$$

$$\frac{\partial}{\partial r}\eta_s(r)\Big|_{r=R} = -\frac{g_{\eta_s} N^2}{8\pi}\left[j_0^2(\omega) + j_1^2(\omega)\right], \tag{2.11}$$

$$\frac{\partial}{\partial r}\eta_p(r)\Big|_{r=R} = -\frac{g_{\eta_p} N^2}{4\pi}j_0(\omega)j_1(\omega),$$

$$\frac{\partial}{\partial r}s_s(r)\Big|_{r=R} = -\frac{g_s N^2}{8\pi}\left[j_0^2(\omega) - j_1^2(\omega)\right].$$

Here ω's are eigenfrequencies determined by boundary conditions (2.12). The normalisation N is defined by (4.1) below. From $\mathcal{L}_{int}$ one can derive

$$\psi(r)\Big|_{r=R_{\mathrm{bag}}} = \mathrm{i}g_\sigma\sigma(r)(\vec{\gamma}\hat{r})\psi(r)\Big|_{r=R_{\mathrm{bag}}} - g_\pi\vec{\tau}\vec{\pi}(r)(\vec{\gamma}\hat{r})\gamma_5\psi(r)\Big|_{r=R_{\mathrm{bag}}} +$$

$$+ g_\eta\eta(r)(\vec{\gamma}\hat{r})\gamma_5\psi(r)\Big|_{r=R_{\mathrm{bag}}} + \mathrm{i}g_s\vec{\tau}\vec{s}(r)(\vec{\gamma}\hat{r})\psi(r)\Big|_{r=R_{\mathrm{bag}}}. \tag{2.12}$$

It takes the following form:

$$\begin{pmatrix} j_0 \\ \mathrm{i}(\vec{\sigma}\hat{r})j_1 \end{pmatrix}\chi_m^f \mathrm{b}_{m,f}^c + \begin{pmatrix} (\vec{\sigma}\hat{r})j_1 \\ \mathrm{i}j_0 \end{pmatrix}\chi_m^f \mathrm{d}_{m,f}^{c\,\dagger} =$$

$$- g_{\pi/p}\pi_p(R)\begin{pmatrix} (\vec{\sigma}\hat{r})j_0 \\ -\mathrm{i}j_1 \end{pmatrix}\chi_m^f \mathrm{b}_{m_1,f_1}^{d\,\dagger}(\vec{\tau}\cdot\vec{\tau})\mathrm{b}_{m_2,f_2}^d[\chi_{m_1}^{f_1\,\dagger}(\vec{\sigma}\hat{r})\chi_{m_2}^{f_2}]\mathrm{b}_{m,f}^c$$

$$+ g_{\pi/p}\pi_p(R)\begin{pmatrix} (\vec{\sigma}\hat{r})j_0 \\ -\mathrm{i}j_1 \end{pmatrix}\chi_m^f \mathrm{d}_{m_2,f_2}^{d\,\dagger}(\vec{\tau}\cdot\vec{\tau})\mathrm{d}_{m_1,f_1}^d[\chi_{m_1}^{f_1\,\dagger}(\vec{\sigma}\hat{r})\chi_{m_2}^{f_2}]\mathrm{b}_{m,f}^c$$

$$- g_{\pi/p}\pi_p(R)\begin{pmatrix} j_1 \\ -\mathrm{i}(\vec{\sigma}\hat{r})j_0 \end{pmatrix}\chi_m^f \mathrm{b}_{m_1,f_1}^{d\,\dagger}(\vec{\tau}\cdot\vec{\tau})\mathrm{b}_{m_2,f_2}^d[\chi_{m_1}^{f_1\,\dagger}(\vec{\sigma}\hat{r})\chi_{m_2}^{f_2}]\mathrm{d}_{m,f}^{c\,\dagger}$$

$$+ g_{\pi/p}\pi_p(R)\begin{pmatrix} j_1 \\ -\mathrm{i}(\vec{\sigma}\hat{r})j_0 \end{pmatrix}\chi_m^f \mathrm{d}_{m_2,f_2}^{d\,\dagger}(\vec{\tau}\cdot\vec{\tau})\mathrm{d}_{m_1,f_1}^d[\chi_{m_1}^{f_1\,\dagger}(\vec{\sigma}\hat{r})\chi_{m_2}^{f_2}]\mathrm{d}_{m,f}^{c\,\dagger}$$

$$+ \mathrm{i}g_\sigma\sigma_s(R)\begin{pmatrix} \mathrm{i}j_1 \\ -(\vec{\sigma}\hat{r})j_0 \end{pmatrix}\chi_m^f \mathrm{b}_{m_1,f_1}^{d\,\dagger}\mathrm{b}_{m_2,f_2}^d \mathrm{b}_{m,f}^c$$

$$+ \mathrm{i}g_\sigma\sigma_s(R)\begin{pmatrix} \mathrm{i}j_1 \\ -(\vec{\sigma}\hat{r})j_0 \end{pmatrix}\chi_m^f \mathrm{d}_{m_1,f_1}^{d\,\dagger}\mathrm{d}_{m_2,f_2}^d \mathrm{b}_{m,f}^c$$

$$- \mathrm{i}g_\sigma f_\pi\begin{pmatrix} \mathrm{i}j_1 \\ -(\vec{\sigma}\hat{r})j_0 \end{pmatrix}\chi_m^f \mathrm{b}_{m,f}^c$$

$$+ \, \mathrm{i} g_\sigma \sigma_s(R) \begin{pmatrix} \mathrm{i}(\vec{\sigma}\hat{r}) j_0 \\ -j_1 \end{pmatrix} \chi_m^f \mathrm{b}_{m_1,f_1}^{d\,\dagger} \mathrm{b}_{m_2,f_2}^{d} \mathrm{d}_{m,f}^{c\,\dagger}$$

$$+ \, \mathrm{i} g_\sigma \sigma_s(R) \begin{pmatrix} \mathrm{i}(\vec{\sigma}\hat{r}) j_0 \\ -j_1 \end{pmatrix} \chi_m^f \mathrm{d}_{m_1,f_1}^{d\,\dagger} \mathrm{d}_{m_2,f_2}^{d} \mathrm{d}_{m,f}^{c\,\dagger}$$

$$- \, \mathrm{i} g_\sigma f_\pi \begin{pmatrix} \mathrm{i}(\vec{\sigma}\hat{r}) j_0 \\ -j_1 \end{pmatrix} \chi_m^f \mathrm{d}_{m,f}^{c\,\dagger}$$

$$- \, g_{\pi/s} \pi_s(R) \begin{pmatrix} (\vec{\sigma}\hat{r}) j_0 \\ -\mathrm{i} j_1 \end{pmatrix} \chi_m^f \mathrm{b}_{m_1,f_1}^{d\,\dagger} (\vec{\tau}\cdot\vec{\tau}) \mathrm{d}_{m_2,f_2}^{d\,\dagger} \mathrm{b}_{m,f}^{c}$$

$$- \, g_{\pi/s} \pi_s(R) \begin{pmatrix} (\vec{\sigma}\hat{r}) j_0 \\ -\mathrm{i} j_1 \end{pmatrix} \chi_m^f \mathrm{d}_{m_1,f_1}^{d} (\vec{\tau}\cdot\vec{\tau}) \mathrm{b}_{m_2,f_2}^{d} \mathrm{b}_{m,f}^{c}$$

$$- \, g_{\pi/s} \pi_s(R) \begin{pmatrix} j_1 \\ -\mathrm{i}(\vec{\sigma}\hat{r}) j_0 \end{pmatrix} \chi_m^f \mathrm{b}_{m_1,f_1}^{d\,\dagger} (\vec{\tau}\cdot\vec{\tau}) \mathrm{d}_{m_2,f_2}^{d\,\dagger} \mathrm{d}_{m,f}^{c\,\dagger}$$

$$- \, g_{\pi/s} \pi_s(R) \begin{pmatrix} j_1 \\ -\mathrm{i}(\vec{\sigma}\hat{r}) j_0 \end{pmatrix} \chi_m^f \mathrm{d}_{m_1,f_1}^{d} (\vec{\tau}\cdot\vec{\tau}) \mathrm{b}_{m_2,f_2}^{d} \mathrm{d}_{m,f}^{c\,\dagger} + \ldots$$

$$+ \, g_{\eta/p} \eta_p(R) \begin{pmatrix} (\vec{\sigma}\hat{r}) j_0 \\ -\mathrm{i} j_1 \end{pmatrix} \chi_m^f \mathrm{b}_{m_1,f_1}^{d\,\dagger} \mathrm{b}_{m_2,f_2}^{d} [\chi_{m_1}^{f_1\,\dagger}(\vec{\sigma}\hat{r})\chi_{m_2}^{f_2}] \mathrm{b}_{m,f}^{c}$$

$$- \, g_{\eta/p} \eta_p(R) \begin{pmatrix} (\vec{\sigma}\hat{r}) j_0 \\ -\mathrm{i} j_1 \end{pmatrix} \chi_m^f \mathrm{d}_{m_2,f_2}^{d\,\dagger} \mathrm{d}_{m_1,f_1}^{d} [\chi_{m_1}^{f_1\,\dagger}(\vec{\sigma}\hat{r})\chi_{m_2}^{f_2}] \mathrm{b}_{m,f}^{c}$$

$$+ \, g_{\eta/p} \eta_p(R) \begin{pmatrix} j_1 \\ -\mathrm{i}(\vec{\sigma}\hat{r}) j_0 \end{pmatrix} \chi_m^f \mathrm{b}_{m_1,f_1}^{d\,\dagger} \mathrm{b}_{m_2,f_2}^{d} [\chi_{m_1}^{f_1\,\dagger}(\vec{\sigma}\hat{r})\chi_{m_2}^{f_2}] \mathrm{d}_{m,f}^{c\,\dagger}$$

$$- \, g_{\eta/p} \eta_p(R) \begin{pmatrix} j_1 \\ -\mathrm{i}(\vec{\sigma}\hat{r}) j_0 \end{pmatrix} \chi_m^f \mathrm{d}_{m_2,f_2}^{d\,\dagger} \mathrm{d}_{m_1,f_1}^{d} [\chi_{m_1}^{f_1\,\dagger}(\vec{\sigma}\hat{r})\chi_{m_2}^{f_2}] \mathrm{d}_{m,f}^{c\,\dagger}$$

$$+ \, \mathrm{i} g_s s_s(R) \begin{pmatrix} \mathrm{i} j_1 \\ -(\vec{\sigma}\hat{r}) j_0 \end{pmatrix} \chi_m^f \mathrm{b}_{m_1,f_1}^{d\,\dagger} (\vec{\tau}\cdot\vec{\tau}) \mathrm{b}_{m_2,f_2}^{d} \mathrm{b}_{m,f}^{c}$$

$$+ \, \mathrm{i} g_s s_s(R) \begin{pmatrix} \mathrm{i} j_1 \\ -(\vec{\sigma}\hat{r}) j_0 \end{pmatrix} \chi_m^f \mathrm{d}_{m_1,f_1}^{d\,\dagger} (\vec{\tau}\cdot\vec{\tau}) \mathrm{d}_{m_2,f_2}^{d} \mathrm{b}_{m,f}^{c}$$

$$+ \, \mathrm{i} g_s s_s(R) \begin{pmatrix} \mathrm{i}(\vec{\sigma}\hat{r}) j_0 \\ -j_1 \end{pmatrix} \chi_m^f \mathrm{b}_{m_1,f_1}^{d\,\dagger} (\vec{\tau}\cdot\vec{\tau}) \mathrm{b}_{m_2,f_2}^{d} \mathrm{d}_{m,f}^{c\,\dagger}$$

$$+ \, \mathrm{i} g_s s_s(R) \begin{pmatrix} \mathrm{i}(\vec{\sigma}\hat{r}) j_0 \\ -j_1 \end{pmatrix} \chi_m^f \mathrm{d}_{m_1,f_1}^{d\,\dagger} (\vec{\tau}\cdot\vec{\tau}) \mathrm{d}_{m_2,f_2}^{d} \mathrm{d}_{m,f}^{c\,\dagger}$$

$$+ \, g_{\eta/s} \eta_s(R) \begin{pmatrix} (\vec{\sigma}\hat{r}) j_0 \\ -\mathrm{i} j_1 \end{pmatrix} \chi_m^f \mathrm{b}_{m_1,f_1}^{d\,\dagger} \mathrm{d}_{m_2,f_2}^{d\,\dagger} \mathrm{b}_{m,f}^{c}$$

$$+ \, g_{\eta/s} \eta_s(R) \begin{pmatrix} (\vec{\sigma}\hat{r}) j_0 \\ -\mathrm{i} j_1 \end{pmatrix} \chi_m^f \mathrm{d}_{m_1,f_1}^{d} \mathrm{b}_{m_2,f_2}^{d} \mathrm{b}_{m,f}^{c}$$

$$+ g_{\eta/s}\eta_s(R) \begin{pmatrix} j_1 \\ -\mathrm{i}(\vec{\sigma}\hat{r})j_0 \end{pmatrix} \chi_m^f \mathrm{b}_{m_1,f_1}^{d\,\dagger} \mathrm{d}_{m_2,f_2}^{d\,\dagger} \mathrm{d}_{m,f}^{c\,\dagger}$$

$$+ g_{\eta/s}\eta_s(R) \begin{pmatrix} j_1 \\ -\mathrm{i}(\vec{\sigma}\hat{r})j_0 \end{pmatrix} \chi_m^f \mathrm{d}_{m_1,f_1}^{d} \mathrm{b}_{m_2,f_2}^{d} \mathrm{d}_{m,f}^{c\,\dagger} \,.$$

After sandwiching [12] one obtains

$$j_0(\omega)[g_\sigma(f_\pi - \sigma_s(R)) + 3g_s s_s(R)] - j_1(\omega)(1 - 3g_{\pi/p}\pi_p(R) + g_{\eta/p}\eta_p(R)) = 0,$$

$$j_0(\omega)(1 + 3g_{\pi/p}\pi_p(R) - g_{\eta/p}\eta_p(R)) - j_1(\omega)[g_\sigma(f_\pi - \sigma_s(R)) - 3g_s s_s(R)] = 0,$$

$$j_0(\omega) - j_1(\omega)(g_\sigma f_\pi + 3g_{\pi/s}\pi_s(R) - g_{\eta/s}\eta_s(R)) = 0,$$

$$j_0(\omega)(g_\sigma f_\pi - 3g_{\pi/s}\pi_s(R) + g_{\eta/s}\eta_s(R)) - j_1(\omega) = 0; \qquad R = R_{\mathrm{Bag}}\,.$$

$$(2.13)$$

In all the above relations flavour and angular-momentum dependent strong coupling constants $g_{\pi/p}$, $g_{\eta/p}$, $g_{\eta/s}$ etc. appear. This reflects chiral symmetry breaking, which appears naturally when the non-linear system (2.2) is solved using the Ansätze (2.8)-(2.10). In order to extract the equations for the $s-$ and $p-$ wave components from Eqs. (2.5), they are "sandwiched" between the final state $\langle f| = \langle q_{f,t}^c| = \langle 0|\mathrm{b}_{f,t}^c$ and initial state $|i\rangle = |q_{i,u}^c\rangle = \mathrm{b}_{i,u}^{c\dagger}|0\rangle$. This choice yields equations for σ, π, η and s fields

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}r^2} + \frac{2}{r}\frac{\mathrm{d}}{\mathrm{d}r}\right]\sigma_s(r) = f_\pi\lambda^2\left(f_\pi^2 - \nu^2\right) +$$
$$+ \lambda^2\left[\sigma_s(r) - f_\pi\right]\left[(\sigma_s(r) - f_\pi)^2 + 3\pi_p(r) + \eta_p^2 + 3s_s^2 - \nu^2\right],$$

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}r^2} + \frac{2}{r}\frac{\mathrm{d}}{\mathrm{d}r}\right]\pi_s(r) = \lambda^2\pi_s(r)\left[f_\pi^2 + 36\pi_s(r) + 12\eta_s^2 - \nu^2\right],$$

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}r^2} + \frac{2}{r}\frac{\mathrm{d}}{\mathrm{d}r} - \frac{2}{r^2}\right]\pi_p(r) = \lambda^2\pi_p\left[(\sigma_s(r) - f_\pi)^2 + 3\pi_p(r) + \eta_p^2 + 3s_s^2 - \nu^2\right],$$

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}r^2} + \frac{2}{r}\frac{\mathrm{d}}{\mathrm{d}r}\right]\eta_s(r) = \lambda^2\eta_s(r)\left[f_\pi^2 + 36\pi_s(r) + 12\eta_s^2 - \nu^2 + \mu^2\right], \qquad (2.14)$$

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}r^2} + \frac{2}{r}\frac{\mathrm{d}}{\mathrm{d}r} - \frac{2}{r^2}\right]\eta_p(r) = \lambda^2\eta_p\left[(\sigma_s(r) - f_\pi)^2 + 3\pi_p(r) + \eta_p^2 + 3s_s^2 - \nu^2 + \mu^2\right].$$

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}r^2} + \frac{2}{r}\frac{\mathrm{d}}{\mathrm{d}r}\right]s_s(r) = \lambda^2 s_s\left[(\sigma_s(r) - f_\pi)^2 + 3\pi_p(r) + \eta_p^2 + 3s_s^2 - \nu^2 + \mu^2\right]$$

The problem is to find a set of solutions of the differential equations (2.14), { σ_s, π_s, π_p, η_s, η_p, s_s }, which satisfy the *mathematical* boundary conditions (2.11). These

solutions must be compatible with (2.13) which is independent of r so one has a strongly correlated algebraic system (2.13) and the system of differential equations.

The parameters (λ, ν, μ) which enter $\mathcal{L}$ (2.2) are restricted by the symmetry - breaking behaviour of the theory. Usually [10, 23], the σ particle is considered to be a 1.2 GeV resonance, whereas the meson "masses" are parameter which, for simplicity (and lack of knowledge), are assigned the values of the physical masses. We have used rounded up values $m_\pi = 0.140 \,\text{GeV}$, $m_\eta = 0.980 \,\text{GeV}$ and $m_s = 0.980 \,\text{GeV}$. Here we have tentatively identified s meson with $a_0(980)$ $\left[I^G(J^{PC}) = 1^-(0^{++}) \right]$ and η with $\eta'(958)$. In the numerics we have also used an alternative value $m_\sigma = 0.450 \,\text{GeV}$ [24]. In the present application, these "experimental" values have been used, although m_σ, m_π etc. can, in principle, be considered as additional parameters.

The usage of the bag-model has to some extent decoupled the equation for the quark expansion functions ϕ_m^f, $\tilde{\phi}_m^f$ etc. (2.4) from the rest. It communicates with the $\pi_n(r), \eta_n(r)$ (n=s, p) and $\sigma(r), s(r)$ functions only through algebraic relations (2.13).

Higher order terms in the expansion, such as the third term in (2.4) for example, would enlarge the system of the coupled equations. As in TDM the whole system would be coupled sector by sector. That would be governed by the number of creation (annihilation) operators and by some additional $|i\rangle$ ($\langle f|$) states besides (2.6) ones. The end results would be analogous to the relations among different sectors in the Fock space in TDM, as one should expect from its reversed picture.

3. Coupling constants and magnetic moment

The results obtained in the leading order of TDIA are used to calculate the proton magnetic moment and the axial vector coupling constants.

The magnetic moment operator is

$$\vec{\mu}(\vec{r}) = \frac{1}{2}(\vec{r} \times \vec{j}_{EM}(\vec{r})). \tag{3.1}$$

Here

$$j^\mu_{EM}(r) = \overline{\psi}(r)\gamma^\mu Q\psi(r) + \epsilon_{3ij}\pi_i(r)\partial^\mu \pi_j(r) + \epsilon_{3ij}s_i(r)\partial^\mu s_j(r), \tag{3.2}$$

$$Q = \frac{2}{3} \cdot \frac{1+\tau_3}{2} - \frac{1}{3} \cdot \frac{1-\tau_3}{2}. \tag{3.3}$$

The quark contribution to the magnetic moment is

$$\mu^{(q)} = \frac{2}{3} \cdot \frac{R}{\omega^4} \cdot \frac{(\omega/2) - (3/8)\sin 2\omega + (\omega/4)\cos 2\omega}{j_0^2(\omega) + j_1^2(\omega) - 2j_0(\omega)j_1(\omega)/\omega}. \tag{3.4}$$

The meson contribution is

$$\mu_p^{(M)} = \frac{16\pi}{3} \cdot \frac{11}{3} \int\limits_{R_{\mathrm{bag}}}^{\infty} r^2 \, \mathrm{d}r \left(\pi_p(r)^2 + s_s(r)^2 \right). \tag{3.5}$$

The proton magnetic moment is given by

$$\mu_p = \mu^{(q)} + \mu_p^{(M)}. \tag{3.6}$$

The axial vector current

$$J_A^\mu = \overline{\psi} \gamma^\mu \gamma_5 \frac{\vec{\tau}}{2} \psi + \sigma \partial^\mu \vec{\pi} - \vec{\pi} \partial^\mu \sigma + \eta \partial^\mu \vec{s} - \vec{s} \partial^\mu \eta \tag{3.7}$$

leads to the quark contribution:

$$\begin{aligned} g_A^{(q)} &= \langle p \uparrow | \int \mathrm{d}^3 \vec{r} \overline{\psi}(\vec{r}) \gamma^3 \gamma^5 \frac{\tau^3}{2} \psi(\vec{r}) | n \uparrow \rangle \\ &= \frac{5}{3} \cdot \frac{1}{3} \cdot \frac{j_0^2(\omega) + j_1^2(\omega)}{j_0^2(\omega) + j_1^2(\omega) - 2 j_0(\omega) j_1(\omega)/\omega} \end{aligned} \tag{3.8}$$

and to the meson contribution:

$$g_A^{(M)} = \frac{5}{3} \cdot \frac{4\pi}{3} \cdot \int\limits_{R_{\mathrm{bag}}}^{\infty} \mathrm{d}r \, r^2 \left[(\sigma_s(r) - f_\pi) \left[\pi_p'(r) + \frac{2\pi_p(r)}{r} \right] - \pi_p(r) \sigma_s'(r) \right]. \tag{3.9}$$

Finally,

$$g_A^{(p)} = g_A^{(q)} + g_A^{(M)}. \tag{3.10}$$

The isoscalar axial vector current

$$J_A^{0\,\mu} = \overline{\psi} \gamma^\mu \gamma_5 \psi + \sigma \partial^\mu \eta - \eta \partial^\mu \sigma + \vec{s} \partial^\mu \vec{\pi} - \vec{\pi} \partial^\mu \vec{s} \tag{3.11}$$

leads to quark contribution to the isoscalar coupling constant

$$g_A^{0\,(q)} = \frac{1}{3} \cdot \frac{j_0^2(\omega) + j_1^2(\omega)}{j_0^2(\omega) + j_1^2(\omega) - 2 j_0(\omega) j_1(\omega)/\omega} \,. \tag{3.12}$$

The corresponding meson contribution is

$$\begin{aligned} g_A^{0\,(M)} &= \frac{4\pi}{3} \cdot \int\limits_{R_{\mathrm{bag}}}^{\infty} \mathrm{d}r \, r^2 \left[\eta_p(r) \sigma_s'(r) - (\sigma_s(r) - f_\pi) \left(\eta_p'(r) + \frac{2\eta_p(r)}{r} \right) + \right. \\ &\quad \left. + 5 s_s \left(\pi_p'(r) + \frac{2\pi_p(r)}{r} \right) - 5 \pi_p(r) s_s'(r) \right]. \end{aligned} \tag{3.13}$$

Finally,

$$g_A^{0\ (p)} = g_A^{0\ (q)} + g_A^{0\ (M)}. \tag{3.14}$$

4. Numerical procedure

Numerics will be illustrated here for a non-linear system of coupled ordinary differential equations which have been derived in Sect. 2.

This system determines fermion and boson radial functions appearing in the Ansätze, (2.8) and (2.10). The boson radial functions had to satify Eqs. (2.11), (2.13) and (2.14).

In (2.13) the normalization constant N can be expressed in terms of spherical Bessel functions and quark eigenfrequencies ω

$$N^2 = \frac{1}{R^3} \left[j_0^2(\omega) + j_1^2(\omega) - \frac{2j_0(\omega)j_1(\omega)}{\omega} \right]^{-1}. \tag{4.1}$$

The radial parts of the quark wave functions appearing in (2.8) are spherical Bessel functions $j_\ell(\omega r/R)$ for any spherical bag with radius R. At the bag boundary, where $r = R$, these functions have to satisfy the relations (2.13) which combine the quark frequency ω with the coupling constants g_σ, g_π, f_π etc.

The linear σ-model parameters satisfy the following relations derived from the symmetry-breaking pattern (see Sect. 2) [10]

$$\lambda^2 = \frac{m_\sigma^2 - m_\pi^2}{2f_\pi^2}, \qquad \nu^2 = f_\pi^2 - \frac{m_\pi^2}{\lambda^2}, \qquad \mu = 2f_\pi^2 \frac{m_\eta^2 - 3m_\pi^2}{m_\sigma^2 - m_\pi^2}. \tag{4.2}$$

The σ meson is expected to have a mass of about 1 GeV [23]. Thus the *parameter* masses m_σ, m_π, m_η and m_s are selected to be 1.2 GeV (0.450 Gev), 0.140 GeV and 0.980 GeV, respectively.

One has to solve simultaneously the system containing non-linear differential equations (2.11) and (2.14) together with the boundary condition

$$
\begin{aligned}
\sigma_s(r)\Big|_{r\to\infty} = 0, \qquad \pi_s(r)\Big|_{r\to\infty} = 0, \qquad \pi_p(r)\Big|_{r\to\infty} = 0, \\
\eta_s(r)\Big|_{r\to\infty} = 0, \qquad \eta_p(r)\Big|_{r\to\infty} = 0, \qquad s_s(r)\Big|_{r\to\infty} = 0
\end{aligned}
\tag{4.3}
$$

and with the algebraic relations (4.2) This determines the meson functions $\sigma(r)$, $\pi_s(r)$, $\pi_p(r)$, $\eta_s(r)$, $\eta_p(r)$ and $s_s(r)$ the quark frequency ω and various coupling (g_π, g_σ, etc.).

This complex system has been solved using the code COLSYS, the **col**location **sys**tem **s**olver, developed by U. Asher, J. Christiansen and R.D. Russel from the *University of British Columbia* and *Simon Fraser University, Canada* [25]. The

boundary conditions are set at $R \gg R_{\text{bag}}$, where R is set to be so large that the fields can be approximated by zero at R. The initial guesses have been supplied. From the asymptotic behaviour and some earlier experience the input was rather simple and convergence has been achieved quickly.

The problem turns out to be rather sensitive to the derivative boundary conditions which in all cases involve the coupling constant(s). Although the asymptotic behaviour of the solutions can be inferred from the system itself (see also [26]), the COLSYS is able to handle rather general initial (guess) solutions.

Upon return the routine gives error estimates for components and its derivatives. The problem parameters can be gradually changed (increased) by using a continuation method in COLSYS which is left to choose the initial mesh points, and in the continuation procedure it refines and redistributes the (former) mesh.

The solutions are compared against the consistency conditions (2.13) and the iterative procedure is continued until the matching is obtained. The iteration consists in performing a self-consistent calculation: the coupling constants g_σ, g_π, g_η and g_s for the chiral quarks are set to be the same at the beginning (their value is set to be equal to 10.00). After every iteration the system (2.13) is checked numerically and iteration is preformed until proscribed tolerance is achived. These new values are replaced in the boundary conditions to calculate new solutions. The procedure converges rather rapidly. When the matching is achieved, the magnetic moment, the axial constant and the *physical* pion mass are calculated from the obtained solutions.

5. Results

Physically acceptable model values for the axial vector constants g_A^3 and g_A^0 and the proton magnetic moment μ_p are shown in Tables 1 and 2. Other Tables, i.e. 3, 4, 5, and 6 show how the results depend on the bag radius R and on the quark eigenfunction ω. As explained in Sect. 2 and 4, the strong coupling constants g_σ, $g_{\pi/s}$, $g_{\pi/p}$, $g_{\eta/s}$, $g_{\eta/p}$ and $g_{s/s}$ were adjusted by a self consistent procedure.

As can be seen from Tables 3, 4, 5 and 6 model is weakly sensitive on m_σ and somewhat more sensitive on R and ω. For the resonable value [11] $R = 5$ GeV^{-1} meson components allows solutions with $\omega < \omega_{\text{bag}}$ ($\omega_{\text{bag}} = 2.04$) for which one obtain a resonable value (Tables 1 and 2)

$$g_A^3 = 1.24 \qquad (\omega = 1.95)\,, \tag{5.1}$$

which is whithin 1.6% from experimental value $g_A^3(exp) = 1.26$. For the same parameters one finds

$$\mu_p = 2.17 \qquad (\omega = 1.95)\,, \tag{5.2}$$

which is 25% smaller than the experimental value $\mu_p(exp) = 2.79$. In both cases (5.1) and (5.2) the contribution of mesonic phase was important, being

$$g_A^3(M) = 9\%\,,$$

$$\mu_p(M) = 16\%\,,$$

The values (5.1) and (5.2) should be compared with the MIT-bag model values $g_A^3 = 1.11$ and $\mu_p = 1.88$. Obviously the chiral bag model works better.

TABLE 1. The results for various ω with $R = 5.0$ GeV^{-1} and $m_\sigma = 1.2$ GeV.

ω	g_σ	g_{π_s}	g_{π_p}	g_{η_s}	g_{η_p}	g_s	g_A^3	g_A^0	μ_p
1.95	10.799	± 4.672	4.659	2.973	2.954	1.958	**1.24**	**0.663**	**2.17**
1.97	10.781	± 4.076	4.154	2.364	2.360	1.579	**1.22**	**0.661**	**2.11**
2.00	10.763	± 3.035	3.218	1.418	1.425	0.969	**1.18**	**0.658**	**2.02**

TABLE 2. Quark and meson contributions. All parameters are as in Table 1.

ω	$g_A^3(q)$	$g_A^3(M)$	g_A^3	$g_A^0(q)$	$g_A^0(M)$	g_A^0	$\mu_p(q)$	$\mu_p(M)$	μ_p
1.95	1.14	0.10	**1.24**	0.681	-0.018	**0.663**	1.87	0.30	**2.17**
1.97	1.12	0.10	**1.22**	0.675	-0.014	**0.661**	1.88	0.23	**2.11**
2.00	1.11	0.07	**1.18**	0.666	-0.008	**0.658**	1.88	0.14	**2.02**

TABLE 3. The results for various R with $\omega = 2.0$ and $m_\sigma = 1.2$ GeV.

R	g_σ	g_{π_s}	g_{π_p}	g_{η_s}	g_{η_p}	g_s	g_A^3	g_A^0	μ_p
4	10.763	± 2.375	2.524	1.045	1.052	0.709	**1.17**	**0.659**	**1.64**
5	10.763	± 3.035	3.218	1.418	1.425	0.969	**1.18**	**0.658**	**2.02**
6	10.763	± 3.727	3.943	1.877	1.836	1.253	**1.20**	**0.657**	**2.40**
7	10.763	± 4.452	4.698	2.270	2.279	1.560	**1.21**	**0.656**	**2.78**

TABLE 4. Quark and meson contributions. All parameters are as in Table 1.

R	$g_A^3(q)$	$g_A^3(M)$	g_A^3	$g_A^0(q)$	$g_A^0(M)$	g_A^0	$\mu_p(q)$	$\mu_p(M)$	μ_p
4	1.11	0.06	**1.17**	0.666	-0.007	**0.659**	1.51	0.13	**1.64**
5	1.11	0.07	**1.18**	0.666	-0.008	**0.658**	1.88	0.14	**2.02**
6	1.11	0.09	**1.20**	0.666	-0.009	**0.657**	2.26	0.14	**2.40**
7	1.11	0.10	**1.20**	0.666	-0.010	**0.656**	2.64	0.14	**2.78**

TABLE 5. The results for $R = 6$GeV^{-1} and $\omega = 2.0$.

m_σ	g_σ	g_{π_s}	g_{π_p}	g_{η_s}	g_{η_p}	g_s	g_A^3	g_A^0	μ_p
1.20	10.763	± 3.727	3.943	1.827	1.836	1.253	**1.20**	**0.657**	**2.40**
0.45	10.763	± 3.509	3.944	1.933	1.953	1.337	**1.20**	**0.657**	**2.40**

TABLE 6. *Quark and meson contributions. All parameters are as in Table 5.*

m_σ	$g_A^3(q)$	$g_A^3(M)$	g_A^3	$g_A^0(q)$	$g_A^0(M)$	g_A^0	$\mu_p(q)$	$\mu_p(M)$	μ_p
1.20	1.11	0.09	**1.20**	0.666	-0.009	**0.657**	2.26	0.14	**2.40**
0.45	1.11	0.09	**1.20**	0.666	-0.009	**0.657**	2.26	0.14	**2.40**

The isoscalar coupling constant g_A^0, whose bag model value is $g_A^0 = 0.666$ is slightly changed by the meson phase. In Table 2 one can find

$$g_A^0 = 0.663 \qquad (\omega = 1.95)\,, \tag{5.3}$$

what seems to be to large by far. One can connect [12] g_A^3 and g_A^0 with the quark densities [27–34]

$$\triangle u = 0.78 \pm 0.06\,,$$
$$\triangle d = -0.48 \pm 0.06\,,$$
$$\triangle s = -0.14 \pm 0.07\,.$$

In our SU(2)×SU(2) model $\triangle s$ can be disregarded and one can identify

$$g_A^3 = \triangle u - \triangle d \cong 1.26\,,$$
$$g_A^0 = \triangle u + \triangle d \cong 0.30\,.$$

Obviously the theoretical explanation for the $g_A^0(exp) \cong 0.30$ must be searched in the enlarged SU(3)×SU(3) chiral quark model.

It is useful to note that enlargement of the model from SU(2) [12] to SU(2) × SU(2) did not change the predicted values [12] for g_A^3 and μ_p significantly. One is tempted to conclude that the low energy strong dynamics can be to a large extent, mimicked by the effective pseudoscalar (i.e. pion) meson fields. Probably that explains why $a_0(980)$ and similar mesons were not very much noticed in the early scattering experiments [3].

References

[1] I. Tamm, J. Phys. (Moscow) **9** (1945) 449.

[2] S. M. Dancoff, Phys. Rev. **78** (1950) 382.

[3] H. A. Bethe and F. de Hoffmann, *Mesons and Fields*, Row, Peterson and Company, Evanston, Il. (1955) Vol. II.

[4] R. J. Perry, A. Harindranath and K. G. Wilson, Phys. Rev. Lett. **65** (1990) 2959.

[5] P. M. Wort, Phys. Rev. D **47** (1993) 608.

[6] S. Glazek et al., Phys. Rev. D **47** (1993) 1599.

[7] K. Harada et al., Phys. Rev. D **54** (1996) 7656.

[8] K. Harada et al., Phys. Rev. D **52** (1995) 2429

[9] E. P. Wigner, Phys. Rev. **94** (1954) 77.

[10] M. C. Birse and M. K. Banerjee, Phys. Lett. B **136** (1984) 284; Phys. Rev. D **31** (1985) 118; M. C. Birse, Phys. Rev D **33** (1986) 1934.

[11] A. W. Thomas, Adv. Nucl. Physics **13**, 1, ed. J. W. Negele and E. Vogt, Plenum (1984); G. E. Brown and M. Rho, Phys. Lett. B **82** (1979) 177; G. E. Brown, M. Rho and V. Vento, Phys. Lett. B **84** (1979) 383; H. Hogaasen and F. Myhrer, Z. Phys. C - Particles and Fields **21** (1983) 73; Xue-Qian Li and Zhen Qi, Commun. Theor. Phys. **18** (1992) 213; G. E. Brown and M. Rho, Comments on Nucl. Part. Phys. **18** (1988) 1; F. Myhrer, in *Quarks and Nuclei*, Int. Rev. Nucl. Phys. **1** (1984), ed. W. Weise, World Scientific, Singapore (1984); A. Hosaka and H. Toki, Phys. Rep. **277** (1996) 65.

[12] D. Horvat, B. Podobnik and D. Tadić, Phys. Rev. D **58** (1998) 034003-1; Fizika B (Zagreb) **7** (1998) 127.

[13] H-Q Zheng, Commun. Theor. Phys. **23** (1995) 465.

[14] M. Lévi, Nuovo Cimento **52 A** (1967) 23.

[15] S. L. Adler and R. F. Dashen, *Current Algebras*, W. A. Benjamin, Inc., New York, Amsterdam (1968).

[16] J. A. McGovern and M. C. Birse, Nucl. Phys. A **506** (1990) 367; A **506** (1990) 393.

[17] N. A. Törnqvist, Phys. Lett. B **426** (1998) 105; Eur. Phys. J. C **11** (1999) 359; hep-ph/9910443 (1999).

[18] J. Schaffner-Bielich and J. Randrup, Phys. Rev. C **59** (1999) 3329.

[19] J. T. Lengahan, D. H. Rischke and J. Schaffner-Bielich, nucl-th/0004006 (2000).

[20] G. Källen, *Quantum Electrodynamics*, Springer-Verlag, New York-Heidelberg-Berlin (1972).

[21] A. Chodos and B. C. Thorn, Phys. Rev D **12** (1975) 2733.

[22] M. C. Birse, Prog. Part. Nucl. Phys. **25**, 1, ed. A. Faessler, Pergamon, Oxford (1990).

[23] J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields*, McGraw-Hill, New York (1964).

[24] T. Hannah, Phys. Rev. D **60** (1999) 017502.

[25] U. Ascher, J. Christiansen and R. D. Russel, Math. Comp. **33** (1979) 659; A. C. M. Trans. Math. Software **7** (1981) 209; SIAM Review **23** (1981) 238.

[26] L. R. Dodd and M. A. Lohe, Phys. Rev D **32** (1985) 1816.

[27] F. E. Close, Preprint RAL-93-034 (1993).

[28] S. D. Bass and A. W. Thomas, Cavendish Preprint HEP 93/4, October 1993.

[29] J. Ellis and M. Karliner, CERN Preprint CERN-TH.7022/93, September 93.

[30] C. H. Chang and C. S. Huang, Commun. Theor. Phys. **19** (1993) 97.

[31] B. Frois and M. Karliner, Phys. World **44**, July 1994.

[32] R. G. Roberts, *The Structure of The Proton*, Cambridge Univ. Press, Cambridge (1990).

[33] Spin Muon Collaboration, Phys. Lett. B **420** (1998) 1.

[34] J.-P. Lin, Comm. Theor. Phys. **26** (1996) 85.

PROŠIREN MODEL KIRALNIH KVARKOVA POTAKNUT
TAMM-DANCOFFOVIM PRIBLIŽENJEM

Postupak potaknut Tamm-Dancoffovom metodom primijenili smo na kiralni kvarkovski model koji smo proširili uključenjem dodatnih stupnjeva slobode: pseudoskalarnog izoskalarnog polja i tripleta skalarno izovektorskog polja. Jednostavniji, tvorbeni σ-model se ranije primjenjivao radi ispitivanja Tamm-Dancoffom potaknutog približenja (TDIA). Ovdje se uzima kiralni kvarkovski model radi ispitivanja mogućih novih učinaka dodatnih stupnjeva slobode i da se pokaže nužnost uvođenja SU(3)-okusa. Predviđanja modela s TDIA za aksijalno-vektorsku konstantu vezanja i za nuklearni magnetski moment uspoređuju se s eksperimentalnim vrijednostima.