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Lepton flavor violation in the standard model extended by heavy singlet Dirac neutrinos

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Low-energy neutrinoless lepton flavor-violating (LFV) processes are studied in an extension of the standard model (SM) by heavy SU(2)×U(1) singlet Dirac neutrinos. An upper-bound procedure is elaborated for the evaluation of amplitudes. A comment on the extraction of heavy-neutrino mixings from astrophysical observations is given. For processes not treated in the model applied, the formalism for evaluating the branching ratios (BR's) is presented. The processes previously studied in the model are examined and some results are improved. The structure of the amplitudes and BR's as well as the relations between BR's of different LFV processes are examined. The decoupling of heavy neutrinos is discussed and it is explicitly shown that very heavy neutrinos decouple when the upper-bound procedure is applied. The LFV decays are shown to be unsuitable for finding upper bounds on ''diagonal'' LFV parameters. Comparing the theoretical BR's with curent experimental upper bounds, a few processes interesting for the search for LFV are proposed. Particularly, *B*-meson LFV processes are suggested for the search of LFV in future *B* factories.

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I. INTRODUCTION

When instanton effects [1] are neglected, the lepton flavor and lepton number are both conserved in the standard model (SM). Recently found atmospheric neutrino oscillations [2] indicate that neutrino masses are nondegenerate and the lepton flavor is not conserved. An independent confirmation of the deviation from the SM is expected to manifest itself as nonconservation of lepton flavor (LFV), nonconservation of lepton number (LNV), as a breaking of lepton universality, in CP-violating processes which are not consistent with SM, etc. The problem of LFV and LNV is related to the physics beyond SM and affects various areas of physics [3]: atomic physics (e.g., muonium-antimuonium conversion), nuclear physics ($\mu \to e$ conversion, double-beta decay), low-energy hadron physics (leptonic and semileptonic decays of mesons and leptons), the problem of CP violation, etc.

LFV has been found in various extensions of the SM [3–6]. Here, LFV is studied within one of the two extensions of the SM by heavy neutrinos with large heavy-neutrino–light-neutrino mixings [7,8], obtained by adding heavy Dirac neutrinos to it. It is referred to here as the V model [8]. Because of the Dirac character of the heavy neutrinos, there are no LNV processes in this model. The other model [7], obtained by extending the SM with additional heavy Majorana neutrinos, has some renormalization problems and light-neutrino mass problems [9]. In addition to the additional heavy Dirac neutrinos, the V model contains three massless neutrinos. It should be noted that in this work the V model is used phenomenologically. Any model with the same gauge properties and about equally large heavy-neutrino masses would give the same results, regardless of whether the light neutrinos are massless or have masses which agree with the present experimental data.

The extensions of the SM by heavy neutrinos contain a Cabibbo-Kobayashi-Maskawa-type matrix for leptons (LCKM). In general, the elements of this matrix are not known. Experimental and theoretical constraints exist only for some specific sums of the matrix elements of the heavy neutrino part of the matrix. Therefore, the LFV amplitudes cannot be evaluated exactly, but only the upper bounds on their values may be found [10]. The evaluation is especially complicated when the amplitudes contain expressions with more than two LCKM matrix elements. In this paper a method for improved evaluation of the upper bounds of amplitudes found in the previous publication [10] is presented. The method gives upper bounds for all values of the model parameters, but, in some directions of the parameter space, it is not very restrictive. It is explicitly shown that the upper-bound procedure leads to the decoupling of the heavy neutrinos in the infinite-mass limit, showing that the ''nondecoupling'' of heavy neutrinos [11,12] is only a transient effect, appearing with an enlargement of the heavy-neutrino mass. It should be noted that this ''proof'' of generalization of the Appelquist-Carrazone theorem is based only on the requirement that the physical system can be described pertubatively, and is independent of the introduction of somewhat undetermined maximal $SU(2)_L$-doublet mass term as in Ref. [12]. To give the feeling of how large an error can be introduced using the upper-bound procedure elaborated here, a few branching ratios obtained by the upper-bound procedure are compared with BR's obtained using ''realistic'' LCKM matrices.

The LFV processes are not very usefull for deriving upper bounds on the matrix elements of LCKM matrix. The amplitudes for these processes are proportional to the sums of products of the LCKM matrix elements and functions of heavy-neutrino masses. Using the freedom to choose unknown phases of the LCKM matrix and heavy-neutrino masses, these sums can always be set to be equal to zero, even if the absolute values of nondiagonal elements of the LCKM matrix are different than zero. The present limits on the LCKM matrix elements are derived from the measurements of lepton-flavor-conserving processes [13], more precisely, from the estimates of deviations of the corresponding

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decay rates from the SM results. For each row (a row corresponds to a specific lepton l) of the LCKM matrix, these data give a limit on the sum of squares of absolute values of the matrix elements corresponding to the heavy neutrinos $(s_L^{\nu_l})^2$. Knowing the upper bounds on $(s_L^{\nu_l})^2$'s one may derive the upper bound for BR of any LFV process. One of the aims of this paper is to derive the upper bounds of BR's for all low-energy LFV processes in the V model. The processes having comparable theoretical and experimental upper bounds of the BR, or a theoretical upper bound larger than the experimental one, are interesting for further experimental investigation.

Neutrino oscillations of two massless neutrinos in supernovae have been shown to give a very strong upper bound of two of the LCKM matrix elements in the part of the matrix corresponding to the massless neutrinos [14]. Here, the analysis has been repeated for three neutrinos, hoping that the upper bounds for other ''massless-neutrino'' LCKM matrix elements may be derived. The knowledge of nondiagonal ''massless-neutrino'' LCKM matrix elements may, in principle, lead to better upper bounds on some combinations of ''heavy-neutrino'' LCKM matrix elements than those obtained from the terrestrial experiments. Unfortunately, the analysis made here shows that the three-neutrino oscillations do not give new constraints on any combination of ''heavy neutrino'' LCKM matrix elements. It only shows that the mixing between massless ''mu'' and ''tau'' neutrinos is smaller than the value obtained from the analysis of Super-Kamiokande data [2], in which ''mu'' and ''tau'' neutrinos were assumed to have small masses.

Until now, many of the low-energy neutrinoless LFV processes have been investigated. Some of them have been examined only within a few models, for instance, LFV decays of heavy mesons. The neutrinoless LFV decays of B mesons have been studied in the frame of SM with an additional Higgs doublet [15], while the neutrinoless LFV decays of D mesons have been studied in the frame of leptoquark models [16] and a flipped left-right symmetric model [17]. Here, they are analyzed in the V model. Among the low-energy LFV processes that have not been studied in the frame of the V model are also the muonium-antimuonium ($M \leftrightarrow \bar{M}$) conversion and neutrinoless LFV-violating decays of the Z boson. The results are given here. Some of the neutrinoless LFV processes have been analyzed in the V model, but the analysis is incomplete [18] or there are some errors in the expressions for amplitudes or decay rates [10–12]. Here, only the corrections to the previous results are given.

On the quark and lepton level there are only a few Feynman diagrams (composite-loop functions) that contribute to any neutrinoless LFV decay amplitude. If two neutrinoless LFV processes contain only one common composite-loop function, the ratio of corresponding BR's is independent of the V-model parameters. Therefore, roughly speaking, knowing one BR, the BR's of processes comprising the same basic Feynman diagram may be evaluated without the knowledge of parameters of the V model. If LFV decay amplitudes contain different loop functions, or more loop functions, the ratio of the BR's depends on V-model parameters. Nevertheless, the mass dependence of the ratio of the BR's simplifies in the limit of large heavy-neutrino masses. Most of the amplitudes become dependent essentially only on one of the composite-loop functions. In that limit, the ratios of the BR's having the same dominant composite-loop function become independent of the V-model parameters. Experimentally, for most neutrinoless LFV processes, only the large heavy-neutrino-mass limit is interesting, because, with few exceptions, only in that limit do BR's assume the values comparable with the present-day experimental limits. A comparitive analysis of the amplitudes and BR's of all neutrinoless LFV processes is presented.

In Sec. II some properties of the V model, relevant for further discussion, are given. A discussion on the limits of the model parameters is given in Sec. III. The amplitudes for the neutrinoless LFV processes not studied in the V model, and some improvements and corrections of the previous results are presented in Sec. IV. The amplitudes and BR's of LFV processes are studied in Sec. V. The numerical results and comparison with experimental limits are also given in Sec. V. The conclusions are summarized in Sec. VI. Appendix comprises the form factors and phase functions relevant for heavy-baryon LFV decays.

II. COMMENTS ON THE MODEL

Here, a model with additional $SU_L(2)\times U(1)$ singlet Dirac neutrinos, which have large mixings with the SM leptons, is used in the calculations. The masses of the singlet neutrinos are not restricted by the $SU_L(2)\times U(1)$-breaking scale. The large mixings and the large masses are the necessary conditions for obtaining observable LFV decay rates.

In the V model considered here [8,19–22], the total lepton number (L) is conserved. For each of n_G SM neutrinos one left-handed and one right-handed singlet neutrino is added. The structure of the mass matrix permits a modification of the V model obtained by adding arbitrary number of pairs (n_R) of left-handed and right-handed neutrinos (Vn_R models). Lepton-number conservation gives a structure to the mass matrix which automatically leads to three massless neutrinos at any order of the perturbation theory [19].

Since the new neutrinos are $SU(2)_L\times U(1)$ singlets, the structure of the lepton interaction vertices in the weak basis is the same as in the SM [19]. However, in a transition to the mass basis, nondegeneracy of the neutrinos leads to the Cabibbo-Kobayashi-Maskawa (CKM) type matrix (B_{ln}) in the charged current (CC) nlW vertices. As only a part of the mass-basis neutrinos interact with the Z boson, neutral current (NC) nnZ vertices (n is neutrino field in the mass basis) are also not flavor diagonal, and contain matrix elements of the nondiagonal matrix (C_{nn}). The NC llZ vertices and the quark vertices are the same as in SM.

The C matrix from the neutrino NC vertex may be expressed in terms of B matrices from the CC lepton vertex. Therefore, beside the SM parameters, the model depends only on the B matrix (or more precisely on the parameters defining the B matrix) and on heavy-neutrino masses. The matrices B and C satisfy a set of relations stemming from the gauge structure (see, e.g., Ref. [11]):

$$\sum_{k=1}^{n_G+n_R} B_{l_1k}B^*_{l_2k}=\delta_{l_1l_2}, \quad \sum_{k=1}^{n_G+n_R} C_{ik}C^*_{jk}=C_{ij},$$

$$\sum_{k=1}^{n_G+n_R} B_{lk}C_{ki}=B_{li}, \quad \sum_{l=1}^{n_G} B^*_{li}B_{lj}=C_{ij}. \tag{1}$$

From the orthogonality relations for B_{ln} matrix elements, phase arbitrariness of leptons and SU(n_R) invariance of massless neutrinos lead to $n_G n_R$ independent angles and $(n_G-1)(n_R-1)$ independent phases of the B matrix [23,24]. Experimentally, only n_G parameters $s_L^{\nu_l}$ may be estimated. Therefore, the B matrix elements are undetermined even for the simplest case with two additional heavy neutrinos, $n_R=2$. Since the B matrix elements are unknown, the amplitudes of LFV processes cannot be evaluated exactly, but only upper bounds of the amplitudes may be found. One should mention that there exists a model with additional heavy Majorana neutrinos for which amplitudes of LFV processes can be evaluated exactly, in the case of $n_R=2$ [7]. Unfortunately, as mentioned before, it is excluded because of some renormalization and light-neutrino-mass problems.

The degeneracy of massless neutrinos allows one to write the B matrix in the following form [6,19,25]:

$$B_{ln_k}=[(UD_A)_{l\nu_i},(UG)_{lN_I}], \quad k=(i,I), \tag{2}$$

where U is a unitary matrix, D_A is a diagonal matrix, and G is a matrix satisfying $D_A^2+GG^\dagger=1$. Indices i and I denote massless (ν) and massive (N) neutrinos, respectively. From the structure of the B matrix, it follows that the massless-neutrino CC in principle is not diagonal, leading to LFV [19,25,26] and nonortogonal effective weak-neutrino states [26], although neutrinos are massless. On the other hand, the massless-neutrino NC, which contains the C matrix elements, is diagonal [19]. Since there are no tree-level flavor-violating neutral currents (FCNCs) in the massless-neutrino sector, the universality of massless-neutrino couplings is not satisfied, because, in general, the elements of the diagonal matrix D_A are not equal. The nonuniversality of these couplings may have some astrophysical implications.

As mentioned in the Introduction, the B matrices are used to define the parameters $s_L^{\nu_l}$, which are a measure of the deviation from SM, in the following way [12,27–30]:

$$(s_L^{\nu_l})^2=\sum_{i=1}^{n_R} B_{lN_i}B^*_{lN_i}. \tag{3}$$

Because the definition of $(s_L^{\nu_l})^2$ contains B_{lN} matrix elements of the same lepton flavor, the term ''diagonal'' mixing(s) will be used sometimes in the text below.

III. LIMITS ON THE MODEL PARAMETERS AND METHODS OF EVALUATION OF AMPLITUDES

A. Experimental limits

The parameters $(s_L^{\nu_l})^2$ have been determined from the global analysis of the low-energy tree level processes [13,27–30]. In these processes, heavy neutrinos may manifest themselves only indirectly, through a change of the light-(massless-) neutrino couplings. These couplings attain additional $c_L^{\nu_l}$ factors, where $(c_L^{\nu_l})^2\equiv 1-(s_L^{\nu_l})^2=\Sigma_{i=1}^{n_G}B_{l\nu_i}B^*_{l\nu_i}$. The changes of the couplings could show up as a nonuniversality of CC couplings, as a deviation from unitarity of the CKM matrix, as a change of the invisible width of the Z boson, etc. [13,28]. The best limits on the mixings $s_L^{\nu_l}$,

$$(s_L^{\nu_e})^2<0.0071, \quad (s_L^{\nu_\mu})^2<0.0014, \quad (s_L^{\nu_\tau})^2<0.033(0.01), \tag{4}$$

were found in Ref. [13]. The value in the brackets is valid for $SU(2)_L\times U(1)$ singlet heavy neutrinos.

More stringent limits on the B_{lN} matrix elements have been searched for investigating the loop effects in the lepton-conserving and lepton-violating processes. Direct limits on the parameters $s_L^{\nu_l}$ are not possible as the expressions derived from the loop amplitudes, which are constrained by experimental data, depend not only on the $s_L^{\nu_l}$ parameters but also on the B_{lN} phases and masses of heavy neutrinos. Lepton-conserving processes, including heavy neutrinos in loops, have been studied by Kalyniak and Melo [31,32]. They studied the loop effects of heavy Dirac neutrinos on muon decay, universality-breaking ratio in $Z\to l\bar{l}$ decays and Δr quantity. They found no new constraints on the $s_L^{\nu_l}$ parameters. The flavor-nondiagonal (LFV) processes without light neutrinos in the final state, have been studied extensively both theoretically [3,4,11,19,20,23,33,35] and experimentally [3,34–37]. The advantage of these processes is that their observation would be a clear and unambiguous signal for LFV. These processes proceed only through loops. Using the independence of the loop functions on the light-neutrino masses and the orthogonality of rows of B matrix, the amplitudes of these processes may always be expressed in terms of heavy-neutrino contributions only. Three of these processes, $\mu\to e\gamma$, $\mu\to 3e$, and e-μ conversion in Ti, gave new very stringent constraints on specific combinations of heavy-neutrino masses and matrix elements B_{eN} and $B_{\mu N}$ [12]. Particularly, near independence of the $\mu\to e\gamma$ amplitude on heavy-neutrino masses enables one to find the following very stringent mass-independent limit:

$$\sum_{i=1}^{n_R} B^*_{\mu N_i}B_{eN_i}<2.4\times 10^{-4}. \tag{5}$$

No other constraints independent of heavy-neutrino masses have been derived from the LFV processes. It should be noted that the limit (5) does not necessarily lead to new limits on the $s_L^{\nu_l}$ parameters. The sum in Eq. (5) may be written in terms of the parameters $s_L^{\nu_\mu}$ and $s_L^{\nu_e}$ and a complex ''cosine'' of the ''angle'' between vectors $\{B_{\mu N_i}\}$ and $\{B_{eN_i}\}$,

$$\sum_{i=1}^{n_R} B^*_{\mu N_i}B_{eN_i}=s_L^{\nu_\mu}s_L^{\nu_e}x^0_{\mu e}, \tag{6}$$

where $x^0_{\mu e}=\Sigma_{i=1}^{n_R}B^*_{\mu N_i}B_{eN_i}/s_L^{\nu_\mu}s_L^{\nu_e}$. Obviously, a reduction of $x^0_{\mu e}$ may assure the fulfillment of the inequality (5) without reducing the $s_L^{\nu_l}$ parameters. Within the V model the explicit estimates of BR's for the processes including more than two B_{lN} matrices were given for the first time in Ref. [10].

B. A comment on astrophysical limits

The masslessness of ''light'' neutrinos in the V model leads to limits on some $B_{l\nu}$ matrix elements which can be derived from astrophysical observations. Valle and collaborators have noticed that the measurements of neutrino flux from the supernova SN87 leads to two very small lepton–massless-neutrino mixings [14],

$$|B_{e\nu_\tau}|,|B_{\tau\nu_e}|<10^{-3}. \tag{7}$$

The result (7) follows from an estimate of the ν_e-ν_τ conversion probability in the V model. To find whether similar upper bounds can be found for other massless-neutrino B matrix elements, their calculation is repeated here for three massless neutrinos. The motivation for such a calculation is the following. Through the orthogonality relations for B matrix elements (1), very stringent limits on the matrix elements $B_{l\nu}$ would lead to better upper bounds on nondiagonal mixings $\Sigma_{i=1}^{n_R}B_{lN_i}B^*_{l'N_i}$ than those obtained by terrestrial experiments.

The Valle *et al.* derivation of the limits (7) is based on an analysis of neutrino oscillations of the two massless neutrinos for which the experimental upper bounds on $s_L^{\nu_l}$ parameters are the weakest, $s_L^{\nu_e}$ and $s_L^{\nu_\tau}$. The oscillations of massless neutrinos are a consequence of an interplay between the charged current (CC) and neutral current (NC) neutrino weak interactions [38]. They appear only if the universality of the NC interactions is not satisfied and if the nondiagonal CC currents are different from zero. Following the notation of Refs. [14,38], the deviation from the universality is described by small parameters h_l (for small h_l, $h_l\approx s_L^{\nu_l}$). The massless-neutrino part of the B matrix is parametrized by one mixing angle θ, which is assumed to be small. The resonance condition is

$$2Y_e=\frac{h_\tau^2-h_e^2}{1+h_e^2}, \tag{8}$$

where $Y_e=n_e/(n_e+n_n)$, $Y_n=1-Y_e$, and n_e and n_n are the electron and the neutron number densities. As the experimental limits on h_e and h_τ are much smaller than 1, the resonance condition can be fulfilled only in a highly neutronized medium, which can be found in supernovae explosions. In Ref. [14] it was shown that the neutrino-sphere appears for the electron fraction $Y_e\approx 6\times10^{-3}$. The experimental upper bounds (4) show that the resonance condition can be fulfilled for $Y_e\lesssim 0.015$, quite close to the Y_e value at the neutrino-sphere. Assuming there is no nonforward scattering of neutrinos [39], the authors of Ref. [14] found the probability for $\nu_e\leftrightarrow\nu_\tau$ and $\overline{\nu_e}\leftrightarrow\overline{\nu_\tau}$ conversions in a simple Landau-Zener approximation [40,41]

$$\begin{aligned} P&\equiv P(\overline{\nu_e}\to\overline{\nu_\tau})\\ &=1-P(\overline{\nu_e}\to\overline{\nu_e})\\ &=\frac{1}{2}-\left[\frac{1}{2}-\exp\left(-\frac{\pi^2}{2}\frac{\delta r}{L_m^{res}}\right)\right]\cos2\theta\cos2\theta_m\\ &\approx 1-\exp\left(-\frac{\pi^2}{2}\frac{\delta r}{L_m^{res}}\right)\equiv 1-P_{\rm LZ}, \end{aligned} \tag{9}$$

where $P_{\rm LZ}$ is the Landau-Zener crossing probability, $L_m^{\rm res}$ is the neutrino oscillation length in matter at resonance, $\theta_m \approx\pi/2$ is the mixing angle in matter at production point (neutrino-sphere), and $\delta r=2\sin2\theta|d\ln Y_e/dr|^{-1}_{\rm res}$. The approximate equality in Eq. (9) is a conseqence of the small mixing angle (θ) approximation. Using that result, the expression for the detected terrestrial flux [42]

$$\phi_{\overline{\nu_e}}=\phi^0_{\nu_e}(1-P)+\phi^0_{\nu_\tau}P \tag{10}$$

($\phi^0_{\nu_e}$ and $\phi^0_{\nu_\tau}$ are $\overline{\nu_e}$ and $\overline{\nu_\tau}$ fluxes in the absence of the neutrino conversion, respectively), the model-independent result for the probability for $\overline{\nu_e}\leftrightarrow\overline{\nu_\tau}$ conversion $P<0.35$ [42], and the density profiles for Y_e from the Wilson supernova model, Valle and his collaborators found the result given in Eq. (7).

Following the procedure of Ref. [14], a similar analysis can be done for the three massless neutrinos. To analyze the terrestrial flux data, one should know only the survival probability of the electron antineutrino $P(\overline{\nu_e}\to\overline{\nu_e})$ [41,43]. Equation (10) is still valid, but $\phi^0_{\nu_\tau}$ represents the sum of $\overline{\nu_\mu}$ and $\overline{\nu_\tau}$ fluxes. In the three-neutrino case there are two resonances: $\overline{\nu_e}\leftrightarrow\overline{\nu_\mu}$ resonance and $\overline{\nu_e}\leftrightarrow\overline{\nu_\tau}$ resonance. According to the limits (4) and the Y_e value at the neutrino sphere, the $\overline{\nu_e}\leftrightarrow\overline{\nu_\mu}$ resonance is within the neutrino sphere. Therefore, the effects of this resonance do not contribute to $P(\overline{\nu_e}\to\overline{\nu_e})$. Taking that into account (or equivalently taking the neutrino-sphere as a source of neutrinos) and using the approximative Kuo-Pantaleone treatment for three-neutrino oscillations [41] adjusted for physical situation studied here, one obtains the following expression for $P\equiv1-P(\overline{\nu_e}\to\overline{\nu_e})$:

$$\begin{aligned} P&=1-(|U_{e1}|^2P_{LZ}+(1-P_{LZ})|U_{e2}|^2))\\ &\quad\times(|U_{e1}|^2+|U_{e2}|^2)-|U_{e3}|^4\\ &=1-P_{LZ}\cos^4\phi\cos2\omega-\cos^4\phi\sin^2\omega-\sin^4\omega \end{aligned} \tag{11}$$

(neutrino states 1, 2, and 3 are mainly ν_e, ν_τ, and ν_μ flavor states, respectively; the angles ω and ϕ perform rotations between 1 and 2 states and 2 and 3 states, respectively). The Landau-Zener crossing probability $P_{\rm LZ}$ can be obtained from the $P_{\rm LZ}$ for two-neutrino oscillations, replacing $\sin2\theta$ with

$2U_{e1}U_{e2}=\cos^2\phi\sin 2\omega$ in the two-neutrino $P_{\rm LZ}$. In the small-angle approximation, assumed in Ref. [14], the probability P tends to zero only if $P_{\rm LZ}$ is almost equal to 1. Using the result of Ref. [42] mentioned above, $P<0.35$, the small-angle approximation, and the analysis of Ref. [14], one finds the limits on mixing angles ω and ϕ

$$\sin^2 2\omega<1\times10^{-6},\quad \phi^2<0.27. \tag{12}$$

The first limit corresponds to the limit (7) obtained in the two-neutrino case. The second one is too weak to give limits on the B_{lN} matrix elements. Therefore, astrophysical measurements give no new limits on the heavy-neutrino part of the B matrix.

The second limit has to be compared with the ν_μ-ν_τ mixing angle obtained from the favorite interpretation of recent Super-Kamiokande results [2], $\theta_{\nu_\mu\nu_\tau}\approx\pi/4$. Obviously, these two results are in a slight contradiction.

C. Theoretical limits

If one wants to work in the perturbative regime of the theory, an additional constraint on the B_{lN} mixings comes from the theoretical argument that the partial-wave unitarity (perturbative unitarity) has to be satisfied. From the perturbative unitarity follows that the decay width of any heavy neutrino has to be smaller than a half of its mass. Written in terms of heavy-neutrino masses and B_{lN}'s, this condition reads [10]

$$m_{N_i}^2\sum_{j=1}^{n_G}|B_{l_jN_i}|^2\leqslant\frac{4}{\alpha_W}M_W^2\equiv m_D^2. \tag{13}$$

m_D represents the upper value the Dirac mass may attain in the neutrino-mass matrix. The perturbative unitarity-bound (PUB) inequalities (13) give an upper limit on a combination of a heavy-neutrino mass m_{N_i} and the matrix elements B_{lN_i}. Using Eq. (3), these relations may be combined into the limit for the lightest heavy-neutrino mass

$$m_{N_1}^2\leqslant(m_{N_1}^0)^2\left(1+\sum_{i=2}^{n_R}\rho_i^{-2}\right), \tag{14}$$

where $(m_{N_1}^0)^2=4M_W^2/[\alpha_W\Sigma_{j=1}^{n_G}(s_L^{\nu_j})^2]$ and $\rho_i=m_{N_i}/m_{N_1}$. Concerning the calculation of BR's, the bound is very effective if the heavy-neutrino masses are equal. If the heavy-neutrino masses differ considerably, the bound is not very restrictive. Namely, if one of the heavy-neutrino masses is smaller than $m_{N_1}^0/n_R^{1/2}$, the others may acquire infinite values not followed by infinitely small values of the corresponding B_{lN_i} mixings. That leads to divergent BR's. Therefore, one has to use the original inequality (13) to restrict model parameters. One cannot obtain closed expressions, since the model has too many free parameters, but one can write two very rough bounds [10]

$$|B_{lN_i}|\leqslant s_L^{\nu_i},$$

$$|B_{lN_i}|\leqslant\frac{2M_W}{\alpha_W^{1/2}m_{N_i}}\equiv B_{lN_i}^{(0)}, \tag{15}$$

originating from Eqs. (3) and (13), respectively, which have to be satisfied simultaneously. If the heavy-neutrino masses differ considerably, the bounds (15) are better for finding upper bounds of BR's than the bound (14).

The ''realistic'' B_{lN_i}'s which automatically satisfy the PUB's (15) and satisfy the relation $\Sigma_i B_{lN_i}B_{lN_i}^*\leqslant(s_L^{\nu_l})^2$ may be obtained by putting

$$B_{lN_i}=[(s_L^{\nu_l})^{-1}+(B_{lN_i}^{(0)})^{-1}]^{-1}n_R^{-1/2}. \tag{16}$$

This choice of B_{lN_i}'s is used below to give an estimate of how large an error can be made in the evaluation of BR's using the rough upper-bound procedure presented above. The B_{lN_i} defined in Eq. (16) begins to differ considerably from the value $s_L^{\nu_l}$ for $m_{N_i}\gtrsim100M_W(0.1/s_L^{\nu_i})$. Therefore, for m_{N_i} values smaller than 2000 GeV, the B_{lN_i}'s are determined by experimental upper bounds (4) and not by the theoretical PUB limits $B_{lN_i}^{(0)}$.

D. Upper-bound procedure for LFV amplitudes

Equations (15) and (16) are the basis for the evaluation of the LFV amplitudes. The evaluation based on Eq. (15) gives the upper bounds on absolute values of the amplitudes [10], which have to be satisfied by any model with additional heavy neutrinos. It uses the Schwartz's inequality for the product of two vectors. It always gives larger estimates for an amplitude than the approach based on Eq. (16). In both approaches the phases of the B_{lN_i}'s are neglected, but in a different manner. In the first approach, the upper-bound value of the amplitude is formed, while in the second the B_{lN_i}'s are taken to be real and positive. Both approaches explicitely show that the very heavy neutrinos are decoupled. That is, they have no influence on the amplitudes of low-energy LFV processes, in accord with the Appelquist-Carazzone theorem and its generalization [44,45].

Here, the improved version of the upper-bound procedure introduced in Ref. [10] is given. The low-energy LFV amplitudes may be written in terms of

$$\sum_{i=1}^{n_R}B_{lN_i}^*B_{l'N_i}f(N_i,\ldots),$$

$$\sum_{j=1}^{n_G}V_{u_jd_a}V_{u_jd_a}^*f(u_j,\ldots),$$

and

$$\sum_{j=1}^{n_G}V_{ud_j}^*V_{ud_j}f(d_j,\ldots), \tag{17}$$

where $f(N_i,\dots)$, $f(u_j,\dots)$, and $f(d_j,\dots)$ are expressions comprising the loop functions. The dots represent the indices not written explicitly. Namely, the amplitudes often contain more than one sum over neutrino or quark flavors. Using the inequalities that can be derived from Schwartz's inequality

$$\left|\sum_i a_i b_i c_i\right| \leq \sum_i |a_i||b_i||c_i|, \tag{18}$$

$$\left|\sum_{i=1}^{n} a_i b_i c_i\right| \leq |\mathbf{a}||\mathbf{b}|\langle c\rangle + |\mathbf{a}||\mathbf{b}|\left(\sum_{i=1}^{n} |c_i - \langle c\rangle|^2\right)^{1/2}, \tag{19}$$

($\langle c\rangle = \Sigma_{i=1}^{n} c_i/n$) and definition of $s_L^{\nu_l}$ (3), one can write the following upper limits for the expressions (17):

$$\left|\sum_{i=1}^{n_R} B_{lN_i}^{*} B_{l'N_i} f(N_i,\dots)\right| \leq s_L^{\nu_l} s_L^{\nu_l'}\Bigg(|\langle f(\cdots)\rangle_N| + \left[\sum_{i=1}^{n_R} (f(N_i,\dots) - \langle f(\cdots)\rangle_N)^2\right]^{1/2}\Bigg), \tag{20}$$

$$\left|\sum_{j=1}^{n_R} V_{u_j d_a} V_{u_j d_a}^{*} f(u_j,\dots)\right| \leq \sum_{j=1}^{n_R} |V_{u_j d_a}||V_{u_j d_a}||f(u_j,\dots)|,$$

$$\left|\sum_{j=1}^{n_R} V_{ud_j} V_{ud_j}^{*} f(d_j,\dots)\right| \leq \sum_{j=1}^{n_R} |V_{ud_j}||V_{ud_j}||f(d_j,\dots)|, \tag{21}$$

where $\langle\rangle_N$ represents the average over heavy neutrinos. The inequality (18) gives the best estimate for the upper limit if the components c_i differ considerably. The inequality (19) gives the better estimate of the upper bound if the components c_i are approximately equal. As the amplitudes $f(u_j,\dots)$ and $f(d_j,\dots)$ depend strongly on quark masses, Eq. (21) give good estimates for the upper bounds. Equation (20) is effective if the heavy-neutrino masses are nearly degenerate, because most of the $f(N_i,\dots)$ functions depend strongly on the heavy-neutrino masses. If one or more heavy-neutrino masses differ considerably from the others, then Eq. (20) may even lead to a divergent result as the heavy-neutrino mass(es) tend to infinity. To avoid such undesirable behavior, one has to use a combination of the upper bounds (18) and (19) for each set of heavy-neutrino mass values in the following manner. First, the heavy-neutrino masses are arranged in increasing order. The arranged masses are divided into two sets, one containing the smaller masses and the other the larger ones. There are $J+1$ such partitions, where J is the number of different heavy-neutrino masses. Then $J+1$ different upper bounds of the expression $\Sigma_{i=1}^{n_R} B_{lN_i}^{*} B_{l'N_i} f(N_i,\cdots)$ are formed combining the upper bounds (18) and (19),

$$\left|\sum_{i=1}^{n_R} B_{lN_i}^{*} B_{l'N_i} f(N_i,\dots)\right| \leq s_L^{\nu_l} s_L^{\nu_l'}\Bigg(|\langle f(\cdots)\rangle_s| + \left[\sum_{i_s} (f(N_{i_s},\dots) - \langle f(\cdots)\rangle_s)^2\right]^{1/2}\Bigg) + \sum_{i_b} B_{lN_{i_b}}^{0} B_{l'N_{i_b}}^{0} |f(N_{i_b},\dots)|, \tag{22}$$

where Σ_{i_s} sums over the lighter heavy-neutrino masses, and Σ_{i_b} over the heavier ones. Finally, the numerical values of the $J+1$ upper bounds (22) are compared and the smallest of them is taken to be the upper-bound value. For amplitudes containing sums over two (heavy-neutrino and/or quark) indices, the procedure is essentially the same. Again, one looks for the minimal upper-bound value between upper bounds obtained for all possible partititions of heavy-neutrino masses. This procedure gives convergent results for absolute values of the amplitudes, and it leads to the decoupling of the very heavy neutrinos.

It should be noted that the above upper-bound procedure gives upper bounds for BR's for neutrinoless LFV processes. Recently, lower-bound limits for τ lepton decays were found using the Super-Kamiokande data on atmospheric deficit of ν_μ, and interpreting it in terms of the best fit to these data [46]. The mild Glashow-Iliopoulos-Maiani (GIM) mechanism suppression, coming from a logarithmic dependence on light-neutrino masses, appearing in $\tau\to\mu l^+ l^-/\mu\rho^0$ decays, leads to the lower bounds of the BR's as large as $\sim 10^{-14}$. As the experimental upper limits on these processes are of the order of $\sim 10^{-6}$, this lower limit is welcome, because it strongly restricts the window for the heavy-neutrino LFV effects. However, these results have to be taken with caution, as the standard interpretation of the Super-Kamiokande data is not the only one [47], although recent papers [48,49] have shown that the energy dependence of the oscillation wavelength strongly supports the standard interpretation. It should be noted that the used V model can easily be modified to include masses for massless neutrinos [20]. The results for the neutrinoless LFV decays almost do not change if light-neutrino masses, consistent with Super-Kamiokande measurements, are introduced.

IV. NEW RESULTS ON LOW-ENERGY NEUTRINOLESS LFV PROCESSES

As mentioned in the Introduction, heavy-meson neutrinoless LFV decays and $M\leftrightarrow\bar{M}$ conversion have not been studied in the V model. They are examined below. Some previous results for neutrinoless LFV decays are extended and/or corrected.

A. Neutrinoless LFV decays of heavy mesons

The LFV decays of heavy mesons were discussed in a few papers in the context of the leptoquark models [16], a flipped left-right symmetric model, [17], and SM with an additional Higgs doublet [15]. In these decays, both lepton and quark flavor are changed. In the V model they can proceed only through box diagrams in which two W bosons are exchanged. The effective Lagrangian on the quark-lepton level reads

$$\mathcal{L}_{eff}=\frac{\alpha_W^2}{16M_W^2}\sum_{l\neq l'}\sum_{Q}\sum_{q_a}F_{\text{box}}^{l'lq_aQ}\overline{l}\gamma_\mu(1-\gamma_5)l'\overline{q_a}\gamma^\mu \times(1-\gamma_5)Q[\delta_{Qc}\delta_{q_au}-\delta_{Qb}(\delta_{q_ad}+\delta_{q_as})]. \tag{23}$$

l and l' are the lepton fields, q_a and Q are the light- and heavy-quark fields, respectively, α_W is the weak fine-structure constant, M_W is the W-boson mass, and $F_{\text{box}}^{l'lq_aQ}$ is the composite-loop function

$$F_{\text{box}}^{l'luc}=\sum_{i=1}^{n_R}\sum_{j=1}^{n_G}B_{l'N_i}^{*}B_{lN_i}V_{ud_j}^{*}V_{cd_j}[H_{\text{box}}(\lambda_{N_i},\lambda_{d_j}) -H_{\text{box}}(\lambda_{N_i},0)-H_{\text{box}}(0,\lambda_{d_j})+H_{\text{box}}(0,0)],$$

$$F_{\text{box}}^{l'lq_ab}=\sum_{i=1}^{n_R}\sum_{j=1}^{n_G}B_{l'N_i}^{*}B_{lN_i}V_{u_jd_a}V_{u_jb}^{*}[F_{\text{box}}(\lambda_{N_i},\lambda_{u_j}) -F_{\text{box}}(\lambda_{N_i},0)-F_{\text{box}}(0,\lambda_{u_j})+F_{\text{box}}(0,0)]. \tag{24}$$

F_{box} and H_{box} are loop functions defined in Ref. [50]. These loop functions have approximately logarithmic dependence on the heavy-neutrino masses.

The dominant processes are those which have maximal value of the CKM matrix elements, maximal LCKM matrix elements, and t-quark mass in the loop function. The main neutrinoless LFV candidates, between the two-prong and three-prong processes studied here, are $\overline{B}_s^0\to\tau^\pm e^\mp$, $B^-\to K^-\tau^\pm e^\mp$, $\overline{B}^0\to\overline{K}^0\tau^\pm e^\mp$, and $\overline{B}_s^0\to\phi\tau^\pm e^\mp$. There are no interesting D-meson candidates for two reasons. One is of dynamical origin—the quark masses involved in loop functions are smaller than in B-meson decays, so loop functions are much smaller. The only larger loop contribution coming from the b quark is suppressed by small CKM matrix elements. The other is kinematical—the difference of τ lepton and D-meson masses is small. The small quark masses in loops and large t quark width makes LFV decays of t quark uninteresting from the experimental point of view.

The matrix element of the neutrinoless LFV decay of a heavy meson H, $H\to Xll'$, contains hadronic matrix element $\langle X|\overline{q_a}(0)\gamma^\mu(1-\gamma_5)Q(0)|H\rangle$. The corresponding matrix elements are usually parametrized in the following way [51,52]:

$$\langle 0|\overline{q_a}(0)\gamma_\mu(1-\gamma_5)Q(0)|H_a(p)\rangle=-if_Hp_\mu,$$

$$\langle P(p')|\overline{q_a}(0)\gamma_\mu(1-\gamma_5)Q(0)|H_a(p)\rangle =\left[\left((p+p')_\mu-\frac{m_H^2-m_P^2}{q^2}q_\mu\right)F_1(q^2) +\frac{m_H^2-m_P^2}{q^2}q_\mu F_0(q^2)\right]N_P^{q_a},$$

$$\langle V(p',\varepsilon)|\overline{q_a}(0)\gamma^\mu(1-\gamma_5)Q(0)|H_a(p)\rangle =\left[-\frac{2V(q^2)}{m_H+m_V}\varepsilon^{\mu\nu\alpha\beta}\varepsilon_\nu^{*}p_\alpha p'_\beta-i\varepsilon^{*}\cdot q\frac{2m_V}{q^2}q_\mu A_0(q^2) -\frac{i\varepsilon^{*}\cdot q}{m_H+m_V}\left((p+p')_\mu-\frac{m_H^2-m_V^2}{q^2}q_\mu\right)A_2(q^2) +i(m_H+m_V)\left(\varepsilon_\mu^{*}-\frac{\varepsilon^{*}\cdot q}{q^2}q_\mu\right)A_1(q^2)\right]N_V^{q_a}. \tag{25}$$

H_a is a heavy pseudoscalar meson containing light quark $\overline{q_a}$, P, and V are a light pseudoscalar meson and a light vector meson, respectively, p and p' are four-momenta of the heavy and light meson, respectively, $q=p-p'$ is the momentum transfer, ε is the polarizaton vector of the light vector meson, f_H is the decay constant of the heavy pseudoscalar meson, F_1, F_2, V, A_0, A_1, and A_2 are form factors and $N_P^{q_a}$ ($N_V^{q_a}$) is a factor in front of the term containing $\overline{q_a}$ in the quark wave function of the P (V) meson. The q^2 dependence of the form factors is a consequence of long-distance (resonance) effects following from strong interactions.

To evaluate the hadronic matrix elements of quark currents and to include the long-distance effects, one has to express the quark currents in terms of the meson states and to introduce a strong-interaction Lagrangian on the meson level. Similar hadronic matrix elements have been extensively studied in radiative, semileptonic and nonleptonic decays of heavy mesons. The combination of heavy-quark effective theory (HQET) and chiral pertubation theory (CHPT) has been applied to these decays [53]. Here, the modification of this formalism [52,54–56] is used. The authors of these papers replaced the HQET propagators by the full heavy-quark propagators, and introduced SU(3) symmetry breaking through physical masses and decay constants of light mesons. The matrix elements in that approach read

$$\langle 0|\overline{q_a}(0)\gamma_\mu(1-\gamma_5)Q(0)|\mathcal{H}(p)\rangle=-if_{\mathcal{H}}p_\mu,$$

$$\langle P(p')|\overline{q}_a(0)\gamma_\mu(1-\gamma_5)Q(0)|\mathcal{H}(p)\rangle=+N_P^{q_a}\left[-\frac{f_\mathcal{H}}{f_P}p_\mu+2\frac{f_{\mathcal{H}'*}}{f_P}(m_\mathcal{H}m_{\mathcal{H}'*})^{1/2}g\left(p'_\mu-\frac{p'\cdot q q_\mu}{m^2_{\mathcal{H}'*}}\right)\frac{m_{\mathcal{H}'*}}{q^2-m^2_{\mathcal{H}'*}}\right],$$

$$\langle V(p',\varepsilon)|\overline{q}_a(0)\gamma^\mu(1-\gamma_5)Q(0)|\mathcal{H}(p)\rangle=N_V^{q_a}\left[2^{3/2}\lambda g_V\left(\frac{m_{\mathcal{H}'*}}{m_\mathcal{H}}\right)^{1/2}f_{H'*}\frac{m_{\mathcal{H}'*}}{q^2-m^2_{\mathcal{H}'*}}\varepsilon_{\mu\nu\alpha\beta}\varepsilon^{*\mu}p^\alpha p'^\beta-i2^{1/2}\beta g_V\left(\frac{m_{\mathcal{H}'}}{m_\mathcal{H}}\right)^{1/2}\right.$$
$$\left.\times f_{\mathcal{H}'}\frac{q\cdot\varepsilon^* q_\mu}{q^2-m'^2_\mathcal{H}}-i2^{1/2}\alpha_1 g_V m^{1/2}_{\mathcal{H}'}\varepsilon^{*\mu}+i2^{1/2}\alpha_2 g_V m^{1/2}_{\mathcal{H}'}\frac{p_\mu p\cdot\varepsilon^*}{m^2_\mathcal{H}}\right]. \quad (26)$$

$\mathcal{H}'$ and $\mathcal{H}'*$ represent heavy-pseudoscalar-meson and heavy-vector-meson resonances, respectively, $f_{\mathcal{H}'}$, $f_{\mathcal{H}'*}$, $m_{\mathcal{H}'}$, $m_{\mathcal{H}'*}$ are the corresponding decay constants and masses, g_V ($\approx 6.0(2/a)^{1/2}$ with $a=2$ in the case of exact vector-meson dominance) is the vector-meson self-interaction coupling constant [57], g and β are the coupling constants in the even part of the strong-interaction Lagrangian [51,52,54,56,58,59], λ is a coupling constant in the odd part of the strong-interaction Lagrangian [51,52,54–56,58], and α_1 and α_2 are coupling constants in the definition of weak current [54,56]. The constants g, β, λ, α_1, and α_2 are free parameters which have to be determined from experimental data.

The matrix elements of $\mathcal{H}\to Xll'$ follow from Eqs. (23), (24), and (26). From these matrix elements follow the corresponding decay rates:

$$B(\mathcal{H}^0_a\to l^-l'^+)=\frac{\alpha_W^4}{2^{10}\pi}\frac{f^2_{\mathcal{H}^0}m^3_{\mathcal{H}^0}}{\Gamma_{\mathcal{H}^0}M_W^4}\frac{\lambda^{1/2}(m^2_{\mathcal{H}^0},m^2_{l'},m^2_l)}{m^2_{\mathcal{H}^0}}\frac{m^2_{\mathcal{H}^0}(m^2_{l'}+m^2_l)-(m^2_{l'}-m^2_l)^2}{m^4_{\mathcal{H}^0}}|F^{l'lq_aQ}_{\rm box}|^2,$$

$$B(\mathcal{H}_a\to Pl^-l'^+)=\frac{\alpha_W^4(N_P^{q_a})^2}{2^{13}\pi^3}\frac{\int_{(m_l+m_{l'})^2}^{(m_\mathcal{H}-m_P)^2}dt[a_P^2Z_{P1}+a_Pb_PZ_{P2}+b_P^2Z_{P3}]}{m^3_\mathcal{H}\Gamma_\mathcal{H}M_W^4}|F^{l'lq_aQ}_{\rm box}|^2,$$

$$B(\mathcal{H}_a\to Vl^-l'^+)=\frac{\alpha_W^4(N_V^{q_a})^2}{2^{12}\pi^3}|F^{l'lq_aQ}_{\rm box}|^2\frac{1}{m^3_\mathcal{H}\Gamma_\mathcal{H}M_W^4}\int_{(m_l+m_{l'})^2}^{(m_\mathcal{H}-m_V)^2}dt[a_V^2Z_{V1}+b_V^2Z_{V2}+c_V^2Z_{V3}+d_V^2Z_{V4}+a_Vc_VZ_{V5}$$
$$+b_Vc_VZ_{V6}+b_Vd_VZ_{V7}+c_Vd_VZ_{V8}]. \quad (27)$$

The form factors a_P, b_P, a_V, b_V, c_V, and d_V, and phase functions Z_{Pi}, $i=1,2,3$, and $Z_{Vi}i=1,\cdots,8$ are defined in the Appendix.

B. Muonium-antimuonium conversion

The CC vertices in the V model have V-A structure. The effective Lagrangian for the $M\leftrightarrow\overline{M}$ conversion comes from the lepton box amplitude. Therefore, the structure of the effective Hamiltonian density for $M\leftrightarrow\overline{M}$ has the same $(V-A)\times(V-A)$ form as in the Feinberg's and Weinberg's papers [60]

$$\mathcal{H}=G_{M\overline{M}}\overline{\psi}_\mu\gamma_\lambda(1-\gamma_5)\psi_e\overline{\psi}_\mu\gamma^\lambda(1-\gamma_5)\psi_e, \quad (28)$$

in which they had elaborated the original idea of Pontecorvo [61]. The constant $G_{M\overline{M}}$ contains information on physics beyond SM. In the frame of the V model it comprises the parameters of the box amplitude for the process $\mu^+e^-\to\mu^-e^+$, which is forbidden in SM,

$$G_{M\overline{M}}=\frac{\alpha_W^2}{16M_W^2}F^{\mu ee\mu}_{\rm box}. \quad (29)$$

$F^{\mu ee\mu}_{\rm box}$ is a composite-loop function having the following structure [11]

$$F^{\mu ee\mu}_{\rm box}=2\sum_{ij=1}^{n_R}B_{eN_i}B_{eN_j}B^*_{\mu N_i}B^*_{\mu N_j}[F_{\rm box}(\lambda_{N_i},\lambda_{N_j})$$
$$-F_{\rm box}(0,\lambda_{N_j})-F_{\rm box}(\lambda_{N_i},0)+F_{\rm box}(0,0)]. \quad (30)$$

Using the expression (30) for large degenerate heavy-neutrino masses, one obtains the limit

$$G_{M\overline{M}}\leqslant 3.9\times 10^{-5}x^0_{\mu ee\mu}G_F, \quad (31)$$

where G_F is the Fermi constant and $x^0_{\mu ee\mu}=F^{\mu ee\mu}_{\rm box}/[0.5\lambda_N(s_L^{\nu_\mu})^2(s_L^{\nu_e})^2]$. From the definition of the composite-loop function and the limit (5) follows that the $x^0_{\mu ee\mu}$ may assume only values smaller than 4.7×10^{-3}. Keeping that in mind, the result (31) has to be compared with the recent experimental upper bound [37,62] which improved the previous experimental result [63] by the factor ~ 50, $G_{M\overline{M}}\leqslant 3.0\times 10^{-3}G_F$. The upper bound (31) is larger than the result found by Swartz [64], estimated within SM with massive Dirac neutrinos, by comparing the effective Hamiltonians for M-$\overline{M}$ conversion and for B^0-$\overline{B}^0$ transition. Having in mind that the upper limit (5) was much weaker than when Swartz wrote his paper, the result obtained here is in fact larger than the numerical results show. The $G_{M\bar{M}}$ was also evaluated in many other models [65]. Depending upon the variant of the model, the value of $G_{M\bar{M}}$ ranges from $10^{-9}G_F$ to $0.1G_F$.

The conversion probability $P(M\to\overline{M})$ is the quantity that is measured in experiments. It is related to the constant $G_{M\bar{M}}$ in the following way [60]:

$$P(M\to\overline{M})=\frac{\delta^2}{2\Gamma_\mu^2}, \tag{32}$$

where

$$\frac{\delta}{2}=\langle\overline{M}|H|M\rangle=\frac{16G_{M\bar{M}}}{\pi a^3} \tag{33}$$

is a transition matrix element between the muonium and antimuonium states (a is the radius of muonium atom) and Γ_μ is the total decay width of muon.

From the point of view of SM extended by heavy neutrinos, M-$\overline{M}$ conversion is not a good place to search for LFV. Roughly speaking, the M-$\overline{M}$ amplitude is proportional to the square of the nondiagonal μ-e mixing $\Sigma_i B_{\mu N_i}B^*_{eN_i}$, which is strongly constrained by the measurements of processes $\mu\to e\gamma$, $\mu\to eee$, and $\mu\to e$ conversion. Amplitudes of the three processes depend approximately linearly on the nondiagonal μ-e mixing. Therefore, if any of the experimental results of measurement of the three processes is improved by a factor a, the experimental result for $P(M\to\overline{M})$ has to be improved by the factor a^2 to be competitive in finding LFV.

C. Extension and correction of some previous results

In this subsection some previous results on neutrinoless LFV processes evaluated within the frame of the V model are extended and/or corrected. The decays of τ lepton into three leptons have been evaluated within the V model in Ref. [18] without including terms with four B_{lN}'s. These terms were shown to dominate for large heavy-neutrino masses in SM extended by two additional heavy Majorana neutrinos [11]. In that model the B_{lN}'s are completely determined by $s_L^{\nu_i}$ parameters and the ratio of the heavy-neutrino masses. Here, the upper bounds of complete amplitudes are evaluated within the V model, and used to find the upper bounds of the corresponding BR's.

Neutrinoless LFV decays of the Z boson were studied in Ref. [11] in SM extended with heavy Majorana neutrinos. The expressions for loop functions are given in Appendix A of that reference, and they are correct except for terms containing

$$\frac{\sqrt{w}}{\lambda_Z}\tan^{-1}\left(\frac{\sqrt{w}}{\lambda_i+\lambda_j-\lambda_Z}\right), \tag{34}$$

which should be replaced with the expression

$$\begin{aligned}\theta(w)\frac{\sqrt{w}}{\lambda_Z}&\left[\tan^{-1}\left(\frac{\sqrt{w}}{\lambda_i+\lambda_j-\lambda_Z}\right)+\pi\theta(\lambda_Z-\lambda_i-\lambda_j)\right]\\&+\theta(-w)\frac{\sqrt{-w}}{\lambda_Z}\left[\frac{1}{2}\ln\left|\frac{\lambda_Z-\lambda_i-\lambda_j+\sqrt{-w}}{\lambda_Z-\lambda_i-\lambda_j-\sqrt{-w}}\right|\right.\\&\left.-i\pi\theta(\lambda_Z-\lambda_i-\lambda_j)\right].\end{aligned} \tag{35}$$

The notation is the same as in Ref. [11]. The theta function in the first square bracket was not taken into account in the analysis in Ref. [11]. As it contributes only for the heavy-neutrino masses smaller than the Z-boson mass, the numerical results given there should not change. For heavy neutrinos lighter than Z-boson mass, the theta function assures the continuity of the loop functions in heavy-neutrino masses. Here, LFV decays of the Z boson are studied in the V model. The terms containing the matrix elements $C^*_{N_iN_j}$, that exist only for heavy Majorana neutrinos, are neglected. In the V model only the upper bounds of the $Z\to ll'$ amplitudes can be found. They are found using the formalism of the Sec. III C.

The only three neutrinoless LFV processes that give additional constraints on B_{lN}'s, $\mu\to e\gamma$, $\mu\to eee$, and $\mu-e$ conversion, have been examined in Ref. [12]. Their analysis has included the ''nondecoupling'' effects of heavy neutrinos, has indicated that a generalization of Appelquist-Carazzone theorem [44,45] is valid for the V model and has determined the limits on specific combinations of B_{lN}'s. The ''proof'' of the generalization of the Appelquist-Carazzone theorem is based on an introduction of a somewhat arbitrary maximal $SU(2)_L$-doublet mass term. The amplitude they present for $\mu\to e$ conversion does not include the photon-exchange and box contributions, and the amplitude for $\mu\to eee$ does not include the photon-exchange term. These terms are included here. Moreover, in their expression for $\mu\to 3e$ BR, obtained in the limit of large heavy-neutrino masses, one has to make replacements $\mathcal{F}_{e\mu}\to 2\mathcal{F}_{e\mu}$ and $\varepsilon_L\equiv -1/2+s_W^2\to -\varepsilon_L$, (the notation of Ref. [12] is used).

The neutrinoless LFV decay of π^0 was studied in Ref. [10] in extensions of SM with additional Majorana and additional Dirac neutrinos. The expressions for the extension with Majorana neutrinos is correct, but the expressions for the extension by Dirac neutrinos is not, because the terms existing only for Majorana neutrinos have been kept in the amplitude. The correct amplitude is obtained neglecting the terms containing the loop function H_Z. When this correction is made, the numerical results for the $\pi\to\mu e$ decay become $\sim$25 times smaller.

V. ON LOW-ENERGY NEUTRINOLESS LFV AMPLITUDES AND DECAY RATES

A. Loop functions included in LFV processes

In the lowest order of perturbation theory, amplitudes of neutrinoless LFV decays are built up from several building

TABLE I. List of neutrinoless LFV processes, the composite-loop functions and the tree level functions contributing to them and the approximations (physics) needed for evaluation of amplitudes. l, P, V, $\mathcal{H}$, and B denote leptons, light pseudoscalar mesons, light vector mesons, heavy pseudocsalar mesons (containing c or b quark), and light baryons, respectively. In the first column, the list of the neutrinoless LFV processes is given, with references only to the calculations made within extensions of SM with heavy neutrinos. The abbreviations cqf = conserved quark flavor, $ncqf$ = nonconserved quark flavor, and H = Higgs-mediated process, serve to distinguish processes with seemingly similar particle content. In the second column, the Feynman diagrams contributing to any specific process are listed. For instance, l-q-box corresponds to the box diagram with one lepton current and one quark current. In the third column the approximations and physics used for calculation of amplitudes are listed. Following abbreviations are used: HQET = heavy-quark effective theory, CHPT = chiral perturbation theory, VMD = vector-meson dominance, GTR = Goldberger-Treiman relation, l = lepton physics, q = quark physics.

process	diagrams	approximations (physics)
$l \to l'\gamma$ [11,18]	γ	l
$\mu \to e$ conversion [12,70,71]	γ, Z and l-q-box	l, q, nuclear
$M \to \overline{M}$ conversion	l-box	l, atomic
$l^- \to l'^- l_1^- l_2^+$ [11,12,18]	γ, Z, and l-box	l
$\tau \to lP^0$ (cqf) [11,18]	γ, Z, and l-q-box	l, q, PCAC
$\tau \to lP^0$ $(ncqf)$ [11]	l-q-box	l, q, PCAC
$\tau \to lV^0$ (cqf) [11]	γ, Z, and l-q-box	l, q, VMD
$\tau \to lV^0$ $(ncqf)$ [11]	l-q-box	l, q, VMD
$Z \to ll'$ [11,67]	γ, Z and l-box	l
$H \to ll'$ [68,69]	H	l
$P^0 \to e\mu$ (cqf) [10]	γ, Z, and l-q-box	l, q, PCAC
$P^0 \to e\mu$ $(ncqf)$ [10]	l-q-box	l, q, PCAC
$\mathcal{H}^0 \to ll'$	l-q-box	l, q, PCAC
$\tau^- \to l'^- P_1 P_2$ (cqf) [66]	all except l-box	l, q, CHPT, PCAC, VMD
$\tau^- \to l'^- P_1 P_2$ $(ncqf)$ [66]	l-q-box and W^+W^-	l, q, CHPT, PCAC, VMD
$\tau^- \to l'^- P_1 P_2$ $(cqf$,H) [66]	H and W^+W^-	l, q, CHPT, PCAC
$P_1 \to P_2 e\mu$ [10]	l-q-box	l, q, VMD, CHPT
$\mathcal{H} \to Pl\overline{l'}$	l-q-box	l, q, VMD, CHPT, HQET
$\mathcal{H} \to Vl\overline{l'}$	l-q-box	l, q, VMD, CHPT, HQET
$B_1 \to B_2 e\mu$ [10]	l-q-box	l, q, PCAC, GTR

blocks (composite-loop functions and tree-level functions) which may be denoted by the exchanged bosons, or by the type of the Feynman diagram: γ, Z, box (box containing only leptons, leptons and u quarks, leptons and d quarks), H and W^+W^-. All functions except the last one are combinations of loop functions and B_{lN}'s [11,10,66]. W^+W^- function is a tree-level function and it is strongly suppressed compared to the others [66]. γ, Z, box, and H functions comprise two-fermion currents. In the γ, Z, and H functions only one of the fermion currents changes flavor, while in box functions flavors may be changed in both fermion currents. The classification of the neutrinoless LFV decays, given in Table I, is made according to the Feynman diagrams they contain and the approximations (physics) one has to use in finding the corresponding amplitudes. The references cited in Table I refer only to the calculations of LFV processes in the extensions of SM by additional heavy neutrinos.

If the heavy-neutrino masses are larger than a few hundred GeV, the expressions for neutrinoless LFV decays simplify considerably. All amplitudes can approximately be expressed in terms of four combinations of masses and B_{lN_i}'s,

$$\mathcal{A}_{ll'} = \sum_{N_i} B^*_{lN_i} B_{l'N_i},$$

$$\mathcal{B}_{ll'} = \sum_{N_i} B^*_{lN_i} B_{l'N_i} \ln \lambda_{N_i},$$

$$\mathcal{C}_{ll'} = \sum_{N_i N_j} B^*_{lN_i} C^*_{N_i N_j} B_{l'N_i} \frac{\lambda_{N_i}\lambda_{N_j}}{\lambda_{N_i}-\lambda_{N_j}} \ln\frac{\lambda_{N_i}}{\lambda_{N_j}},$$

$$\mathcal{D}_{ll'l_1l_2} = \frac{1}{2}\sum_{N_i N_j} B^*_{lN_i} B^*_{l_2N_j} (B_{l'N_i} B_{l_1N_j} + B_{l_1N_i} B_{l'N_j}) \frac{\lambda_{N_i}\lambda_{N_j}}{\lambda_{N_i}-\lambda_{N_j}} \ln\frac{\lambda_{N_i}}{\lambda_{N_j}}, \qquad (36)$$

where $\lambda_{N_i} = m^2_{N_i}/m^2_W$. The building blocks mentioned above, expressed in terms of combinations (36), read

$$G_\gamma^{ll'} \approx \frac{1}{2}\mathcal{A}_{ll'},$$

$$F_\gamma^{ll'} \approx -\frac{1}{6}\mathcal{B}_{ll'},$$

$$F_Z^{ll'} \approx -\frac{3}{2}\mathcal{B}_{ll'} - \frac{1}{2}\mathcal{C}_{ll'},$$

$$F_{\rm box}^{ll'l_1l_2} \approx -(\mathcal{A}_{ll'}\delta_{l_1l_2} + \mathcal{A}_{ll_1}\delta_{l'l_2}) + \frac{1}{2}\mathcal{D}_{ll'l_1l_2},$$

$$F_{\rm box}^{ll'u_au_b} \approx \Bigg[-4\delta_{u_au_b} + \Bigg(-\frac{9}{4}\frac{\lambda_b}{1-\lambda_b} + \frac{-\lambda_b^3 + 8\lambda_b^2 - 16\lambda_b}{4(1-\lambda_b)}\ln\lambda_b \Bigg) V^*_{u_ab}V_{u_bb}\Bigg]\mathcal{A}_{ll'} + \Bigg[\frac{\lambda_b}{4}V^*_{u_ab}V_{u_bb}\Bigg]\mathcal{B}_{ll'},$$

$$F_{\rm box}^{ll'd_ad_b} \approx \Bigg[-\delta_{d_ad_b} + \sum_{u_i=c,t}\Bigg(\frac{3}{4}\frac{\lambda_{u_i}}{1-\lambda_{u_i}} + \frac{-\lambda_{u_i}^3 + 8\lambda_{u_i}^2 - 4\lambda_{u_i}}{4(1-\lambda_{u_i})}\ln\lambda_{u_i} \Bigg) V_{u_id_a}V^*_{u_id_b}\Bigg]\mathcal{A}_{ll'} + \Bigg[\sum_{u_i=c,t}\frac{\lambda_{u_i}}{4}V_{u_id_a}V^*_{u_id_b}\Bigg]\mathcal{B}_{ll'},$$

$$F_H^{ll'} \approx G_H^{ll'} \approx \frac{5}{8}\mathcal{A}_{ll'} + \frac{\lambda_H}{4}\mathcal{B}_{ll'} + \frac{3}{4}\mathcal{C}_{ll'},$$

$$F_{W^+W^-} \approx \Bigg(\sum_{i=1}^{n_G} V_{u_id_a}V^*_{u_id_b} \Bigg)\mathcal{A}_{ll'}, \tag{37}$$

where $\lambda_x = m_x^2/m_W^2, \quad x=b,t,H.$

For the important case of degenerate ($\lambda_{N_i}=\lambda_N$) and large heavy-neutrino masses the functions (36) can be written in terms of parameters $s_L^{\nu_l}$ and $x^0_{ll'}$,

$$\mathcal{A}_{ll'} = s_L^{\nu_l}s_L^{\nu_{l'}}x^0_{ll'},$$

$$\mathcal{B}_{ll'} = s_L^{\nu_l}s_L^{\nu_{l'}}x^0_{ll'}\ln\lambda_N,$$

$$\mathcal{C}_{ll'} = s_L^{\nu_l}s_L^{\nu_{l'}}\sum_{i=1}^{n_G}(s_L^{\nu_i})^2 x^0_{ll_i}x^0_{l_il'}\lambda_N,$$

$$\mathcal{D}_{ll'l_1l_2} = \frac{1}{2}s_L^{\nu_l}s_L^{\nu_{l'}}s_L^{\nu_{l_1}}s_L^{\nu_{l_2}}(x^0_{ll'}x^0_{l_1l_2} + x^0_{ll_1}x^0_{l'l_2})\lambda_N. \tag{38}$$

It is convenient to introduce four combinations of B_{lN}'s, heavy-neutrino masses $\lambda_N^{\rm PUB}$, and upper-bound values for $s_L^{\nu_l}$ parameters (4), denoted by $\tilde{s}_L^{\nu_l}$:

$$x_{ll'} = \mathcal{A}_{ll'}(\tilde{s}_L^{\nu_l}\tilde{s}_L^{\nu_{l'}})^{-1},$$

$$z_{ll'} = \mathcal{B}_{ll'}(\tilde{s}_L^{\nu_l}\tilde{s}_L^{\nu_{l'}}\ln\lambda_N^{\rm PUB})^{-1},$$

$$y_{ll'} = \mathcal{C}_{ll'}\Bigg(\tilde{s}_L^{\nu_l}\tilde{s}_L^{\nu_{l'}}\sum_{i=1}^{n_G}(\tilde{s}_L^{\nu_i})^2\lambda_N^{\rm PUB}\Bigg)^{-1},$$

$$y_{ll'l_1l_2} = \mathcal{D}_{ll'l_1l_2}(\tilde{s}_L^{\nu_l}\tilde{s}_L^{\nu_{l'}}\tilde{s}_L^{\nu_{l_1}}\tilde{s}_L^{\nu_{l_2}}\lambda_N^{\rm PUB})^{-1}. \tag{39}$$

Any of these combinations is always smaller than 1.

A few comments are in order here. First, it is obvious that $|\mathcal{D}_{ll'l_1l_2}| \leqslant |\mathcal{C}_{ll'}|$ (the relation is also valid for large, nondegenerate heavy-neutrino masses). Second, for degenerate neutrino masses, the function $\mathcal{C}_{ll'}$ becomes larger than the functions $\mathcal{A}_{ll'}$ and $\mathcal{B}_{ll'}$ if

$$\lambda_N \gtrsim \frac{1}{\sum_{i=1}^{n_G}(s_L^{\nu_i})^2} \quad \text{and} \quad \lambda_N \gtrsim \frac{\ln\lambda_N}{\sum_{i=1}^{n_G}(s_L^{\nu_i})^2}, \tag{40}$$

respectively. The dominance of the functions with quadratic mass dependence of the amplitude leads to the transient, so called ''nondecoupling behavior'' of amplitudes. As explained in Sec. III D, decoupling follows from PUB inequalities (13). A typical mass value for which the quadratic terms become larger than the logarithmic terms is $m_N \sim 1500$ GeV for $s_L^{\nu_l}$ values of the order of the present experimental bounds (4). Third, at the maximal λ_N value permitted by the PUB ($\lambda_N^{\rm PUB}$), the function $\mathcal{C}_{ll'}$ depends essentially only on two diagonal mixing parameters, $s_L^{\nu_l}$ and $s_L^{\nu_{l'}}$,

$$\mathcal{C}_{ll'}(\lambda_N^{\rm PUB}) = \frac{4n_R\sum_i (s_L^{\nu_i})^2 x^0_{ll_i}x^0_{l_il'}}{\alpha_W\sum_i (s_L^{\nu_i})^2} s_L^{\nu_l}s_L^{\nu_{l'}} \lesssim \frac{4n_R}{\alpha_W}s_L^{\nu_l}s_L^{\nu_{l'}}x_{ll'} = \frac{4n_R}{\alpha_W}\mathcal{A}_{ll'}. \tag{41}$$

Therefore, at $m_N = m_N^{\rm PUB}$ all amplitudes depend essentially only on $s_L^{\nu_l}$ and $s_L^{\nu_{l'}}$. If both the logarithmic and quadratic mass terms are present in LFV amplitude, at $m_N^{\rm PUB}$ logarithmic terms contribute up to $\sim$10% of the total amplitude. Fourth, if the t-quark contribution is multiplied by small CKM matrix elements, box amplitudes may have large contribution coming from the c quark in the loop expressions. For instance, in the processes $\tau \to eP^0/\mu P^0$ c-quark contribution to the amplitude is $\sim$13%. Fifth, the processes containing only the function $\mathcal{A}_{ll'}$ are most suitable for obtaining new information on B_{lN_i} parameters, because they are almost independent of heavy-neutrino masses. Sixth, for degenerate heavy neutrinos the dependence of LFV amplitudes on LCKM matrix elements appears only through six sums $\Sigma_i B_{lN_i}B^*_{l'N_i}$, $l \neq l'$, and $\Sigma_i |B_{lN_i}|^2$ (diagonal and nondiagonal mixings). Writing the sums in terms of $s_L^{\nu_l}$'s and $x^0_{ll'}$'s, one can easily show that if some LFV amplitude tends to zero for $s_L^{\nu_l}\to 0$, then the amplitude tends to zero for $x^0_{ll'}\to 0$, $l\neq l'$, too. (Strictly speaking, reducing a parameter $s_L^{\nu_l}$ by a factor a is equivalent to the reduction $x^0_{ll}\to a^2 x^0_{ll}$ and $x^0_{ll'}\to a x^0_{ll'}$, $l\neq l'$, but, by definition, $x^0_{ll}=1$.) This analysis shows that LFV amplitudes may be reduced without changing the diagonal mixing parameters $s_L^{\nu_l}$. It also indicates that the absolute values of LFV amplitudes may attain any value between zero and the upper-bound value. Therefore, LFV processes are unsuitable for finding the limits on the diagonal mixing parameters $s_L^{\nu_l}$.

B. Approximative expressions for BR's in the large-mass limit and relations between them

Keeping only the leading terms in the large-mass limit of heavy neutrinos, the expressions for BR's of neutrinoless LFV decays may be expressed in terms of the functions (36). In the following, these expressions are listed. The definitions of unknown quantities are given below.

$$B(l\to l'\gamma)\approx\frac{\alpha_W^3 s_W^2}{2^{10}\pi^2}\,\frac{m_l^5}{M_W^4\Gamma_l}|\mathcal{A}_{ll'}|^2, \tag{42}$$

$$B(l^-\to l'^- l_1^- l_2^+, l_1=l_2\neq l')\approx\frac{\alpha_W^4}{3\times 2^{15}\pi^3}\,\frac{m_l^5}{M_W^4\Gamma_l}(|\mathcal{D}_{ll'l_1l_1}-(1-2s_W^2)\mathcal{C}_{ll'}|^2+|2s_W^2\mathcal{C}_{ll'}|^2),$$

$$B(l^-\to l'^- l_1^- l_2^+, l'=l_1=l_2)\approx\frac{\alpha_W^4}{3\times 2^{16}\pi^3}\,\frac{m_l^5}{M_W^4\Gamma_l}\left(|\mathcal{D}_{ll'l'l'}-2(1-2s_W^2)\mathcal{C}_{ll'}|^2+\frac{1}{2}|4s_W^2\mathcal{C}_{ll'}|^2\right),$$

$$B(l^-\to l'^- l_1^- l_2^+, l_2\neq l', l_1)\approx\frac{\alpha_W^4}{3\times 2^{16}\pi^3}\,\frac{m_l^5}{M_W^4\Gamma_l}|\mathcal{D}_{ll'l_1l_2}|^2, \tag{43}$$

$$B(Z\to l^- l'^+ + l^+ l'^-)\approx\frac{\alpha_W^3}{3\times 2^8 c_W^3}\,\frac{M_W}{\Gamma_Z}|\mathcal{C}_{ll'}|^2, \tag{44}$$

$$R(\mu^-\mathrm{Ti}\to e^-\mathrm{Ti})\approx\frac{\alpha_W^4\alpha_{em}^3}{2^{10}\pi^2}\,\frac{Z_{\mathrm{eff}}^4}{Z}|F(-m_\mu^2)|^2 Q_W^2\frac{m_\mu^5}{M_W^4\Gamma_{\mathrm{capture}}}|\mathcal{C}_{\mu e}|^2, \tag{45}$$

$$|G_{M\bar{M}}|\approx\frac{\alpha_W^2}{2^5 M_W^2}|\mathcal{D}_{\mu ee\mu}|, \tag{46}$$

$$B(\tau\to lP^0, cqf)\approx\frac{\alpha_W^4(\alpha_{P^0}^Z)^2}{2^{13}\pi}\left(1-\frac{m_{P^0}^2}{m_\tau^2}\right)^2\frac{m_\tau^3 f_{P^0}^2}{M_W^4\Gamma_\tau}|\mathcal{C}_{\tau l}|^2, \tag{47}$$

$$B(\tau\to lV^0, cqf)\approx\frac{\alpha_W^4(\alpha_{V^0}^Z)^2}{2^{13}\pi\gamma_{V^0}^2}\left(1-\frac{m_{V^0}^2}{m_\tau^2}\right)^2\left(1+2\frac{m_{V^0}^2}{m_\tau^2}\right)\frac{m_\tau^3 m_{V^0}^2}{M_W^4\Gamma_\tau}|\mathcal{C}_{\tau l}|^2, \tag{48}$$

$$B(\tau\to lP^0, ncqf)\approx\frac{\alpha_W^4(\alpha_{P^0}^{\mathrm{box},ds})^2}{2^{11}\pi}\left(1-\frac{m_{P^0}^2}{m_\tau^2}\right)^2\frac{m_\tau^3 f_{P^0}^2}{M_W^4\Gamma_\tau}|F_{\mathrm{box}}^{\tau lds}|^2, \tag{49}$$

$$B(\tau\to lV^0, ncqf)\approx\frac{\alpha_W^4(\alpha_{V^0}^{\mathrm{box},ds})^2}{2^{11}\pi\gamma_{V^0}^2}\left(1-\frac{m_{V^0}^2}{m_\tau^2}\right)^2\left(1+2\frac{m_{V^0}^2}{m_\tau^2}\right)\frac{m_\tau^3 m_{V^0}^2}{M_W^4\Gamma_\tau}|F_{\mathrm{box}}^{\tau lds}|^2, \tag{50}$$

$$B(P^0\to e\mu, cqf)\approx\frac{\alpha_W^4(\alpha_{P^0}^Z)^2}{2^{12}\pi}\left(1-\frac{m_\mu^2}{m_{P^0}^2}\right)^2\frac{m_{P^0} m_\mu^2 f_{P^0}^2}{M_W^4\Gamma_{P^0}}|\mathcal{C}_{\mu e}|^2, \tag{51}$$

$$B(P^0\to e\mu,ncqf)\approx\frac{\alpha_W^4(\alpha_{P^0}^{\mathrm{box},ds})^2}{2^{10}\pi}\left(1-\frac{m_\mu^2}{m_{P^0}^2}\right)^2\frac{m_{P^0}m_\mu^2f_{P^0}^2}{M_W^4\Gamma_{P^0}}|F_{\mathrm{box}}^{\mu eds}|^2, \tag{52}$$

$$B(\mathcal{H}^0\to ll')\approx\frac{\alpha_W^4}{2^{10}\pi}\left(1-\frac{m_l^2}{m_{\mathcal{H}^0}^2}\right)^2\frac{m_{\mathcal{H}^0}m_l^2f_{\mathcal{H}^0}^2}{M_W^4\Gamma_{\mathcal{H}^0}}|F_{\mathrm{box}}^{ll'q_aQ}|^2, \tag{53}$$

$$B(\tau\to lP_1P_2,cqf)\approx\frac{\alpha_W^4}{2^{16}\pi^3}\frac{\int_{(m_1+m_2)^2}^{(m_\tau-m_l)^2}dt\,\alpha\left|\sum_{V^0}p_{BW}^{V^0}(q)\alpha_{V^0}^ZC_{V^0P_1P_2}\right|^2}{M_W^4m_\tau^3\Gamma_\tau}|\mathcal{C}_{\tau l}|^2, \tag{54}$$

$$B(\tau\to lP_1P_2,ncqf)\approx\frac{\alpha_W^4(\alpha_{K^{*0}}^{\mathrm{box},sd})^2|C_{K^{*0}P_1P_2}|^2}{2^{14}\pi^3}|F_{\mathrm{box}}^{\tau lsd}|^2 \times\frac{\int_{(m_1+m_2)^2}^{(m_\tau-m_l)^2}dt\{\alpha+2[(m_1^2-m_2^2)/m_{K^{*0}}^2]\beta+[(m_1^2-m_2^2)/m_{K^{*0}}^2]^2\gamma\}|p_{BW}^{K^{*0}}(q)|^2}{M_W^4m_\tau^3\Gamma_\tau}, \tag{55}$$

$$B(\tau\to lP_1P_2,cqfH)\approx\frac{\alpha_W^4}{2^{16}\pi^3}\frac{M_{HP_1P_2}^4\int_{(m_1+m_2)^2}^{(m_\tau-m_l)^2}dt\,\iota}{M_H^4M_W^4m_\tau\Gamma_\tau}\left|\frac{3}{2}\mathcal{C}_{\tau l}\right|^2, \tag{56}$$

$$B(K_W\to\pi\mu^\mp e^\pm)\approx\frac{\alpha_W^4\tilde{c}_{K^{*0}K_W\pi}^2}{2^{14}\pi^3}\frac{\int_{(m_\mu-m_e)^2}^{(m_{P_1}-m_{P_2})^2}dt[A_{++}f_+^2+A_{+-}f_+f_-+A_{--}f_-^2]}{M_W^4m_K^3\Gamma_{K_W}}|F_{\mathrm{box}}^{\mu lsd}|^2, \tag{57}$$

$$B(\mathcal{H}_a\to Pl'^-l^+)\approx\frac{\alpha_W^4(N_P^{q_a})^2}{2^{13}\pi^3}\frac{\int_{(m_l+m_{l'})^2}^{(m_{\mathcal{H}_a}-m_P)^2}dt[a_P^2Z_{P1}+a_Pb_PZ_{P2}+b_P^2Z_{P3}]}{M_W^4m_{\mathcal{H}_a}^3\Gamma_{\mathcal{H}_a}}|F_{\mathrm{box}}^{ll'Qq_a}|^2, \tag{58}$$

$$B(\mathcal{H}_a\to Vl'^-l^+)\approx\frac{\alpha_W^4(N_V^{q_a})^2}{2^{12}\pi^3}|F_{\mathrm{box}}^{ll'Qq_a}|^2\frac{1}{M_W^4m_{\mathcal{H}_a}^3\Gamma_{\mathcal{H}_a}}\int_{(m_l+m_{l'})^2}^{(m_{\mathcal{H}_a}-m_V)^2}dt[a_V^2Z_{V1}+b_V^2Z_{V2}+c_V^2Z_{V3}+d_V^2Z_{V4}+a_Vc_VZ_{V5} +b_Vc_VZ_{V6}+b_Vd_VZ_{V7}+c_Vd_VZ_{V8}], \tag{59}$$

$$B(B\to B'e\mu)\approx\frac{\alpha_W^4}{2^{10}\pi^3}|F_{\mathrm{box}}^{\mu eds}|^2\frac{1}{M_W^4m_B^3\Gamma_B}\int_{(m_\mu+m_e)^2}^{(m_B-m_{B'})^2}dt[A_1(f_1^2+g_1^2)+A_2(f_1^2-g_1^2)+A_3(f_1g_1)+A_4(g_1g_3) +A_5(g_3^2)]. \tag{60}$$

All these expressions are written in terms of products of dimensionless factors. For the expressions containing the dominant term $\mathcal{C}_{ll'}$, the error one makes by keeping only the dominant term is of the order $\gtrsim 20\%$, because the term $\mathcal{C}_{ll'}$ is always accompanied with the $\mathcal{B}_{ll'}$ term giving $\sim 10\%$ contribution to the amplitude at m_N^{PUB}. The following abbreviations are used: $s_W=\sin\theta_W$, $c_W=\cos\theta_W$ (θ_W is Weinberg's angle), $s_P=\sin\theta_P$, $c_P=\cos\theta_P$ (θ_P is the mixing angle for psudoscalar nonet states), and $s_V=\sin\theta_P$, $c_V=\cos\theta_P$ (θ_V is the mixing angle for vector nonet states). In Eq. (45) $\alpha_{em}=1/137$ is the fine structure constant, Z is atomic number (for ${}^{48}_{22}$Ti $Z=22$, $N=A-Z=26$), $Z_{\mathrm{eff}}=17.6$ [70–72] is the effective atomic number of Ti [73], $F(-m_\mu^2)=0.54$ is its nuclear form factor [74,75] at momentum transfer $q^2\approx -m_\mu^2$ [70], $Q_W=Z(1-4s_W^2)-N$ is the coherent nuclear charge associated with coupling of Z boson to nucleus [71] and $\Gamma_{\mathrm{capture}}$ is the capture rate for negative muons on Ti [71,76,77]. In Eqs. (47)–(53) f_P and $f_{\mathcal{H}}$ are decay constants

TABLE II. (a) Coefficients defining the meson content in axial-vector quark currents with denoted quark content and normalization given by Eq. (61). Two additional coefficients are different from zero: $\alpha_{K^0}^{\mathrm{box},ds}=1$ and $\alpha_{\overline{K^0}}^{\mathrm{box},sd}=1$. (b) Coefficients defining the meson content in vector quark currents with denoted quark content and normalization given in Eq. (61). Two additional coefficients are different from zero: $\alpha_{K^{*0}}^{\mathrm{box},ds}=-1$ and $\alpha_{\overline{K^{*0}}}^{\mathrm{box},sd}=-1$.

(a)

P^0	$\alpha_{P^0}^{Z}$	$\alpha_{P^0}^{\mathrm{box},uu}$	$\alpha_{P^0}^{\mathrm{box},dd}$	$\alpha_{P^0}^{\mathrm{box},ss}$
π^0	$-\sqrt{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	0
η	$-\frac{\sqrt{2}c_P}{\sqrt{3}}-\frac{s_P}{\sqrt{3}}$	$-\frac{c_P}{\sqrt{6}}+\frac{s_P}{\sqrt{3}}$	$\frac{c_P}{\sqrt{6}}-\frac{s_P}{\sqrt{3}}$	$-\frac{\sqrt{2}c_P}{\sqrt{3}}-\frac{s_P}{\sqrt{3}}$
η'	$\frac{c_P}{\sqrt{3}}-\frac{\sqrt{2}s_P}{\sqrt{3}}$	$-\frac{c_P}{\sqrt{3}}-\frac{s_P}{\sqrt{6}}$	$\frac{c_P}{\sqrt{3}}+\frac{s_P}{\sqrt{6}}$	$\frac{c_P}{\sqrt{3}}-\frac{\sqrt{2}s_P}{\sqrt{3}}$

(b)

V^0	$\alpha_{V^0}^{Z}$	$\alpha_{V^0}^{\mathrm{box},uu}$	$\alpha_{V^0}^{\mathrm{box},dd}$	$\alpha_{V^0}^{\mathrm{box},ss}$	$\beta_{V^0}^{\gamma}$	$\gamma_{V^0}^{\gamma}$
ρ^0	$\sqrt{2}c_{2W}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	0	$2\sqrt{2}s_W^2$	$-2\sqrt{2}s_W^2$
ϕ	$\frac{\sqrt{2}c_Vc_{2W}}{\sqrt{3}}+\frac{s_V}{\sqrt{3}}$	$\frac{c_V}{\sqrt{6}}-\frac{s_V}{\sqrt{3}}$	$-\frac{c_V}{\sqrt{6}}+\frac{s_V}{\sqrt{3}}$	$\frac{\sqrt{2}c_V}{\sqrt{3}}+\frac{s_V}{\sqrt{3}}$	$\frac{2\sqrt{2}c_Vs_W^2}{\sqrt{3}}$	$-\frac{2\sqrt{2}c_Vs_W^2}{\sqrt{3}}$
ω	$-\frac{c_V}{\sqrt{3}}+\frac{\sqrt{2}s_Vc_{2W}}{\sqrt{3}}$	$\frac{c_V}{\sqrt{3}}+\frac{s_V}{\sqrt{6}}$	$-\frac{c_V}{\sqrt{3}}-\frac{s_V}{\sqrt{6}}$	$-\frac{c_V}{\sqrt{3}}+\frac{\sqrt{2}s_V}{\sqrt{3}}$	$\frac{2\sqrt{2}s_Vs_W^2}{\sqrt{3}}$	$-\frac{2\sqrt{2}s_Vs_W^2}{\sqrt{3}}$

of light and heavy pseudoscalar mesons respectively, and γ_{V^0} are constants defining the decay constants for light vector mesons, $f_V=m_V/\gamma_V$. The normalizations used here are

$$\mathcal{A}_P^{\mu}(x)=if_P\partial^{\mu}P(x),$$

$$\mathcal{V}_V^{\mu}(x)=\begin{cases}\dfrac{m_V^2}{\gamma_V}V^{\mu}(x), & \text{for light vector mesons,}\\ f_Vm_VV^{\mu}(x), & \text{for heavy vector mesons,}\end{cases} \tag{61}$$

where $\mathcal{A}_P^{\mu}$ and $\mathcal{V}_V^{\mu}$ are the axial-vector (vector current) with the same quark content as corresponding pseudoscalar-meson $P(x)$ and vector-meson $V(x)$ fields, respectively. $\alpha_{P^0}^{Z}$, $\alpha_{V^0}^{Z}$, $\alpha_{P^0}^{\mathrm{box},d_ad_b}$, and $\alpha_{V^0}^{\mathrm{box},d_ad_b}$ are constants defining the meson content in vector and axial-vector quark currents contained in quark combinations in the Z and box amplitudes. They are defined in Table II. The mass of the lighter lepton is neglected in the expressions (47)–(53). As the composite-loop function $F_{\mathrm{box}}^{ll'q_aq_b}$ contains two terms of approximately equal magnitude, for brevity the expresions of these BR's are not written in terms of the functions (38). The expressions (54)–(60) contain three-body phase integrals. The phase functions may be found in the following references: α, β, γ, and ι in Ref. [66]; A_{++}, A_{+-}, and A_{--} in Ref. [10]; A_1, A_2, A_3, A_4, and A_5 in Ref. [10]. In Eq. (54), $C_{V^0P_1P_2}=g_{\rho^0\pi^+\pi^-}c_{V^0P_1P_2}$ are constants stemming from the gauged chiral $\mathrm{U}(3)_L\times\mathrm{U}(3)_R/\mathrm{U}(3)_V$ Lagrangian [66] (e.g., $c_{\rho^0\pi^+\pi^-}=1$), and

$$p_{\mathrm{BW}}^{V^0}(q)=\frac{1}{\gamma_{V^0}}\frac{m_{V^0}^2-i\Gamma_{V^0}m_{V^0}}{m_{V^0}^2-i\Gamma_{V^0}m_{V^0}-q^2}\equiv\frac{1}{\gamma_{V^0}}p_{\mathrm{BW}}^{V^0,\mathrm{norm}}(q) \tag{62}$$

is the Breit-Wigner propagator for a vector meson V^0 multiplied by slightly modified expression m_V^2/γ_V. The modification of the expression m_V^2/γ_V is made to obtain $p_{\mathrm{BW}}^{V^0,\mathrm{norm}}(0)=1$ [66,78–80]. The constant $g_{\rho^0\pi^+\pi^-}$ is equal to the ρ self coupling constant g_V from Sec. IV A. In Eq. (56) $M_{HP_1P_2}^2$ are mass parameters contained in the effective Higgs-meson Lagrangian [66],

$$\begin{aligned}\mathcal{L}_{HMM}=\frac{g_W}{4M_W}\Bigg[&m_\pi^2((\pi^0)^2+2\pi^+\pi^-)+2m_{K^+}^2K^+K^-\\&+2m_{K^0}^2K^0\overline{K^0}+\frac{m_{K^+}^2+m_{K^0}^2+m_\pi^2}{3}\eta_1^2\\&+\frac{2m_{K^+}^2+2m_{K^0}^2-m_\pi^2}{3}\eta_8^2\\&+\frac{2^{3/2}(2m_\pi^2-m_{K^+}^2-m_{K^0}^2)}{3}\eta_8\eta_1\Bigg],\end{aligned} \tag{63}$$

obtained by comparing the quark mass Lagrangian with the corresponding term in the chiral Lagrangian [81] [e.g., $M_{H\pi^+\pi^-}^2=2m_\pi^2\equiv2(2m_{\pi^+}^2+m_{\pi^0}^2)/3$]. In Eq. (57) $\tilde{c}_{K^{*0}K_W\pi}=ac_{K^{*0}\overline{K}_S\pi}+bc_{\overline{K^{*0}}K_S\pi}$ ($K_W=a\overline{K}_S+bK_S$ is a weak kaon eigenstate, and K_S is a mass eigenstate). In Eq. (60) f_1, g_1, and g_3 are baryon form factors. The other baryon form factors do not contribute, because they belong to the second class currents, or give a contribution proportional to the difference of baryon masses. The form factors f_1 and g_1 can be defined in terms of two SU(3) Clebsh-Gordan coefficients and two reduced matrix elements corresponding to symmetric and antisymmetric octet SU(3) representations. These reduced matrix elements are almost independent of momentum transfer and are usually identified with their value at zero momentum transfer, D and F. The functions g_1 and g_3 are not independent, but correlated through the Goldberger-Treiman relation [10].

All approximate expressions for the BR's (42)–(60), valid in the large-mass limit, except $B(l^-\to l'^- l_1^- l_2^+, l_1=l_2\neq l')$ and $B(l^-\to l'^- l_1^- l_2^+, l'=l_1=l_2)$, depend only on one of the functions (36). In the following, the smaller of two dominant functions $\mathcal{D}_{ll'l_1l_2}$ will be neglected in the two exceptional expressions. The maximal error one makes in the evaluation of BR's of the exceptional processes is $\sim 40\%$. With those approximations, the ratios of BR's having the same dominant function (36) become independent of the V-model parameters:

(a) BR's with $F_Z^{\mu e}(\mathcal{C}_{\mu e})$:
$$R(\mu\mathrm{Ti}\to e\mathrm{Ti}):B(\mu\to ee^-e^+):B(Z\to\mu^\mp e^\pm):B(\pi^0\to\mu^\mp e^\pm):B(\eta\to\mu^\mp e^\pm)$$
$$=1:5.60\times10^{-2}:3.77\times10^{-2}:6.05\times10^{-10}:1.69\times10^{-11}, \tag{64}$$

(b) BR's with $F_Z^{\tau l}$ $(\mathcal{C}_{\tau l})$:
$$B(Z\to\tau^\mp l^\pm):B(\tau\to l\pi^0):B(\tau\to l\rho^0):B(\tau\to l\pi^+\pi^-):B(\tau\to l\phi)$$
$$:B(\tau\to ll^-l^+):B(\tau\to ll_1^-l_1^+):B(\tau\to lK^+K^-):B(\tau\to lK^0\overline{K^0}):B(\tau\to l\eta')$$
$$:B(\tau\to l\eta):B(\tau\to l\omega):B(\tau\to l\eta\eta):B(\tau\to l\pi^0\pi^0)$$
$$=1:3.40\times10^{-1}:3.17\times10^{-1}:2.83\times10^{-1}:2.81\times10^{-1}:2.64\times10^{-1}:1.64\times10^{-1}$$
$$:1.20\times10^{-1}:7.43\times10^{-2}:6.15\times10^{-2}:4.72\times10^{-2}$$
$$:8.78\times10^{-3}:4.34\times10^{-12}:5.50\times10^{-13}, \tag{65}$$

(c) BR's with $F_{\mathrm{box}}^{\mu esd}$:
$$B(K_L\to\mu^\mp e^\pm):B(K^+\to\pi^+\mu^\mp e^\pm):B(\Sigma^+\to p\mu^\mp e^\pm):B(\Xi^0\to\Lambda\mu^\mp e^\pm):B(\Lambda\to n\mu^\mp e^\pm)$$
$$:B(\Xi^-\to\Sigma^-\mu^\mp e^\pm):B(\Xi^0\to\Sigma^0\mu^\mp e^\pm):B(\Sigma^0\to n\mu^\mp e^\pm)=1:3.01\times10^{-2}$$
$$:1.30\times10^{-4}:1.21\times10^{-4}:8.66\times10^{-5}$$
$$:6.40\times10^{-7}:4.07\times10^{-7}:6.31\times10^{-14}, \tag{66}$$

(d) BR's with $F_{\mathrm{box}}^{\mu ebd}$:
$$B(B^-\to\pi^-\mu^\mp e^\pm):B(B^0\to\mu^\mp e^\pm)=1:3.76\times10^{-4}, \tag{67}$$

(e) BR's with $F_{\mathrm{box}}^{\mu ebs}$:
$$B(B^-\to K^{*-}\mu^\mp e^\pm):B(\overline{B^0}\to K^{*0}\mu^\mp e^\pm):B(\overline{B_s^0}\to\eta'\mu^\mp e^\pm):B(\overline{B_s^0}\to\phi\mu^\mp e^\pm):B(B^-\to K^-\mu^\mp e^\pm)$$
$$:B(\overline{B^0}\to\overline{K^0}\mu^\mp e^\pm):B(\overline{B_s^0}\to\eta\mu^\mp e^\pm):B(\overline{B_s^0}\to\mu^\mp e^\pm)=1:9.34\times10^{-1}:8.83\times10^{-1}$$
$$:8.57\times10^{-1}:7.92\times10^{-1}:7.47\times10^{-1}$$
$$:3.31\times10^{-1}:4.93\times10^{-4}, \tag{68}$$

(f) BR's with $F_{\mathrm{box}}^{\tau lds}$:
$$B(\tau\to e\pi^+K^-):B(\tau\to eK^{*0}):B(\tau\to eK^0)=1:7.32\times10^{-1}:2.99\times10^{-1}, \tag{69}$$

(g) BR's with $F_{\mathrm{box}}^{\tau lbd}$:
$$B(B^-\to\pi^-\tau^\mp e^\pm):B(\overline{B^0}\to\tau^\mp e^\pm)=1:1.14\times10^{-1}, \tag{70}$$

(h) BR's with $F_{\mathrm{box}}^{\tau lbs}$:
$$B(B^-\to K^{*-}\tau^\mp e^\pm):B(\overline{B^0}\to\overline{K}^{*0}\tau^\mp e^\pm):B(\overline{B_s^0}\to\phi\tau^\mp e^\pm):B(\overline{B_s^0}\to\eta'\tau^\mp e^\pm)$$
$$:B(B^-\to K^-\tau^\mp e^\pm):B(\overline{B^0}\to\overline{K^0}\tau^\mp e^\pm):B(\overline{B_s^0}\to\eta\tau^\mp e^\pm):B(\overline{B_s^0}\to\tau^\mp e^\pm)=1:9.37\times10^{-1}$$
$$:8.10\times10^{-1}:6.57\times10^{-1}:6.54\times10^{-1}:6.16\times10^{-1}:2.76\times10^{-1}:1.63\times10^{-1}. \tag{71}$$

For each group of BR's the BR's are lined up in the descending order. For instance, $\mu \to e$ conversion is the most suitable for finding LFV in the group containing the composite-loop function $F_Z^{\mu e}$. The position in the group depends on the coupling constants, phase factors and the total decay rate of the decaying particle. For instance, the BR's for $\tau \to l P_1 P_2$ processes containing Z-boson amplitude are $\sim 10^{12}$ times larger than BR's of the processes $\tau \to l P_1 P_2$ containing only Higgs amplitude, because of the small Higgs-meson couplings, although the dominant composite-loop functions are essentially the same. The ratios (64)–(71) are given for measured processes and LFV processes that have not been studied in models with additional heavy neutrinos before. The $l \to l' \gamma$ decays are not included in the above ratios, because each process $l \to l' \gamma$ forms a group for itself, depending only on the function $\mathcal{A}_{ll'}$. The numerical results for the ratios of BR's agree quite well with the exact ratios for degenerate heavy-neutrino masses obtained at $m_N = m_N^{\rm PUB}$. That allows one to consider only one of the decays of each group when comparing theoretical and experimental results.

In addition to the ratios of BR's having the same dominant function (36), it is usefull to have relations that relate BR's of different groups of decays. These relations generally depend on the matrix elements of B matrix and CKM matrix. For instance,

$$
\begin{aligned}
&B(Z \to \tau^\mp e^\pm):B(Z \to \tau^\mp \mu^\pm):B(Z \to \mu^\mp e^\pm) = |F_Z^{\tau e}|^2 : |F_Z^{\tau\mu}|^2 : |F_Z^{\mu e}|^2,\\
&B(\tau \to e P^0):B(\tau \to \mu P^0):B(P^0 \to e^\mp \mu^\pm) = \begin{cases} |F_Z^{\tau e}|^2 : |F_Z^{\tau\mu}|^2 : \dfrac{1.45\times 10^9 s^{-1}}{\Gamma_{P^0}} \dfrac{m_{P^0}}{m_\mu} \dfrac{(1-m_\mu^2/m_{P^0}^2)^2}{(1-m_{P^0}^2/m_\tau^2)^2} : |F_Z^{\mu e}|^2 & \text{for } cqf,\\ |F_{\rm box}^{\tau eds}|^2 : |F_{\rm box}^{\tau\mu ds}|^2 : \dfrac{1.45\times 10^9 s^{-1}}{\Gamma_{P^0}} \dfrac{m_{P^0}}{m_\mu} \dfrac{(1-m_\mu^2/m_{P^0}^2)^2}{(1-m_{P^0}^2/m_\tau^2)^2} |F_{\rm box}^{\mu eds}|^2 & \text{for } ncqf, \end{cases}\\
&B(\tau \to e\gamma):B(\tau \to \mu\gamma):B(\mu \to e\gamma) = |B_{\tau N}^* B_{eN}|^2 : |B_{\tau N}^* B_{\mu N}|^2)^2 : 5.63 |B_{\mu N}^* B_{eN}|^2,\\
&B(\tau \to e e^\mp e^\pm):B(\tau \to \mu\mu^\mp \mu^\pm):B(\mu \to e e^\mp e^\pm) \approx |F_Z^{\tau e}|^2 : |F_Z^{\tau\mu}|^2 : 5.63 |F_Z^{\mu e}|^2,\\
&B(\tau \to e K^+ \pi^-):B(\tau \to \mu K^+ \pi^-):B(K^- \to \pi^- \mu^\mp e^\pm) = |F_{\rm box}^{\tau eds}|^2 : 0.983 |F_{\rm box}^{\tau\mu ds}|^2 : 6.82 |F_{\rm box}^{\mu eds}|^2,\\
&B(B_i \to l_1^\mp l_2^\pm):B(B_j \to l_3^\mp l_4^\pm) = |F_{\rm box}^{l_2 l_1 q_i b}|^2 : |F_{\rm box}^{l_4 l_3 q_j b}|^2, \quad q_i, q_j = u,d,s. \qquad (72)
\end{aligned}
$$

BR's of processes having only the logarithmic dependence on mass are several orders of magnitude smaller than BR's containing the quadratic mass-dependent terms. In the processes containing quarks in the final state, the presence of small CKM matrix elements additionally reinforces this difference. For example, at $m_N^{\rm PUB}$

$$
\begin{aligned}
&B(l \to l' \gamma):B(l \to l' l_1 l_2) \lesssim 10^{-2},\\
&B(\tau^- \to e^- K^0):B(\tau^- \to e^- \pi^0) \approx B(\tau^- \to e^- \pi^+ K^-):B(\tau^- \to e^- \pi^- \pi^+) \lesssim 10^{-9}. \qquad (73)
\end{aligned}
$$

Between the LFV decays having the box contribution only, the B-meson decays have the largest CKM matrix elements. For that reason they might be the most suitable box-dominated processes for finding LFV in the future B factories.

C. Numerical results, comparison with experiment and discussion

In this subsection the experimental upper bounds for the measured neutrinoless LFV BR's are compared with the theoretical upper bounds obtained in the V model. For some interesting unmeasured processes, the theoretical upper bounds are given, too. The results are discussed. The limit on the nondiagonal μ-e mixing is updated. The decoupling of very heavy neutrinos is shown explicitely. The possible error one can make using the upper-bound procedure given in the Sec. III D is estimated.

Theoretical results depend on the V-model parameters: ''diagonal'' mixings $s_L^{\nu_l}$, phases of B_{lN}'s and heavy-neutrino masses. The parameters $s_L^{\nu_l}$ must satisfy the experimental upper bounds (4), the heavy-neutrino masses are bound by the PUB inequalities (13) and (14), while the phases of B_{lN} matrices are undetermined. The numerical results are in principle largest for degenerate neutrino masses at maximal values of $s_L^{\nu_l}$ parameters and maximal neutrino mass $m_N^{\rm PUB}$. For degenerate heavy-neutrino masses the phase dependence of B_{lN} matrices is contained in the parameters $x_{ll'}$.

The numerical values for BR's and $G_{M\bar{M}}$ depend on a number of ''SM'' particle properties, too: decay rates of particles, masses of the particles included in the decays, CKM matrix elements, decay constants of mesons, quark masses included in loops, mixing angles, various couplings, etc. Almost all these quantities are taken from Ref. [36], or derived from the data given there. For instance, masses of the u, d, s, c, and b quarks are taken to be equal to the average of the upper- and lower-bound values. The CKM matrix elements are derived in the same way. The t-quark mass is set to be equal to the experimental value obtained from the direct observations of t quark. For pseudoscalar-meson decay constants of light mesons, we took the values partly from Ref. [36] and partly from Ref. [82],

$$f_{\pi^+}=130.7\ \text{MeV},\quad f_{K^+}=159.8\ \text{MeV},$$
$$f_{\pi^0}=119\ \text{MeV},$$
$$f_\eta=131\ \text{MeV},\quad \text{and}\quad f_{\eta'}=118\ \text{MeV}. \tag{74}$$

Due to the isospin symmetry, $f_{K^0}=f_{\bar{K}^0}=f_{K^\pm}$. The constants γ_{V^0}, defining the decay constants of light vector mesons, are extracted from the $V^0\to e^+e^-$ decay rates

$$\gamma_{\rho^0}=2.518,\quad \gamma_\phi=2.933,\quad \gamma_\omega=3.116, \tag{75}$$

or estimated using the SU(3) octet symmetry, $\gamma_{K^{*0}}=\gamma_{\rho^0}$. For all decay constants of D and D^* mesons, the conservative value 200 MeV is taken. The decay constants of B and B^* mesons are derived using the scaling law for decay constants derived from HQET,

$$f_{\mathcal{H}}\sim\ m_{\mathcal{H}}^{-1/2}. \tag{76}$$

The weak fine-structure constant is defined as $\alpha_W=\alpha_{em}/\sin^2\theta_W$, with $\cos\theta_W=M_W/M_Z$. The $\rho-\pi-\pi$ coupling constant (which is equal to the ρ-meson self-coupling constant) is derived from the $\rho\to2\pi$ coupling width. Other vector-meson–pseudoscalar-meson couplings of light mesons are fixed by one of the chiral model Lagrangians [57,66]. The mixing of the vector-meson nonet states is determined from the quadratic Gell-Mann–Okubo mass formula $\theta_V=39.3°$. The mixing of the pseudoscalar-meson nonet states is extracted from the $e^+e^-\to e^+e^-\gamma\gamma^*\to e^+e^-(P\to\gamma\gamma)$ experiments [83], $\theta_P=-23°$. The only ''SM'' parameters that are not firmly established are ''HQET +CHPT'' parameters describing the semileptonic LFV decays of the B mesons g, β, λ, α_1, and α_2 (see Sec. IV A). The corresponding parameters for D mesons have been determined by fitting the theory to the experimental values of the semileptonic decays of D mesons [56,84,85]. The B-meson parameters λ, α_1, and α_2 may be derived from the D-meson parameters from the scaling laws for the vector and axial-vector current [51]. The parameter g is independent of a heavy-quark mass, and the value of parameter β is consistent with zero. The best B-meson parameters obtained using the above procedure are [85]

$$g=0.2,\quad \beta=0,\quad \lambda=-0.34\ \text{GeV}^{-1},$$
$$\alpha_1=-0.13\ \text{GeV}^{1/2},\quad \alpha_2=-0.36\ \text{GeV}^{1/2}. \tag{77}$$

That way, all parameters are defined.

For measured processes, the experimental and theoretical upper bounds of the exact BR's are compared in Table III(a). For some interesting processes that have not been measured, the theoretical upper bounds are given in Table III(b). In both tables, the numerical part of the theoretical results is evaluated for degenerate heavy-neutrino masses and the maximal heavy-neutrino mass permited by PUB, maximal $s_L^{\nu_l}$ values and neglecting the B_{lN} phases. The factors $x_{ll'}$, $y_{ll'}$, $y_{ll'l_1l_2}$, and $z_{ll'}$ describe the deviation of BR's from these values, when the model parameters assume other values. The factors $y_{ll'}$ and $y_{ll'l_1l_2}$ give only the behavior of the dominant, m_N^2-dependent term, on the model parameters. For m_N values for which the terms quadratic in B_{lN} matrices begin to dominate ($m_N\sim1000-1500$ GeV), the $z_{ll'}$ terms begin to dominate.

Comparing the theoretical upper bounds for the processes of the same type with different leptons in the initial and final state, one can see that they are often comparable in magnitude. For instance, upper bounds for BR's of the processes $l\to l'\gamma$, $l\to l'l_1l_2$, and $Z\to ll'$ are of the order $\sim10^8$, $\sim10^{-6}$, and $\sim10^{-6}$, respectively. For that reason, the muon LFV processes which have been measured with the greatest precision, are the most attractive for finding LFV. A process with weaker experimental bounds may be interesting only if the parameter(s) $x_{ll'}^0$ for that process is (are) large.

If, for a specific process, the theoretical upper bound is larger than the experimental one, then the process gives the better bound on a specific combination of B_{lN}'s than the limit (4). The processes for which this ratio is larger than one are $\mu\to e\gamma$, $\mu\to eee$, $\mu\text{Ti}\to e\text{Ti}$, $\tau\to e\rho^0$, $\tau\to e\pi^+\pi^-$, and $Z\to e\tau$. For the last three processes the ratio is very close to one. As their amplitudes are dominated by m_N^2 part of the amplitude, the new limits on B_{lN} combinatons contain m_N^2 mass dependence, too, and therefore are uninteresting. For the first three processes the ratio is much larger than one, and they do give new limits on specific combinations of B_{lN}'s as shown in Ref. [12]. Since that paper was published, the limits on $B(\mu\to e\gamma)$ and $R(\mu\,\text{Ti}\to e\,\text{Ti})$ improved by factors 1.3 (4.1 [86]) and 7, respectively. The improvement of $B(\mu\to e\gamma)$ gives a new limit on nondiagonal μ-e mixing,

$$\sum_{i=1}^{n_G} B_{\mu N_i}B_{eN_i}^*\leqslant 2.15\times10^{-4}\ \ (1.19\times10^{-4}). \tag{78}$$

To obtain the limit on the nondiagonal τ-e and τ-μ mixings, the present experimental sensitivities of $\tau\to l$ decays should improve by two orders of magnitude. It is interesting that the $\mu\text{Ti}\to e\text{Ti}$ conversion also gives very good mass-independent limit on the sum $\sum_{i=1}^{n_G}B_{\mu N_i}B_{eN_i}^*$. Namely, $\mu\text{Ti}\to e\,\text{Ti}$ amplitude contains the mass-independent part coming from the photon exchange. If the terms in the $\mu\,\text{Ti}\to e\,\text{Ti}$ amplitude do not cancel completely, one can make an esti-

TABLE III. (a) The comparison of experimental and theoretical upper bounds on LFV BR's. Experimental upper bounds for unmarked processes are taken from Ref. [36], while those denoted by # are from Ref. [37]. The newest value $B^{UB}(\mu^-\to e^-\gamma)=1.2\times10^{-11}$ is given in Ref. [86]. (b) Theoretical upper bounds for some interesting BR's, for which experimental upper bounds have not been found.

(a)

Process	$B_{\rm exp}^{UB}$	$B_{\rm th}^{UB}$
$^{\#}\mu^-\to e^-\gamma$	3.8×10^{-11}	$8.08\times10^{-9}x_{\mu e}^2$
$\tau^-\to e^-\gamma$	2.7×10^{-6}	$3.38\times10^{-8}x_{\tau e}^2$
$\tau^-\to\mu^-\gamma$	3.0×10^{-6}	$6.68\times10^{-9}x_{\tau\mu}^2$
$\mu^-\to e^-e^+e^-$	1.0×10^{-12}	$6.41\times10^{-7}y_{\mu e}^2$
$\tau^-\to e^-e^+e^-$	2.9×10^{-6}	$2.69\times10^{-6}y_{\tau e}^2$
$\tau^-\to\mu^-\mu^+\mu^-$	1.9×10^{-6}	$4.48\times10^{-7}y_{\tau\mu}^2$
$\tau^-\to e^-\mu^+\mu^-$	1.8×10^{-6}	$1.44\times10^{-6}y_{\tau e}^2$
$\tau^-\to\mu^-e^+e^-$	1.7×10^{-6}	$3.71\times10^{-7}y_{\tau\mu}^2$
$\tau^-\to e^+\mu^-\mu^-$	1.5×10^{-6}	$1.32\times10^{-9}y_{\tau\mu\mu e}^2$
$\tau^-\to\mu^+e^-e^-$	1.5×10^{-6}	$6.67\times10^{-9}y_{\tau ee\mu}^2$
$\tau^-\to e^-\pi^0$	3.7×10^{-6}	$2.77\times10^{-6}y_{\tau e}^2$
$\tau^-\to\mu^-\pi^0$	4.0×10^{-6}	$5.40\times10^{-7}y_{\tau\mu}^2$
$\tau^-\to e^-\eta$	8.2×10^{-6}	$4.01\times10^{-7}y_{\tau e}^2$
$\tau^-\to\mu^-\eta$	9.6×10^{-6}	$7.81\times10^{-8}y_{\tau\mu}^2$
$\tau^-\to e^-\rho^0$	2.0×10^{-6}	$2.70\times10^{-6}y_{\tau e}^2$
$\tau^-\to\mu^-\rho^0$	6.3×10^{-6}	$5.27\times10^{-7}y_{\tau\mu}^2$
$\tau^-\to e^-\phi$	6.9×10^{-6}	$2.30\times10^{-6}y_{\tau e}^2$
$\tau^-\to\mu^-\phi$	7.0×10^{-6}	$4.46\times10^{-7}y_{\tau\mu}^2$
$\tau^-\to e^-\pi^+\pi^-$	2.2×10^{-6}	$2.67\times10^{-6}y_{\tau e}^2$
$\tau^-\to\mu^-\pi^+\pi^-$	8.2×10^{-6}	$5.19\times10^{-7}y_{\tau\mu}^2$
$\tau^-\to e^-K^+K^-$	6.0×10^{-6}	$1.07\times10^{-6}y_{\tau e}^2$
$\tau^-\to\mu^-K^+K^-$	15×10^{-6}	$2.07\times10^{-7}y_{\tau\mu}^2$
$\pi^0\to e^-\mu^+$	1.72×10^{-8}	$5.54\times10^{-15}y_{\mu e}^2$
$\eta\to e^-\mu^+$	6×10^{-6}	$1.61\times10^{-16}y_{\mu e}^2$
$Z\to e^\mp\mu^\pm$	1.7×10^{-6}	$3.43\times10^{-7}y_{\mu e}^2$
$^{\#}Z\to e^\mp\tau^\pm$	7.3×10^{-6}	$8.08\times10^{-6}y_{\tau e}^2$
$^{\#}Z\to\mu^\mp\tau^\pm$	10×10^{-6}	$1.59\times10^{-6}y_{\tau\mu}^2$
$^{\#}\mu^-\mathrm{Ti}\to e^-\mathrm{Ti}$	6.1×10^{-13}	$1.01\times10^{-5}y_{\mu e}^2$
$\tau^-\to e^-K^0$	1.3×10^{-3}	$9.82\times10^{-16}x_{\tau e}^2$
$\tau^-\to\mu^-K^0$	1.0×10^{-3}	$1.93\times10^{-16}x_{\tau\mu}^2$
$\tau^-\to e^-K^{*0}$	5.1×10^{-6}	$2.40\times10^{-15}x_{\tau e}^2$
$\tau^-\to\mu^-K^{*0}$	7.5×10^{-6}	$4.68\times10^{-16}x_{\tau\mu}^2$
$\tau^-\to e^-\overline{K}^{*0}$	7.4×10^{-6}	$2.40\times10^{-15}x_{\tau e}^2$
$\tau^-\to\mu^-\overline{K}^{*0}$	7.5×10^{-6}	$4.68\times10^{-16}x_{\tau\mu}^2$
$\tau^-\to e^-\pi^+K^-$	6.4×10^{-6}	$3.29\times10^{-15}x_{\tau e}^2$
$\tau^-\to\mu^-\pi^+K^-$	6.5×10^{-6}	$6.37\times10^{-16}x_{\tau\mu}^2$
$\tau^-\to e^-\pi^-K^+$	3.8×10^{-6}	$3.29\times10^{-15}x_{\tau e}^2$
$\tau^-\to\mu^-\pi^-K^+$	7.4×10^{-6}	$6.37\times10^{-16}x_{\tau\mu}^2$
$^{\#}K_L\to e^\mp\mu^\pm$	2×10^{-11}	$3.16\times10^{-14}x_{\mu e}^2$
$^{\#}K_L\to\pi^0e^\mp\mu^\pm$	3.2×10^{-9}	0
$^{\#}K^+\to\pi^+e^\mp\mu^\pm$	4.0×10^{-11}	$9.72\times10^{-16}x_{\mu e}^2$
$\overline{B}^0\to e^\mp\mu^\pm$	5.9×10^{-5}	$3.07\times10^{-15}x_{\mu e}^2$
$\overline{B}^0\to e^\mp\tau^\pm$	5.3×10^{-4}	$1.61\times10^{-11}x_{\tau e}^2$
$\overline{B}^0\to\mu^\mp\tau^\pm$	8.3×10^{-4}	$3.18\times10^{-12}x_{\tau\mu}^2$
$\overline{B}_s^0\to e^\mp\mu^\pm$	4.1×10^{-3}	$6.11\times10^{-14}x_{\mu e}^2$
$\overline{B}^-\to\pi^-e^\mp\mu^\pm$	6.4×10^{-3}	$8.16\times10^{-12}x_{\mu e}^2$
$\overline{B}^-\to K^-e^\mp\mu^\pm$	6.4×10^{-3}	$1.02\times10^{-10}x_{\mu e}^2$
$\overline{B}^0\to\overline{K}^0e^\mp\mu^\pm$	1.8×10^{-5}	$9.57\times10^{-11}x_{\mu e}^2$

TABLE III. (*Continued*).

(a)

Process	$B_{\rm exp}^{UB}$	$B_{\rm th}^{UB}$
$\tau^-\to e^-\pi^0\pi^0$	6.5×10^{-6}	$4.02\times10^{-18}y_{\tau e}^2$
$\tau^-\to \mu^-\pi^0\pi^0$	14×10^{-6}	$7.91\times10^{-19}y_{\tau\mu}^2$
$\tau^-\to e^-\eta\eta$	35×10^{-6}	$3.16\times10^{-17}y_{\tau e}^2$
$\tau^-\to \mu^-\eta\eta$	60×10^{-6}	$5.94\times10^{-18}y_{\tau\mu}^2$
$\tau^-\to e^-\pi^0\eta$	22×10^{-6}	0
$\tau^-\to \mu^-\pi^0\eta$	24×10^{-6}	0

(b)

Process	$B_{\rm th}^{UB}$	Process	BthUB
$\tau^-\to e^-K^0\overline{K}^0$	$6.625\times10^{-7}z_{\tau e}^2$	$B^-\to K^{*-}e^{\mp}\mu^{\pm}$	$1.19\times10^{-10}z_{\mu e}^2$
$\tau^-\to \mu^-K^0\overline{K}^0$	$1.282\times10^{-7}z_{\tau\mu}^2$	$B^-\to K^{*-}e^{\mp}\tau^{\pm}$	$1.96\times10^{-9}z_{\tau e}^2$
$B_s^0\to e^{\mp}\tau^{\pm}$	$3.34\times10^{-10}z_{\tau e}^2$	$B^-\to K^{*-}\mu^{\mp}\tau^{\pm}$	$3.85\times10^{-10}z_{\tau\mu}^2$
$B_s^0\to \mu^{\mp}\tau^{\pm}$	$6.62\times10^{-11}z_{\tau\mu}^2$	$\overline{B}^0\to K^{*0}e^{\mp}\mu^{\pm}$	$1.12\times10^{-10}z_{\mu e}^2$
$B^-\to\pi^-e^{\mp}\tau^{\pm}$	$1.14\times10^{-10}z_{\tau e}^2$	$\overline{B}^0\to K^{*0}e^{\mp}\tau^{\pm}$	$1.82\times10^{-9}z_{\tau e}^2$
$B^-\to\pi^-\mu^{\mp}\tau^{\pm}$	$2.24\times10^{-11}z_{\tau\mu}^2$	$\overline{B}^0\to K^{*0}\mu^{\mp}\tau^{\pm}$	$3.60\times10^{-10}z_{\tau\mu}^2$
$B^-\to K^-e^{\mp}\tau^{\pm}$	$1.34\times10^{-9}z_{\tau e}^2$	$\overline{B}_s^0\phi e^{\mp}\mu^{\pm}$	$1.01\times10^{-10}z_{\mu e}^2$
$B^-\to K^-\mu^{\mp}\tau^{\pm}$	$2.63\times10^{-10}z_{\tau\mu}^2$	$\overline{B}_s^0\phi e^{\mp}\tau^{\pm}$	$1.56\times10^{-9}z_{\tau e}^2$
$\overline{B}^0\to\overline{K}^0e^{\mp}\tau^{\pm}$	$1.26\times10^{-9}z_{\tau e}^2$	$\overline{B}_s^0\phi\mu^{\mp}\tau^{\pm}$	$3.06\times10^{-10}z_{\tau\mu}^2$
$\overline{B}^0\to\overline{K}^0\mu^{\mp}\tau^{\pm}$	$2.48\times10^{-10}z_{\tau\mu}^2$	$\Sigma^+\to pe^{\mp}\mu^{\pm}$	$4.09\times10^{-18}z_{\mu e}^2$
$\overline{B}_s^0\to\eta e^{\mp}\mu^{\pm}$	$4.24\times10^{-11}z_{\mu e}^2$	$\Lambda\to ne^{\mp}\mu^{\pm}$	$2.74\times10^{-18}z_{\mu e}^2$
$\overline{B}_s^0\to\eta e^{\mp}\tau^{\pm}$	$5.64\times10^{-10}z_{\tau e}^2$	$\Xi^0\to\Lambda e^{\mp}\mu^{\pm}$	$3.18\times10^{-18}z_{\mu e}^2$
$\overline{B}_s^0\to\eta\mu^{\mp}\tau^{\pm}$	$1.11\times10^{-10}z_{\tau\mu}^2$	$\Xi^0\to\Sigma^0e^{\mp}\mu^{\pm}$	$1.29\times10^{-20}z_{\mu e}^2$
$\overline{B}_s^0\to\eta' e^{\mp}\mu^{\pm}$	$1.13\times10^{-10}z_{\mu e}^2$	$\Xi^-\to\Sigma^-e^{\mp}\mu^{\pm}$	$2.02\times10^{-20}z_{\mu e}^2$
$\overline{B}_s^0\to\eta' e^{\mp}\tau^{\pm}$	$1.35\times10^{-9}z_{\tau e}^2$	$\Sigma^0\to ne^{\mp}\mu^{\pm}$	$1.99\times10^{-27}z_{\mu e}^2$
$\overline{B}_s^0\to\eta'\mu^{\mp}\tau^{\pm}$	$2.64\times10^{-10}z_{\tau\mu}^2$		

mate of the sum by attributing the whole amplitude to the photon-exchange part of the amplitude. That way one can only make a worse estimate of the sum. The limit one obtains that way is

$$\sum_{i=1}^{n_G} B_{\mu N_i}B_{eN_i}^{*}\leqslant 3.93\times10^{-4}. \tag{79}$$

For all processes whose amplitudes comprise only the box amplitude, the theoretical upper bounds are several orders of magnitude smaller than the experimental upper bounds. For the $K_L\to e^{\mp}\mu^{\pm}$ decay the ratio of theoretical and experimental upper bound is largest, $1.58\times10^{-3}x_{\mu e}^2$. As the present experimental limit is 2×10^{-11} [37], its significant improvement cannot be expected. Although the experimental upper bounds for semileptonic LFV B-meson processes are weak, the corresponding theoretical upper bounds are of the order $\sim10^{-9}$. Therefore, B-meson decays are interesting for finding LFV decays in the near future.

The recent Super-Kamiokande experiment shows there is a large mixing between ν_μ and some other light neutrino, very probably ν_τ. If the additional heavy neutrinos exist, this ight suggest a large ''angle'' parameter $x_{\tau\mu}^0$. Therefore, the Super-Kamiokande result might be a sign to search for LFV among processes with tauon and muon in the final (and initial) state.

To estimate how large an error one can make using the upper-bound procedure from Sec. III D, the BR's for the processes $\mu\,{\rm Ti}\to e\,{\rm Ti}$, $Z\to\mu^{\mp}\tau^{\pm}$, $K_L\to e^{\mp}\mu^{\pm}$, and $B^-\to K^{*-}\mu^{\mp}\tau^{\pm}$ are evaluated using both the upper-bound procedure and the ''realistic'' B_{lN}'s (16). These processes are chosen because they have the maximal BR within the group of processes with the same dominant composite-loop function. The first two of these processes contain $F_Z^{ll'}$ function and the last two contain $F_{\rm box}^{ll'd_ad_b}$ function only. The BR's are evaluated for degenerate heavy-neutrino masses and two sets of $s_L^{\nu_l}$ and $x_{ll'}^0$ parameters for which the maximal theoretical value for $B(\mu\,{\rm Ti}\to e\,{\rm Ti})$ is equal to the present experimental upper bound. The first set is obtained from the ''maximal set'' [$s_L^{\nu_l}$'s from Eq. (4) and all $x_{ll'}=1$] by replacing the maximal value for $(s_L^{\nu_e})^2$ with the value $(s_L^{\nu_e})^2=4.29\times10^{-10}=7.1\times10^{-3}\times(2.459\times10^{-4})^2$. The second set is obtained from the ''maximal set'' by putting $x_{\mu e}=x_{\tau e}=2.459\times10^{-4}$. The first set is used both within the upper-bound procedure and with the ''realistic'' B_{lN} matrix elements introduced in Eq. (16). The second can be applied

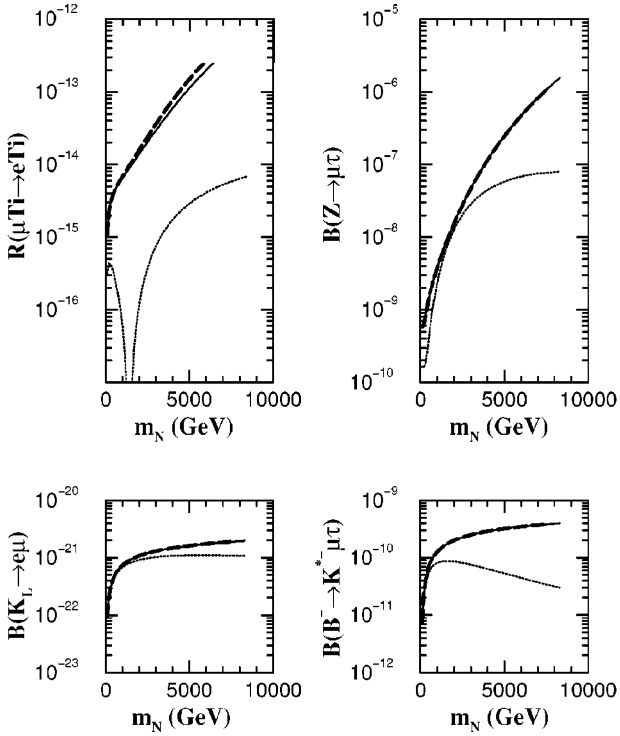

FIG. 1. The BR's and the upper bounds of the BR's for four leading processes of the groups of processes given in Eqs. (64), (65), (66), and (71). Each of these four processes is shown in one of four panels. The BR's are evaluated for degenerate heavy-neutrino masses, m_N. The model parameters are adjusted so that the maximal BR values are smaller than the present experimental upper bounds. It is assumed that the ''angle'' parameter $x_{\mu\tau}\approx 1$, in accord with the Super-Kamiokande measurements. The full line represents the upper-bound calculation keeping the parameters $x_{ll'}$ equal to one, and adjusting the $s_L^{\nu_l}$ parameters: $(s_L^{\nu_e})^2=4.29\times 10^{-10}$, $(s_L^{\nu_\mu})^2=1.4\times 10^{-3}$, and $(s_L^{\nu_\tau})^2=3.3\times 10^{-2}$. The heavy, long-dashed line represents the upper-bound calculation keeping the $s_L^{\nu_l}$ equal to the present experimental upper-bound values (4), and adjusting the $x_{ll'}$ parameters: $x_{\mu e}=x_{\tau e}=2.459\times 10^{-4}$, $x_{\tau\mu}=1$. The dotted line represents the calculation with the ''realistic'' B_{lN} matrix elements, and with the same parameters as for the full-line calculation.

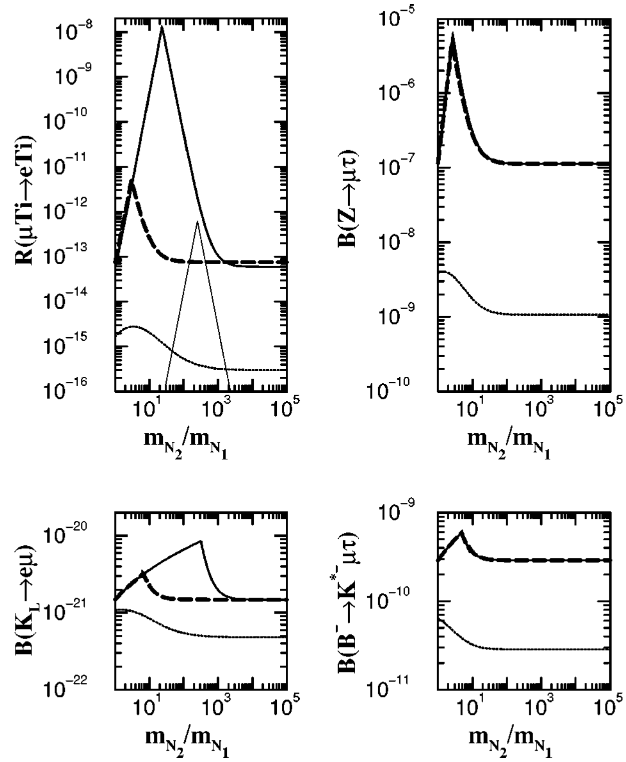

FIG. 2. The BR's and upper bounds for BR's for the same processes as in Fig. 1, but now evaluated as a function of the ratio of two heavy-neutrino masses m_{N_2}/m_{N_1}. For all curves the first and third masses are taken to be degenerate, $m_{N_1}=m_{N_3}=4000$ GeV, while the second mass assumes values within the interval $1 \leqslant m_{N_2}/m_{N_1}\leqslant 10^5$. The types of lines represent the same sets of parameters as in Fig. 1. In the first panel, representing the $\mu\to e$ conversion on Ti, additional curve is added, to show that one can always achieve theoretical values smaller than the present experimental bounds. The calculation for that curve was made within the upper-bound procedure, for $x_{ll'}=1$, $(s_L^{\nu_e})^2=(s_L^{\nu_\mu})^2=0.5\times 10^{-10}$, and $(s_L^{\nu_\tau})^2=0.033$.

only within the upper-bound procedure, because the procedure with the ''realistic'' B_{lN} matrix elements has fixed $x_{ll'}$ values. In all calculations $x_{\mu\tau}$ is kept to be equal to one in accord with the Super-Kamiokande results. The BR's are presented in Fig. 1. as functions of the common heavy-neutrino mass. The figures illustrate the following properties of the BR's. First, for all m_N values, the upper-bound procedure gives larger value than the ''realistic'' B_{lN}'s. Second, while the BR's evaluated in the upper-bound procedure increase in the whole region of m_N values permitted by PUB, the BR's evaluated with the ''realistic'' B_{lN}'s may have a maximum below the m_N^{PUB}. The maximum is a consequence of the mass dependence of the ''realistic'' B_{lN}'s. All BR's of processes with the *box*-amplitude only have the maximum, but it can appear in the BR's having the Z amplitude, too. Third, by a reduction of $x_{ll'}$'s one obtains results which are numerically equivalent to the results obtained by a reduction of $s_L^{\nu_l}$ parameters. Fourth, a strong cancellation of the amplitude terms may appear in the BR's evaluated with the ''realistic'' B_{lN}'s, as in the case of $\mu\,\mathrm{Ti}\to e\,\mathrm{Ti}$. Fifth, the error one can make in the evaluation of the maximum of BR's using the upper-bound procedure is $\lesssim 10$ for processes with the box amplitude only, and $\lesssim 100$ for processes with Z amplitudes. The flat behavior of $Z\to l^{\mp}l'^{\pm}$ at $m_N\sim 100$ GeV ($\sim m_Z$) is a consequence of treshold effects.

As shown in Sec. III C, all heavy-neutrino masses, except one, can assume any value between zero and infinity. BR's should not assume values larger than one in the whole parameter space permitted by the model. The illustration of convergence and of good behavior of branching ratios evaluated using the upper-bound procedure and ''realistic'' B_{lN}'s is given in Fig. 2. BR's are evaluated keeping two masses equal, while the third one is assumeed to take very large variable values. In Fig. 2 the BR's for the same processes as in Fig. 1 are given, but here as a function of ratio of the large mass (m_{N_2}) and mass which is kept constant ($m_{N_1}=m_{N_3}$). Graphs in Fig. 2 show that the very heavy neutrinos decouple, and therefore, that the nondecoupling of heavy neutrinos is only a transient effect. Within the upper-bound procedure, the decoupling of the very heavy neutrino(s) manifests as the equality of BR values for degenerate heavy neutrinos and when some of masses tend to infinity, while for ''realistic'' B_{lN}'s BR's reduce in magnitude. Figure 2 also illustrates that the upper-bound procedure is very crude in the transient region where the upper bounds (15) and upper bound (14) are almost equally effective as the second (15) bound. To show that, with the proper choice of the parameters, experimental limits are always satisfied, in the first panel of Fig. 2 the additional BR curve is added, evaluated in the upper-bound procedure for parameters for which the maximal BR value is smaller than the present experimental upper bound for $R(\mu\,\mathrm{Ti}\to e\,\mathrm{Ti})$. Only the top of the curve is seen in the figure. The curves obtained using the ''realistic'' B_{lN}'s are much smoother than the curves obtained from the upper-bound procedure. Therefore, for nondegenerate heavy neutrinos good knowledge of the B_{lN} matrix elements is necessary to obtain reasonable estimate of the BR values.

VI. CONCLUSIONS

All low-energy neutrinoless LFV processes are studied in an extension of SM by heavy $\mathrm{SU}(2)_L\times\mathrm{U}(1)$ singlet Dirac neutrinos. The structure of amplitudes and relations between BR's are carefully analyzed. It is shown that, in principle, the neutrinoless LFV decays cannot give new limits on the ''diagonal'' mixings $s_L^{\nu_l}$. The approximate expressions for all BR's are listed, keeping only the dominant terms of the corresponding amplitudes in the large heavy-neutrino mass limit. The approximate BR's are compared within the groups of processes with the same dominant composite-loop function, and within each group the experimentally most interesting process are found: $\mu\mathrm{Ti}\to e\mathrm{Ti}$, $Z\to\tau^{\mp}l^{\pm}$, $K_L\to e^{\mp}\mu^{\pm}$, $B^-\to K^{*-}\mu^{\mp}e^{\pm}$, $\tau\to e\pi^+K^-$, and $B^-\to K^{*-}\tau^{\mp}e^{\pm}$. The upper bounds of exact BR's are evaluated using the improved version of the upper-bound procedure found in the previous publication [10]. The results are compared with the present experimental upper bounds. For maximal values of model parameters, only six processes have the theoretical upper bounds larger than the experimental ones: $\mu\to e\gamma$, $\mu\to eee$, $\mu\,\mathrm{Ti}\to e\,\mathrm{Ti}$, $\tau\to e\rho^0$, $\tau\to e\pi^+\pi^-$, and $Z\to e\tau$. For these processes, new limits on combinations of B_{lN} matrices are obtained. The first three have been studied before [12] and they give a new limit on the nondiagonal μ-e mixing. The limit is updated here. For the last three, the ratio of the theoretical and experimental upper bounds are very close to one and the limit obtained is mass dependent. Therefore, it is not useful. A two-orders-of-magitude improvement of experimental sensitivities is needed to obtain mass-independent limits on the nondiagonal τ-e and τ-μ mixings from $\tau\to l\gamma$ decays. Concerning the processes with the box amplitude only, the $K_L\to e^{\mp}\mu^{\pm}$ decay has the best ratio of theoretical to experimental upper bound. Nevertheless, neutrinoless LFV B-meson decays have BR's of the order $\sim 10^{-9}$, which makes them interesting for finding LFV in future experiments. If the structure of the massless part of the B matrix is as suggested by the Super-Kamiokande experiment, one may expect that in the future the processes containing τ and μ leptons in the final (and initial) state will be most interesting for finding LFV. In addition to BR's for the low-energy neutrinoless LFV decays, the constant characteristic for the muonium–antimuonium conversion $G_{M\bar{M}}$ is evaluated. The result obtained is too small to be interesting experimentally.

All the above results depend only on the gauge structure of the model used and masses of heavy neutrinos. The results do not change if the massless neutrinos are replaced with the light neutrinos satisfying the present experimental limits. A comment on extraction of heavy-neutrino mixings from astrophysical observations is given. Following the V-model assumption of massless ''light'' neutrinos, an analysis of oscillations of three massless neutrinos in the supernovae is done. The analysis gives the limits on mixings in the massless-neutrino sector that are in a slight contradiction with the Super-Kamiokande results.

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APPENDIX A

The form factors a_P and b_P and a_V, b_V, c_V, and d_V follow directly from matrix elements of corresponding hadronic currents (26). They read

$$a_P=-\frac{f_{\mathcal{H}}}{f_P}-2g\frac{f_{\mathcal{H}'*}}{f_P}\frac{p_2\cdot q}{m^2_{\mathcal{H}'*}}\frac{(m_1m^3_{\mathcal{H}'*})^{1/2}}{q^2-m^2_{\mathcal{H}'*}},$$

$$b_P=2g\frac{f_{\mathcal{H}'*}}{f_P}\left(1+\frac{p_2\cdot q}{m^2_{\mathcal{H}'*}}\right)\frac{(m_1m^3_{\mathcal{H}'*})^{1/2}}{q^2-m^2_{\mathcal{H}'*}} \tag{A1}$$

and

$$a_V=2^{3/2}\lambda g_V f'_{\mathcal{H}}\frac{(m^3_{\mathcal{H}'*}m^{-1}_{\mathcal{H}})^{1/2}}{q^2-m^2_{\mathcal{H}'*}},$$

$$b_V=-2^{1/2}\beta g_V f'_{\mathcal{H}}\frac{(m^3_{\mathcal{H}'*}m^{-1}_{\mathcal{H}})^{1/2}}{q^2-m^2_{\mathcal{H}'*}},$$

$$c_V = -2^{1/2}\alpha_1 g_V (m'_{\mathcal{H}})^{1/2},$$

$$d_V = 2^{1/2}\alpha_2 \frac{(m'_{\mathcal{H}}{}^*)^{1/2}}{m_1^2}. \tag{A2}$$

The phase functions Z_{Pi}, $i=1,2,3$, and Z_{Vi}, $i=1,\ldots,8$ in the square bracket expressions in Eq. (27) read

$$Z_{P1} = \int_{s_{13}^{\min}}^{s_{13}^{\max}} ds_{13}[2p_1\cdot p_3 p_1\cdot p_4 - m_1^2 p_3\cdot p_4],$$

$$Z_{P2} = \int_{s_{13}^{\min}}^{s_{13}^{\max}} ds_{13}[2(p_1\cdot p_3 p_2\cdot p_4 + p_1\cdot p_4 p_2\cdot p_3 - p_1\cdot p_2 p_3\cdot p_4)],$$

$$Z_{P3} = \int_{s_{13}^{\min}}^{s_{13}^{\max}} ds_{13}[2p_2\cdot p_3 p_2\cdot p_4 - m_2^2 p_3\cdot p_4], \tag{A3}$$

for $\mathcal{H}\to Pll'$ decays and

$$Z_{V1} = \int_{s_{13}^{\min}}^{s_{13}^{\max}} ds_{13}[p_1\cdot p_3 p_1\cdot p_2 p_2\cdot p_4 + p_2\cdot p_3 p_1\cdot p_4 p_1\cdot p_2 - m_1^2 p_2\cdot p_3 p_2\cdot p_4 - m_2^2 p_1\cdot p_3 p_1\cdot p_4],$$

$$Z_{V2} = \int_{s_{13}^{\min}}^{s_{13}^{\max}} ds_{13}\left[-q^2 p_3\cdot q p_4\cdot q + \frac{1}{2}q^2 p_3\cdot p_4 + \frac{1}{m_2^2}(p_2\cdot q)^2\left(p_3\cdot q p_4\cdot q - \frac{1}{2}q^2 p_3\cdot p_4\right)\right],$$

$$Z_{V3} = \int_{s_{13}^{\min}}^{s_{13}^{\max}} ds_{13}\left[p_3\cdot p_4 + \frac{1}{m_2^2}\left(p_2\cdot p_3 p_2\cdot p_4 - \frac{1}{2}m_2^2 p_3\cdot p_4\right)\right],$$

$$Z_{V4} = \int_{s_{13}^{\min}}^{s_{13}^{\max}} ds_{13}\left[-q^2 p_1\cdot p_3 p_1\cdot p_4 + \frac{1}{2}m_1^2 q^2 p_3\cdot p_4 + \frac{1}{m_2^2}(p_2\cdot q)^2\left(p_1\cdot p_3 p_1\cdot p_4 - \frac{1}{2}m_1^2 p_3\cdot p_4\right)\right],$$

$$Z_{V5} = \int_{s_{13}^{\min}}^{s_{13}^{\max}} ds_{13}[2p_1\cdot p_3 p_2\cdot p_4 - 2p_1\cdot p_4 p_2\cdot p_3],$$

$$Z_{V6} = \int_{s_{13}^{\min}}^{s_{13}^{\max}} ds_{13}\left[-2p_3\cdot q p_4 q + q^2 p_3\cdot p_4 + \frac{1}{m_2^2}p_2\cdot q(p_2\cdot p_3 p_4\cdot q + p_2\cdot p_4 p_3\cdot q - p_3\cdot p_4 p_2\cdot q)\right],$$

$$Z_{V7} = \int_{s_{13}^{\min}}^{s_{13}^{\max}} ds_{13}\left[-q^2 p_3\cdot q p_1\cdot p_4 - q^2 p_4\cdot q p_1\cdot p_3 + q^2 p_1\cdot q p_3\cdot p_4 + \frac{1}{m_2^2}(p_2\cdot q)^2(p_3\cdot q p_1\cdot p_4\cdot + p_4\cdot q p_1\cdot p_3 - p_1\cdot q p_3\cdot p_4)\right],$$

$$Z_{V8} = \int_{s_{13}^{\min}}^{s_{13}^{\max}} ds_{13}\left[-p_3\cdot q p_1\cdot p_4 - p_4\cdot q p_1\cdot p_3 + p_1\cdot q p_3\cdot p_4 + \frac{1}{m_2^2}p_2\cdot q(p_2\cdot p_3 p_1\cdot p_4 + p_2\cdot p_4 p_1\cdot p_3 - p_1\cdot p_2 p_3\cdot p_4)\right], \tag{A4}$$

for $\mathcal{H}\to Vll'$ decays. The p_1, p_2, p_3, and p_4 are four-momenta of a heavy meson ($\mathcal{H}$), a light meson (P of V), a lepton (l) and antilepton (l'), respectively. The corresponding masses are m_1, m_2, m_3, and m_4. The phase functions contain integration over Mandelstam variable $s_{13}=(p_1-p_3)^2$. The limits of integration are defined in the standard way [36].

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