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THE DOUBLE RADIATIVE ANNIHILATION OF THE HEAVY-LIGHT FERMION BOUND STATES

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Dedicated to Professor Kseno Ilakovac on the occasion of his 70$^{\mathbf{th}}$ birthday

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We consider the double-radiative decays of heavy-light QED and QCD atoms, $\mu^+e^- \to \gamma\gamma$ and $\bar{B}_s^0 \to \gamma\gamma$. Especially, we take under scrutiny contributions coming from operators that vanish on the free-quark mass shell. We show that by field redefinitions, these operators are converted into contact terms attached to the bound state dynamics. A net off-shell contribution is suppressed with respect to the effect of the well known flavour-changing magnetic-moment operator by the bound-state binding factor. The negligible off-shellness of the weakly-bound QED atoms becomes more relevant for strongly bound QCD atoms. We analyse this off-shellness in model-approaches to QCD, one of them enabling us to keep close contact to the related effect in QED. We also comment on the off-shell effect in the corresponding process $\bar{B}_d \to K^*\gamma$, and discuss possible hindering of the claimed beyond-standard-model discovery in this decay mode.

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1. Uses of a comparative study of the off-shell effects in QED and QCD atoms

Off-shell effects are known to be quite elusive. The most famous measured effect, dubbed Lamb shift [1, 2], appears in atomic physics. It is represented by the atomic level shift on account of a tiny difference in the self-energies of the free electron and of the electron bound in the H-atom. Half a century after its discovery, investigations of Lamb shift still provide a precision test of bound-state QED [3]. Now, an experimental uncertainty of 3 ppm in laser experiments is essentially smaller than the 10 ppm theoretical inaccuracy due to poor knowledge of the proton charge radius. In this situation, the study of unstable leptonic atoms becomes competitive to the study of hydrogen.

In a study of the inter-nucleon potential, there have been some early expectations [4] to reveal the off-shell parts of the nucleon-nucleon bremsstrahlung amplitude. Fearing and Scherer [5, 6] excluded this possibility by subsuming such off-shell amplitudes into redundant terms [7] that can be rotated by field redefinitions into so-called contact terms. The parity violating anapole terms [8] might be one exception, deserving a separate study.

However, particle physics provides new microscopic interactions leading to potentially interesting new contact terms. The most famous one is the anomalous $\pi^0\gamma\gamma$ coupling. As explained in some detail in Ref. [9], this coupling can be viewed as an off-shell effect. On top of the QCD binding of the quark-antiquark atom, the triangle quantum loop dominated by far off-shell quarks produces an anomalous coupling responsible for the π^0 decay. Such manifestation of the off-shellness motivates us to study the two-photon annihilation of atoms in general.

The simplest "total disintegration" of an "atom" occurs when it consists of a particle-antiparticle pair, like in the case of the true QED-atom, positronium[1]. Actually, a comparative study of QED and QCD atoms has been very fruitful in the early days of quarkonia. A total disintegration of an atom consisting of different fermions is more subtle. It happens on account of the flavour changing (FC) processes familiar from weak interactions. While the transitions among charged quarks are well known, the lepton-flavour violating (LFV) transitions among charged leptons is an open urgent issue, stimulated by accumulated indication of the neutrino oscillations. In order to benefit from the cross-fertilization of different fields, we pursue here the comparative study of annihilation in the *heavy-light* QED and QCD systems.

Such a comparative study throws a new light on the off-shell nonperturbative effects of valence quarks, studied first by two of us in the case of the double decays of the K_L [12, 13] and $\bar{B}_s$ meson [14]. Subsequently, this study has been continued within the specific bound state models, both for $K_L \to 2\gamma$ [15] and for $\bar{B}_s^0 \to 2\gamma$ [16]. In these papers, it was explicitly demonstrated that operators that vanish by using the perturbative equations of motion gave nonzero contributions for processes involving bound quarks. The purpose of the present paper is to elaborate to more detail our more recent study [17] which accounts for similar effects for the bound leptons.

1.1. The relativistic QED atom

Our starting point is the Lagrangian density for the two fermions in the absence of the FC transitions. Thus the light particle (electron e^- of mass m) and a

[1]A revival of positronium [10] appeared after the discovery of the QCD atoms, together with the recognition that the first proposal of the positronium (termed "electrum") has been given as early as in 1934 [11], immediately after the discovery of the positron by Anderson.

heavy positively charged particle (say muon μ^+ of mass M) interact only through electromagnetic field, as given by the last term in

$$\mathcal{L} = \mathcal{L}_e + \mathcal{L}_\mu - \frac{1}{4} F^{\alpha\beta} F_{\alpha\beta} - J^\alpha A_\alpha \ . \tag{1}$$

The Dirac Lagrangian

$$\mathcal{L}_i = \bar{\psi}_i \left[\frac{\mathrm{i}}{2} \gamma^\alpha \frac{\overleftrightarrow{\partial}}{\partial x^\alpha} - m_i \right] \psi_i \ ; \quad \overleftrightarrow{\partial} = \overrightarrow{\partial} - \overleftarrow{\partial} \ , \tag{2}$$

for a given particle ($i = e, \mu$) leads to the Dirac equations for ψ_i and $\bar{\psi}_i$ treated as independent fields

$$\bar{\psi}_i (\mathrm{i} \overleftarrow{\not\partial} + m_i) = 0 \ , \quad (\mathrm{i}\not\partial - m_i)\psi_i = 0 \ . \tag{3}$$

Imposing the Coulomb (radiation) gauge, $\nabla \cdot \boldsymbol{A} = 0$, one can solve for A^0 (eliminate it from the Lagrangian), leading to

$$\mathcal{L} = \mathcal{L}_e + \mathcal{L}_\mu + \frac{1}{2} (\boldsymbol{E}_\perp^2 - \boldsymbol{B}^2) + \boldsymbol{J} \cdot \boldsymbol{A} - \frac{1}{2} \int \frac{\mathrm{d}^3 r'}{4\pi} \frac{\rho(\boldsymbol{r}, t) \rho(\boldsymbol{r}', t)}{|\boldsymbol{r} - \boldsymbol{r}'|} \ . \tag{4}$$

Here the last two terms can be expressed in terms of ρ and $\boldsymbol{J}$ components of the fermion current

$$J^\alpha = e(\bar{\psi}_\mu \gamma^\alpha \psi_\mu - \bar{\psi}_e \gamma^\alpha \psi_e) \ . \tag{5}$$

The corresponding Hamiltonian, after neglecting the self-energy terms in the Coulomb interaction, has the form [18]

$$H(x) = H(x)_{\mathrm{Atom}} + H(x)_{\mathrm{Rad}} + H(x)_{\mathrm{Coulomb-inst}} + H(x)_{\mathrm{int}} \ , \tag{6}$$

where

$$H_{\mathrm{Atom}} = H_\mu + H_e \tag{7}$$

contains the relativistic fermion contributions

$$H_{e(\mu)} = \int \mathrm{d}^3 r \, \mathcal{H}^0_{e(\mu)} = \int \mathrm{d}^3 r \, \psi^\dagger_{e(\mu)}(x) \big[-\mathrm{i}\boldsymbol{\alpha} \cdot \nabla + m_{e(\mu)} \beta \big] \psi_{e(\mu)}(x) \ . \tag{8}$$

The electromagnetic piece splits into the radiation part,

$$H_{\mathrm{Rad}} = \frac{1}{2} \int \mathrm{d}^3 r \big[\boldsymbol{E}_\perp^2(x) + \boldsymbol{B}^2(x) \big] \ , \tag{9}$$

containing the relevant electric and magnetic fields

$$\boldsymbol{E}_\perp = -\frac{\partial \boldsymbol{A}}{\partial t} \ , \quad \boldsymbol{B} = \nabla \times \boldsymbol{A} \ , \tag{10}$$

and the instantaneous Coulomb term

$$H_{\mathrm{Coulomb-inst}} = \frac{1}{4\pi}\int \frac{\mathrm{d}^3r\, d^3r'}{|\boldsymbol{r}-\boldsymbol{r}'|} J^0_\mu(\boldsymbol{r},t) J^0_e(\boldsymbol{r}',t) \ . \tag{11}$$

The relativistic QED atoms can be treated to all orders by solving exactly the Dirac equation with a Coulomb interaction. This means solving the Dirac equation with $V(x) = \gamma_0 V_c(x)$ (V_c denotes the Coulomb potential)

$$\left[\mathrm{i}\partial\!\!\!/ + V(x) - m_i\right]\psi_i = 0 \ . \tag{12}$$

Correspondingly, the fermion propagator in external field reads

$$\left[\mathrm{i}\partial\!\!\!/ + V(x) - m_e\right] S^e_F(x,y) = \delta^{(4)}(x-y) \ . \tag{13}$$

Thus, in contrast to the free-particle propagator, the propagator for the bound fermion,

$$\mathrm{i}S_F(x,y) = \theta(x_0 - y_0)\sum_{n,\sigma}\psi^{(+)}_{n,\sigma}(x)\bar{\psi}^{(+)}_{n,\sigma}(y) - \theta(y_0 - x_0)\sum_{n,\sigma}\psi^{(-)}_{n,\sigma}(x)\bar{\psi}^{(-)}_{n,\sigma}(y) \ , \tag{14}$$

should require the sum over all possible excited states, which appear when decomposing the fermion field in terms of a complete set of positive and negative energy eigenfunctions:

$$\psi_e(x) = \sum_{n,\sigma}\left\{ b_{n,\sigma}\psi^{(+)}_{n,\sigma}(x) + d^\dagger_{n,\sigma}\psi^{(-)}_{-n,-\sigma}(x)\right\} \ . \tag{15}$$

The solutions of the free Hamiltonian

$$H_0 = H_{\mathrm{Atom}} + H_{\mathrm{Rad}} + H_{\mathrm{Coulomb-inst}} \tag{16}$$

form a complete set of stationary states $|a,N\rangle$, expressed as a direct product of atomic wave functions ψ_a and the photon Fock states

$$|a,N\rangle = \psi_a(r)|N\rangle \ . \tag{17}$$

When the interaction is turned on, one should make a replacement

$$H_0 \to H = H_0 + H_I(t)\,, \tag{18}$$

and the relevant states cease to remain stationary. Their evolution in time in practice means that the excited states decay under the influence of the QED interaction

$$H_I(t) = -\int \mathrm{d}^3r \left[\boldsymbol{J}_p(\boldsymbol{r},t) + \boldsymbol{J}_e(\boldsymbol{r},t)\right]\cdot\boldsymbol{A}(\boldsymbol{r},t) \tag{19}$$

into other states $|b, N\rangle$, where N photons are emitted.

In addition to the ordinary interaction (19), in the next subsection we shall consider possible additional interactions (25), involving the flavour changing $\mu \leftrightarrow e$ transition. This will enable the atom to disintegrate completely. The lowest order disintegration requires μ and e overlap, happening when $l = 0$, i.e. a decay from S states. The decay from $l > 0$ corresponds to a cascade down to the S state, followed by the decay from there — a higher order process which we do not need to consider in what follows.

1.2. Some motivation for scrutinizing muonium

There has been a considerable revival of the interest in muonium (Mu $= \mu^+ e^-$ system) in view of the very precise measurements in this system. At the same time, the theoretical predictions are plagued by the nonperturbative bound-state effects. The only known way to achieve the required precision for the bound states is by expanding around a nonrelativistic limit. Such methods, like non-relativistic quantum electrodynamics [19, 20] start from the bound state described by a Schrödinger wave function, and build up corrections in terms of the relative velocity of the components.

For the muonium at hand, our analysis and results bear a close analogy to the correction to the muon lifetime due to muonium formation, reported in Ref. [21]. In this system, electron and muon have r.m.s. velocities $\beta_e = \alpha \approx 1/137$, and $\beta_\mu \approx \alpha m_e/m_\mu \approx 3.5 \times 10^{-5}$. In terms of these parameters, the bound-state corrections acquire a form $\alpha^n (m_e/m_\mu)^m$, where the corrections up to $n + m = 4$ matter in practice. The current world average for the muon lifetime measurements [22]

$$\tau_\mu = 2.19703(4) \times 10^{-6}\,\mathrm{s}\,, \tag{20}$$

has an uncertainty of only 18 ppm. In order to benefit from an improvement of the measurement of τ_μ (and thereby of G_μ) by a factor of 20 (i.e. reducing its uncertainty to only ± 1 ppm), a knowledge of modification of τ_μ due to the formation of muonium is required.

The reexamination of the muonium bound state effect [21] showed only a tiny effect, the correction of about 6×10^{-10} to the lifetime

$$\tau_{\mathrm{Mu}} = \tau_\mu \left(1 + \tfrac{\alpha^2}{2} \tfrac{m_e^2}{m_\mu^2}\right).$$

This negligible overall shift is in contrast to the relatively large $\mathcal{O}(\alpha \frac{m_e}{m_\mu})$ velocity effects on the spectrum [21]. In the present work we are pointing out the off-shell effects (53) in the radiative annihilation of muonium which are in between of these two.

For completeness, let us mention that besides the mentioned radiative annihilation, there is also the W-exchange annihilation Mu$\rightarrow \nu_e \bar{\nu}_\mu$ (the analog of $\mu^- p$

capture), with the rate

$$\Gamma(\mathrm{Mu} \to \nu_e \bar{\nu}_\mu) = 48\pi \left(\frac{\alpha m_e}{m_\mu}\right)^3 \Gamma(\mu^+ \to e^+ \nu_e \bar{\nu}_\mu) \; . \tag{21}$$

Still, it leads to a miniscule branching ratio $\approx 7 \times 10^{-12}$, and moreover is restricted to the orthomuonium decay, which is out of our scope here. Let us note that some interest in radiative orthomuonium decay might come from the three-photon analog decays: the puzzling discrepancy in orthopositronium (a brief sketch of the recent status can be found in Ref. [23]) and the surprising suppression in the theoretical estimate for $K_L \to 3\gamma$ [24]. These three-photon decays provide three-party entanglement similar to the one in quantum optics [25].

Of course, the muonium annihilation involves the LFV transition which is a matter of the beyond the standard model (BSM) physics. Such lepton-number violating interaction induces simultaneously a $\mu \to e\gamma$ transition [26], so that the unknown details will cancel in the ratio of these two processes.

For the heavy-light muonium system $\mu^+ e^-$ (where $m_\mu \equiv M \gg m_e \equiv m$), the bound-state calculation corresponds to that of the relativistic hydrogen. Thereby we distinguish between the Coulomb field responsible for the binding, and the radiation field [27] participating in the flavour-changing transition at the relevant high-energy scale. In this way, the radiative disintegration of an atom becomes tractable by implementing the two-step treatment [28]: "neglecting at first annihilation to compute the binding and then neglecting binding to compute annihilation". This factorization of scales was introduced for the first time by Wheeler [29]. For the muonium atom at hand, the binding problem is analogous to a solved problem of the H-atom. In this way, we avoid the relativistic bound state problem, which is a difficult subject, and we have no intention to contribute to it here.

The mentioned two-step method is known to work well for the disintegration (annihilation) of the simplest QED atom, positronium. Generalization of this procedure to muonium means that the two-photon decay width of muonium is obtained by using

$$\Gamma = \frac{|\psi(0)|^2 \, |\mathcal{M}(\mu^+ e^- \to \gamma\gamma)|^2}{64\pi M m} \; , \tag{22}$$

where $|\psi(0)|^2$ is the square of the bound-state wave function at the origin. After this factorization has been performed, the rest of the problem reduces to the evaluation of the scattering-annihilation invariant amplitude $\mathcal{M}$. In the case of positronium, this expression will involve equal masses ($M{=}m$), and the invariant amplitude, which for a positronium annihilation at rest has the textbook form [30]

$$\mathcal{M} = \frac{\mathrm{i}e^2}{2m^2} \bar{v}_s(p_2) \Big\{ \not\epsilon_2^* \not\epsilon_1^* \not k_1 + \not\epsilon_1^* \not\epsilon_2^* \not k_2 \Big\} u_r(p_1) \; . \tag{23}$$

Only the antisymmetric piece in the decomposition of the product of three gamma matrices above

$$\Big\{ \quad \Big\} \to \mathrm{i}\epsilon^{\mu\nu\alpha\beta} \gamma_5 \gamma_\beta (k_1 - k_2)_\alpha (\epsilon_1^*)_\mu (\epsilon_2^*)_\nu \; , \tag{24}$$

contributes to the spin singlet parapositronium two-photon annihilation. This selects $(\boldsymbol{\epsilon}_1^* \times \boldsymbol{\epsilon}_2^*)$, a CP-odd configuration of the final two-photon state. We will see that for muonium annihilation also the CP-even $\boldsymbol{\epsilon}_1^* \cdot \boldsymbol{\epsilon}_2^*$ configuration contributes.

The paper is organized as follows: In Sect. 2 we consider the quantum field treatment of the annihilation process $\mu^+ e^- \to \gamma\gamma$ in arbitrary external field(s). In Sect. 3 we relate the binding forces to the external fields of Sect. 2. In Sect. 4 we perform the calculation of $\bar{B}_s \to \gamma\gamma$ in several different QCD models. In addition we consider the related off-shell bound-state effects in $\bar{B}_d \to K^*\gamma$ decay. In Sect. 5 we present our conclusions.

2. *Flavour-changing operators for* $\mu^+ e^- \to \gamma\gamma$

Augmenting the electroweak theory by LFV enables the one- and two-photon radiative decays $\mu \to e\gamma$ and $\mu \to e\gamma\gamma$. Accordingly, the double-radiative transition is triggered by two classes of one-particle irreducible diagrams (Figs. 1a and b), related by the Ward identities. After integrating out the heavy particles in the

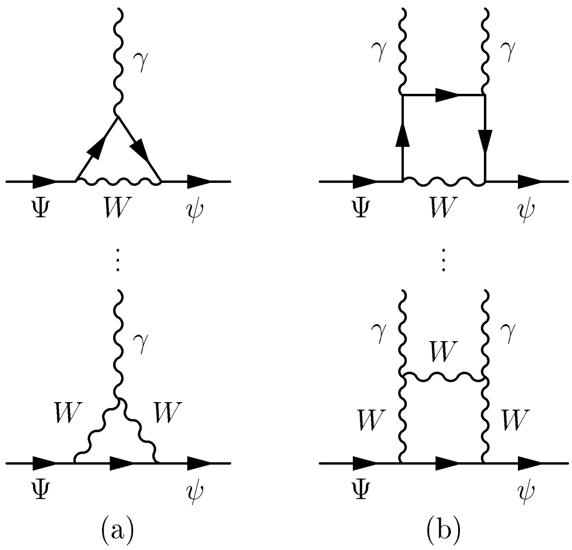

Fig. 1. The examples of the one-particle-irreducible diagrams leading to the double-radiative flavour-changing transitions. Only the second-row diagrams exist for the leptonic case.

loops, these one-loop electroweak transitions can be combined into an effective Lagrangian [13],

$$\mathcal{L}(e \to \mu)_\gamma \,=\, B\, \epsilon^{\mu\nu\lambda\rho} F_{\mu\nu}\, (\bar{\Psi}\ \mathrm{i}\ \overleftrightarrow{D}_\lambda\ \gamma_\rho L\, \psi) \,+ \mathrm{h.c.}\,, \tag{25}$$

where muon and electron are described by quantum fields $\Psi = \psi_\mu$ and $\psi = \psi_e$. Correspondingly, for $\bar{B}_s^0 \to 2\gamma$, the involved fields are $\psi_s = s$ and $\psi_b = b$.

In our case, we do not need to specify the physics behind the lepton-flavour-violating transition in (25). For instance, the strength B might contain Maki-Nakagawa-Sakata [31] parameters, analogous to the Cabibbo-Kobayashi-Maskawa parameters $\lambda_{\rm CKM}$ in the quark sector.

Keeping in mind that the fermions in the bound states are not on-shell, we are not simplifying the result of the electroweak loop calculation by using the perturbative equation of motion. Thus the effective Lagrangian (25) obtained within perturbation theory splits into the on-shell magnetic transition operator $\mathcal{L}_\sigma$

$$\mathcal{L}_\sigma(1\gamma) = B_\sigma \bar{\Psi}\,(M\sigma\cdot FL + m\,\sigma\cdot FR)\,\psi\, + \text{h.c.}\,, \tag{26}$$

and an off-shell piece $\mathcal{L}_F$ [13]

$$\mathcal{L}_F = B_F\,\bar{\Psi}[(\mathrm{i}\,\overleftarrow{\not{D}} - M)\,\sigma\cdot FL + \sigma\cdot FR(\mathrm{i}\,\not{D} - m)]\psi\, + \text{h.c.}\,, \tag{27}$$

where $\sigma\cdot F$ denotes $\sigma_{\mu\nu}F^{\mu\nu}$, while $L=(1-\gamma_5)/2$ and $R=(1+\gamma_5)/2$ denote left-hand and right-hand projectors. To lowest order in QED (or QCD), $B_F = B_\sigma = B$, but in general they are different due to different anomalous dimensions of the operators in (26) and (27). Let us note that the off-shell part $\mathcal{L}_F$ has zero anomalous dimension [14].

By decomposing the covariant derivative, $\mathrm{i}\not{D} = \mathrm{i}\not{\partial} - e\not{A}$, in the off-shell operator (27), we separate the one-photon piece,

$$\mathcal{L}_F(1\gamma) = B_F\,\bar{\Psi}[(\mathrm{i}\,\overleftarrow{\not{\partial}} - M)\,\sigma\cdot FL + \sigma\cdot FR(\mathrm{i}\not{\partial} - m)]\psi\, + \text{h.c.}\,, \tag{28}$$

from the two-photon piece

$$\mathcal{L}_F(2\gamma) = B_F\,\bar{\Psi}[-e\not{A}\sigma\cdot FL + \sigma\cdot FR(-e\not{A})]\psi\, + \text{h.c.} \tag{29}$$

The amplitude for the two-photon diagram (Fig. 2) is given by

$$A_a = \mathrm{i}\int \mathrm{d}^4x\,\mathcal{L}_F(2\gamma) = A_a^L + A_a^R\,, \tag{30}$$

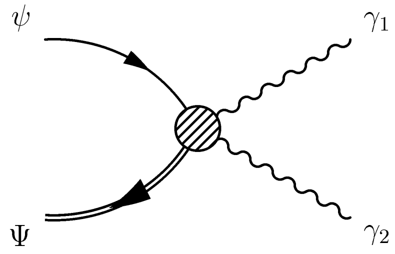

Fig. 2. The two-photon contact (seagull) diagram that can be rotated away by a field redefinition.

in an obvious notation. The single-photon off-shell Lagrangian $\mathcal{L}_F(1\gamma)$ leads to the amplitude with the heavy particle in the propagator

$$\begin{aligned} A_b &= \mathrm{i}\, B_F \iint \mathrm{d}^4x\, \mathrm{d}^4y\, \bar{\Psi}(y)\Big[-\mathrm{i}e\not{A}_2(y)\Big]\, \mathrm{i}S_F^{(\mu)}(y,x) \\ &\times\Big[(\mathrm{i}\overleftarrow{\not{\partial}}_x - M)\,\sigma\cdot F_1(x)L + \sigma\cdot F_1(x)R(\mathrm{i}\not{\partial}_x - m)\Big]\psi(x)\ , \end{aligned} \tag{31}$$

and a similar amplitude with the light particle in the propagator

$$\begin{aligned} A_c &= \mathrm{i}\, B_F \iint \mathrm{d}^4x\, \mathrm{d}^4y\, \bar{\Psi}(x)\Big[(\mathrm{i}\overleftarrow{\not{\partial}}_x - M)\,\sigma\cdot F_1(x)L + \\ &\sigma\cdot F_1(x)R(\mathrm{i}\not{\partial}_x - m)\Big]\times\, \mathrm{i}S_F^{(e)}(x,y)\Big[-\mathrm{i}e\not{A}_2(y)\Big]\psi(y)\ . \end{aligned} \tag{32}$$

The subscripts 1 and 2 distinguish between the two photons. It is understood that a term with the $1 \leftrightarrow 2$ subscript interchange should be added in order to make our result symmetric in the two photons.

Within the quantum field formalism, the sum of the equations (30), (31) and (32) describes the processes $\mu^+e^- \to \gamma\gamma$, or $\mu \to e\gamma\gamma$.

Let us now be very general and assume that both particles (e and μ) feel some kind of external field(s) represented by $V_{(e)}$ and $V_{(\mu)}$, and obey one-body Dirac equations

$$\big[\mathrm{i}\not{\partial} - V_{(i)}(x) - m_{(i)}\big]\psi_{(i)} = 0\ , \tag{33}$$

for $i = e$ or μ (in general $V_{(i)} = \gamma_\alpha V^\alpha_{(i)}$), and accordingly the particle propagators $S_F^{(i)}$ satisfy:

$$\big[\mathrm{i}\not{\partial} - V_{(i)}(x) - m_i\big]S_F^{(i)}(x,y) = \delta^{(4)}(x-y)\ . \tag{34}$$

Our photon fields enter via perturbative QED, switched on by the replacement $\partial_\mu \to D_\mu = \partial_\mu + \mathrm{i}eA_\mu$ in (12). It should be emphasized that $A_\mu(x)$ represents the radiation field and does not include binding forces, which will in the next section be related to the external fields $V_{(i)}$.

Now, using relations (12) and (13), we obtain

$$A_b = -A_a^L + \Delta A_b\ , \qquad A_c = -A_a^R + \Delta A_c\ , \tag{35}$$

resulting in a partial cancellation when the amplitudes are summed

$$A_a + A_b + A_c = \Delta A_b + \Delta A_c\ . \tag{36}$$

This shows that the local off-diagonal fermion seagull transition of Fig. 2 cancels, even if the external fermions are off-shell. The left-over quantities ΔA_b and ΔA_c

involve the integrals over the Coulomb potential and represent the net off-shell effect.

There are also amplitudes A_d and A_e which are counterparts of A_b and A_c when $\mathcal{L}_F(1\gamma)$ is replaced by $\mathcal{L}_\sigma$. The total contribution from our flavour-changing Lagrangian ($\mathcal{L}_F$ and $\mathcal{L}_\sigma$ parts) is then given by

$$\begin{aligned} A_d + \Delta A_b \quad = \quad & \mathrm{i} \iint \mathrm{d}^4x\, \mathrm{d}^4y\, \bar{\Psi}(y) \Big[-\mathrm{i}e\not{A}_2(y) \Big] \\ & \times \mathrm{i}S_F^{(\mu)}(y,x)\, Q(x)\, \psi(x)\ , \end{aligned} \tag{37}$$

represented by Fig. 3, and a similar one

$$\begin{aligned} A_e + \Delta A_c \quad = \quad & \mathrm{i} \iint \mathrm{d}^4x\, \mathrm{d}^4y\, \bar{\Psi}(x)\, Q(x)\, \mathrm{i}S_F^{(e)}(x,y) \\ & \times \Big[-\mathrm{i}e\not{A}_2(y) \Big] \psi(y)\ , \end{aligned} \tag{38}$$

corresponding to Fig. 3b.

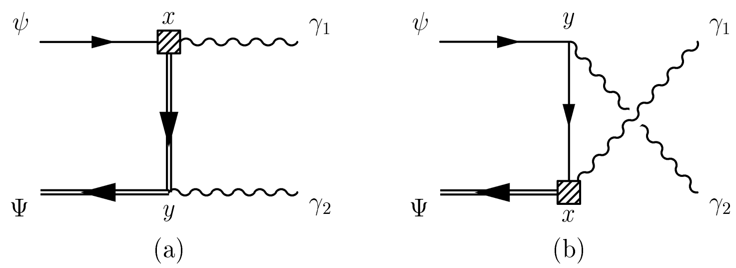

Fig. 3. The shaded boxes indicate the combination of the unrotated off-shell transition (proportional to B_F) and the on-shell magnetic moment transition (proportional to B_σ), giving the effective vertex in Eq. (47).

The operator $Q(x)$ in these expressions reads

$$\begin{aligned} Q(x) = & \left[B_\sigma M + B_F V_{(\mu)}(x) \right] \sigma \cdot F_1(x) L \\ & + \sigma \cdot F_1(x) R \left[B_\sigma m + B_F V_{(e)}(x) \right] . \end{aligned} \tag{39}$$

The result given by Eqs. (37)–(39) can also be understood in terms of the following field redefinition. Equation (12) can be obtained from the Lagrangian

$$\mathcal{L}_D(\Psi,\psi) = \bar{\Psi}\big[\mathrm{i}\not{D} - V_{(\mu)} - M\big]\Psi + \bar{\psi}\big[\mathrm{i}\not{D} - V_{(e)} - m\big]\psi\ . \tag{40}$$

Now, by defining new fields

$$\Psi' = \Psi + B_F\, \sigma \cdot F\, L\psi\ , \qquad \psi' = \psi + B_F^*\, \sigma \cdot F L \Psi\ , \tag{41}$$

we obtain

$$\mathcal{L}_D(\psi,\Psi) + \mathcal{L}_F \;=\; \mathcal{L}_D(\psi',\Psi') \;+ \Delta\mathcal{L}_B \;, \tag{42}$$

which shows that $\mathcal{L}_F$ can be transformed away from the perturbative terms, but a relic of it,

$$\Delta\mathcal{L}_B \;=\; B_F\,\bar{\Psi}\big[V_{(\mu)}\sigma\cdot FL \;+\; \sigma\cdot FR\,V_{(e)}\big]\psi \;+\mathrm{h.c.}\;, \tag{43}$$

remains in the bound-state dynamics as a contact term. Thus, the off-shell effects are non-zero for bound external fermions. Combining $\Delta\mathcal{L}_B$ and $\mathcal{L}_\sigma$, we obtain

$$\Delta\mathcal{L}_B \;+\; \mathcal{L}_\sigma \;=\; \bar{\Psi}\,Q\,\psi \;+\mathrm{h.c.}\;, \tag{44}$$

where Q is given by (39). This shows how the upper field redefinition rotates away the contact term shown in Fig. 2, leaving us with the result given by Eqs. (37)–(39).

3. Off-shellness in the muonium annihilation amplitude

The preceding section shows how far we can push the problem within quantum field theory. Up till now we have made no approximations except for standard perturbation theory. Now we apply the obtained results to the double radiative annihilation of muonium. Naively, the product $\bar{\Psi}\,\psi$ corresponds to the bound state of μ^+ and e^-, which might be true only for the asymptotic free fields. However, relativistic bound state physics is a difficult subject, which we circumvent by sticking to the two-step procedure [28] as explained in Sect. 1. We perform the calculations in the muonium rest frame (CM frame of μ^+ and e^-) where we put the external field(s) equal to a mutual Coulomb field, $V_{(i)} \to \gamma_0\,V_C$ (where $V_C = -e^2/4\pi r$). In calculating the $\mu^+e^- \to \gamma\gamma$ amplitude in the momentum space, we take for V_C the average over solutions in the Coulomb potential, which is $\langle V_C\rangle = -(m\alpha^2/2)$. In this way, the muonium-decay invariant amplitude acquires the form which is a straightforward generalization of the positronium-decay invariant amplitude (23) in momentum space.

The amplitudes $A_d + \Delta A_b$ from Eq. (37), together with $A_e + \Delta A_c$ from (38), transformed to the momentum space, take the form

$$\mathcal{M} \;\;=\;\; \frac{2eB_\sigma}{m}\bar{v}_\mu(p_2)\Big\{\frac{m}{M}\not{k}_2\not{\epsilon}_2^*\mathrm{P} - \mathrm{P}\not{\epsilon}_2^*\not{k}_2 + (1\leftrightarrow 2)\Big\}u_e(p_1), \tag{45}$$

where v_μ and u_e are muon and electron spinors, and $\epsilon^*_{1,2}$ are photon polarization vectors. The factor, incorporating the binding in the form of a four-vector $U^\alpha = (\rho,\mathbf{0})$,

$$\mathrm{P} \equiv (1 - x\not{U})\not{k}_1\not{\epsilon}_1^* L + x\not{k}_1\not{\epsilon}_1^* R(1-\not{U})\;, \tag{46}$$

accounts for the aforementioned factorization of a binding and a decay, and is represented by the shaded box of Fig. 3:

$$\left[M(1 - x\rho\gamma^0)\sigma \cdot F_1 L + m\,\sigma \cdot F_1 R(1 - \rho\gamma^0)\right] . \tag{47}$$

Here we introduced abbreviations for two small constant parameters,

$$x \equiv \frac{m}{M} , \qquad \rho \equiv - \frac{B_F \left\langle V_C \right\rangle}{m B_\sigma} , \tag{48}$$

in terms of which the sought off-shell effect will be expressed. Note that in the effective interaction (47), the left-handed part corresponding to $V_{(\mu)}$ has gotten an extra suppression factor $x = m/M$ in front of the binding factor ρ, in agreement with the expectation that the heavy particle (μ^+) is approximately free, and the light particle (e^-) is approximately the reduced particle, in analogy with the H-atom.

The annihilation amplitude (45) can now be evaluated explicitly. The usual procedure of squaring the amplitude and using the Casimir trick for converting spinors into Dirac matrices would give us expressions with traces of up to twelve Dirac matrices, making the calculation unnecessarily extensive. It is much easier to proceed by going into the frame in which the muonium is at rest and photons are emitted along the z-axis, i. e.

$$k_1 = \begin{pmatrix} \omega \\ 0 \\ 0 \\ \omega \end{pmatrix} , \quad k_2 = \begin{pmatrix} \omega \\ 0 \\ 0 \\ -\omega \end{pmatrix} , \quad \epsilon_\pm = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ \pm \mathrm{i} \\ 0 \end{pmatrix} , \tag{49}$$

where $\omega = (m + M)/2 \approx M/2$ is the photon energy. In this frame $\not{k}_i$ and $\not{\epsilon}_j^*$ $(i, j = 1, 2)$ formally anticommute

$$\not{k}_i \not{\epsilon}_j^* = \omega(\gamma^0 \pm \gamma^3) \frac{1}{\sqrt{2}} (\gamma^1 \pm \mathrm{i}\gamma^2) = -\not{\epsilon}_j^* \not{k}_i , \tag{50}$$

so we can group them together and calculate

$$\not{k}_1 \not{k}_2 \not{\epsilon}_1^* \not{\epsilon}_2^* = -2\omega^2 \begin{pmatrix} \boldsymbol{\epsilon}_2^* \cdot \boldsymbol{\epsilon}_1^* - (\boldsymbol{\epsilon}_2^* \times \boldsymbol{\epsilon}_1^*) \cdot \hat{\boldsymbol{k}}_1\, \sigma^3 & \boldsymbol{\epsilon}_2^* \cdot \boldsymbol{\epsilon}_1^*\, \sigma^3 - (\boldsymbol{\epsilon}_2^* \times \boldsymbol{\epsilon}_1^*) \cdot \hat{\boldsymbol{k}}_1 \\ \boldsymbol{\epsilon}_2^* \cdot \boldsymbol{\epsilon}_1^*\, \sigma^3 - (\boldsymbol{\epsilon}_2^* \times \boldsymbol{\epsilon}_1^*) \cdot \hat{\boldsymbol{k}}_1 & \boldsymbol{\epsilon}_2^* \cdot \boldsymbol{\epsilon}_1^* - (\boldsymbol{\epsilon}_2^* \times \boldsymbol{\epsilon}_1^*) \cdot \hat{\boldsymbol{k}}_1\, \sigma^3 \end{pmatrix} . \tag{51}$$

It is now easy to multiply this by the appropriate chiral projectors L and R, $\rho\gamma^0$ terms, and $\bar{v}_\mu(p_2)$ and $u_e(p_1)$ spinors. Now, taking into account that muonium leading to the two-photon final state is in the spin singlet, we get the result

$$\begin{aligned}\mathcal{M} &= -2eB_\sigma M^2\sqrt{\frac{2M}{m}}\Big[(1-x^2+x\rho+x^2\rho)\boldsymbol{\epsilon}_2^*\cdot\boldsymbol{\epsilon}_1^* \\ &\quad + \mathrm{i}(1+2x+x^2+x\rho-x^2\rho)(\boldsymbol{\epsilon}_2^*\times\boldsymbol{\epsilon}_1^*)\cdot\hat{\boldsymbol{k}}_1\Big] . \qquad (52)\end{aligned}$$

In comparison to the expressions (23) and (24) for parapositronium, we notice that in addition to $\boldsymbol{\epsilon}_2^*\times\boldsymbol{\epsilon}_1^*$, there appears also $\boldsymbol{\epsilon}_2^*\cdot\boldsymbol{\epsilon}_1^*$, a CP-even two-photon configuration.

The explicit expression for ρ depends on some assumptions. As explained previously, we use $\langle V_C\rangle = -m\alpha^2/2$ which gives $\rho = \alpha^2/2$ for $B_\sigma = B_F = B$, which is a good approximation in the leptonic case.

Equation (22) finally gives

$$\Gamma = \frac{2\alpha M^4}{m^2}|\psi(0)|^2|B_\sigma|^2\,(1+2x\rho) \ , \qquad (53)$$

where we have kept only the leading term in ρ and x. Since the wave function at the origin appears as a prefactor, it is not necessary to know the precise value of $|\psi(0)|^2 \approx (m\alpha)^3/\pi$, in order to know the relative off-shell contribution. Thus, for muonium, the sought off-shell contribution is only a tiny correction, $2x\rho = \alpha^2 m/M \approx 2.6\times 10^{-7}$, to the magnetic moment dominated rate.

We may note in passing that we have checked our results also by the direct calculation of the squared Feynman amplitude (45) on the computer using the `FeynCalc` Mathematica package for algebraic manipulation of expressions involving Dirac matrices and spinors [32, 33]. Here the explicit Lorentz covariance was preserved at all steps of the calculation and the final result was in agreement with the one obtained by calculation made by hand.

4. *Off-shellness in* $\bar{B}_s^0 \to \gamma\gamma$

In comparison to a tiny effect in the preceding section, we expect the corresponding off-shellness in a strongly bound QCD system to be significantly larger. We also take into account the B_F/B_σ correction in (53), when considering the $\bar{B}_s^0 \to \gamma\gamma$ decay.

The expressions (25) to (29) apply to the $b\to s\gamma\gamma$ induced $\bar{B}_s^0 \to 2\gamma$ decay amplitude by simple replacements $\mu\to s$ and $e\to b$. Then one has to scale the operators $\mathcal{L}_{F,\sigma}$ defined at the M_W scale, down to the B-meson scale. The coefficients B_F of $\mathcal{L}_F$, and B_σ of $\mathcal{L}_\sigma$, in Eqs. (27) and (26), both being equal to B at the W scale, may evolve differently down to the $\mu = m_b$ scale. This difference between B_F and B_σ is due to different anomalous dimensions of the respective operators. Within the standard model (SM) one can write

$$B_{\sigma,F} = \frac{4G_F}{\sqrt{2}}\,\lambda_{\rm CKM}\,\frac{e}{16\pi^2}\,C_7^{\sigma,F} \ . \qquad (54)$$

The coefficient C_7^σ has been studied by various authors [34–38]. The coefficient C_7^F was considered in [14], where at the b-quark scale we obtained

$$\frac{C_7^F}{C_7^\sigma} \approx 4/3 \qquad (\mu = m_b) \,. \tag{55}$$

Although the off-shell effect for $\bar{B} \to 2\gamma$ is expected to be suppressed by the ratio binding energy to m_b, it could still be numerically interesting.

The conventional procedure when evaluating the pseudoscalar meson decay amplitudes is to express them in terms of the meson decay constants, by using the PCAC relations

$$\langle 0|\bar{s}\gamma_\mu\gamma_5 b|\bar{B}_s^0(P)\rangle \;=\; -\mathrm{i} f_B P_\mu \,, \tag{56}$$

$$\langle 0|\bar{s}\gamma_5 b|\bar{B}_s^0(P)\rangle \;=\; \mathrm{i} f_B M_B \,. \tag{57}$$

These relations will be useful after reducing our general expression (45) containing the terms with products of up to five Dirac matrices. After some calculation, we arrive at the expression for the $\bar{B}_s$ meson decay at rest, which is analogous to, and in fact confirms our previous relation (52) obtained in a different way,

$$\begin{aligned}\mathcal{M}^B \;=\; & -\mathrm{i}\frac{e}{3}B_\sigma f_B M^2 \frac{(1+x)^2}{x}\Big[(1 - x^2 + x\tau + x^2\tau)\,\boldsymbol{\epsilon}_2^* \cdot \boldsymbol{\epsilon}_1^* + \\ & + \; \mathrm{i}(1 + 2x + x^2 + x\tau - x^2\tau)(\boldsymbol{\epsilon}_2^* \times \boldsymbol{\epsilon}_1^*) \cdot \hat{\boldsymbol{k}}_1\Big].\end{aligned} \tag{58}$$

Here, the parameter τ represents the off-shell effect in the QCD problem at hand, and will be more model dependent than its QED counterpart ρ. With the amplitude (58), keeping only the leading terms in τ and x, we arrive at the total decay width

$$\Gamma = \frac{\alpha M^5}{18 m^2} f_B^2 |B_\sigma|^2 \,(1 + 2x\tau) \;, \tag{59}$$

where by switching off τ we reproduce the result of Ref. [39].

4.1. Coulomb-type QCD model

In order to estimate the value of the off-shell contribution τ, in this subsection we assume a QED-like QCD model with the Coulombic wave function [40, 41] $\psi(r) \sim \exp(-mr\alpha_{\mathrm{eff}})$. Thus we rely again on an exact solution corresponding to effective potential $V(r) = -4\alpha_{\mathrm{eff}}/(3r)$, with effective coupling $\alpha_{\mathrm{eff}}(r) = -(4\pi b_0 \ln(r\Lambda_{\mathrm{pot}}))^{-1}$. Here $b_0 = (1/8\pi^2)(11 - (2/3)N_f)$. The mass scale Λ_{pot}, appropriate to the heavy-light quark $\bar{Q}q$ potential, is related to the more familiar QCD scale parameter, e.g. $\Lambda_{\mathrm{pot}} = 2.23\,\Lambda_{\overline{\mathrm{MS}}}$ (for $N_f = 3$). Within this model, we obtain

$$\tau = \frac{2}{3}\alpha_{\mathrm{eff}}^2 \frac{C_7^F}{C_7^\sigma} \,. \tag{60}$$

By matching the meson decay constant f_B and the wave function at the origin

$$N_c \frac{|\psi_B(0)|^2}{M} = \left(\frac{f_B}{2}\right)^2 \; ; \quad |\psi_B(0)|^2 = \frac{(m\alpha_{\rm eff})^3}{\pi} \; , \tag{61}$$

we obtain the value for the strong interaction fine structure strength $\alpha_{\rm eff} \approx 1$. Then, including (55) for the QCD case, the correction factor

$$x\tau \approx 0.1 \tag{62}$$

is much larger than $x\rho$ in the corresponding QED case. Correspondingly, one expects even more significant off-shell effects in light-quark systems, in compliance with our previous results [13, 12, 15].

4.2. A constituent quark calculation

As an alternative to the Coulomb-type QCD model described above, now we adopt a variant of the approach in Refs. [14, 16]. One might use the PCAC relations (56)–(57) together with a kinematical assumption for the $\bar{s}$-quark momentum, similar to those in Refs. [39, 42]. We assume the bound $\bar{s}$ and b quarks in $\bar{B}_s^0$ to be on their respective *effective* mass-shells. Note that even if one is using (56) and (57), the amplitude will still explicitly depend on the $\bar{s}$-quark momentum $p_{\bar{s}}$. This is put on the effective mass-shell by using the relation $p_{\bar{s}}^\mu = -M_s(k_1+k_2)^\mu/M_b$, where $M_q = m_q + m_0$ (for $q = b, s$) are the effective (total) masses, m_q are the current masses, and m_0 the constituent mass of the order of a few hundred MeV. The structure of the amplitude now comes out essentially as in (58) with a relative off-shell contribution

$$x\,\tilde{\tau} = \frac{2m_0}{m_b} \approx 0.1 \; , \tag{63}$$

of the same order as in (62). However, unlike (58), the off-shell effect is now only in the CP-odd term $(\boldsymbol{\epsilon}_1^* \times \boldsymbol{\epsilon}_2^*)$, the square bracket in (58) being replaced by

$$\left[\boldsymbol{\epsilon}_2^* \cdot \boldsymbol{\epsilon}_1^* + {\rm i}(1 + 2x + x\tilde{\tau})(\boldsymbol{\epsilon}_2^* \times \boldsymbol{\epsilon}_1^*)\right] \; . \tag{64}$$

This may be different in other approaches [12], showing the model dependence of the off-shell effect. For instance, potential-QCD models in general, besides a vector Coulomb potential, also contain a scalar potential.

4.3. A bound state quark model

In our previous accounts [14, 16], we applied a bound state model for $\bar{B}_s^0 \to 2\gamma$. Then the potentials V_i in (12) are replaced by a quark-meson interaction Lagrangian

$$\mathcal{L}_\Phi(s,b) \; = \; G_B \bar{b}\,\gamma_5\, s\,\Phi + {\rm h.c.} \; , \tag{65}$$

where Φ is the B-meson field. In this case, the term $\mathcal{L}_F$ can be transformed away by means of the field redefinitions:

$$s' = s + B_F\,\sigma\cdot F\,L\,b\;, \qquad b' = b + B_F^*\,\sigma\cdot FL\,s\;. \tag{66}$$

However, its effect reappears in a new bound-state interaction $\Delta\mathcal{L}_\Phi$,

$$\mathcal{L}_\Phi(s,b) + \mathcal{L}_F \;=\; \mathcal{L}_\Phi(s',b') \;+ \Delta\mathcal{L}_\Phi\;, \tag{67}$$

where, after using $R\gamma_5 = R$ and $L\gamma_5 = -L$,

$$\Delta\mathcal{L}_\Phi \;=\; B_F\,G_B\left[\bar{b}'\sigma\cdot FL\,b' \;-\; \bar{s}'\,\sigma\cdot FR\,s'\right]\Phi + \text{h.c.}\;. \tag{68}$$

The two terms in this equation correspond to two contact amplitudes displayed in Fig. 4. Also in this case, net off-shell effects are found [14, 16]. Further calculations of $B\to 2\gamma$ within bound state models of the type in (65) will be presented elsewhere.

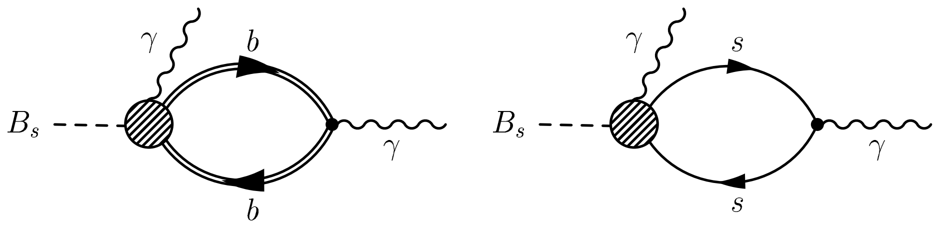

Fig. 4. The two-photon transition amplitude from a contact term (68) left over after the field redefinitions.

Note that in bound-state models based on heavy-quark effective theory, the expression (65) is slightly modified such that the b quark field will be replaced by the product of the reduced heavy-quark field and its projector $P_+(v) = (1+\gamma\cdot v)/2$, where v is the velocity of the heavy quark [43–46].

4.4. Link to $\bar{B}_d \to K^*\gamma$

Although we focused till now only to the two-photon processes, the interaction in (27) contributes to the one-photon couplings as well.

Actually, we observe that off-diagonal one-photon couplings contained in the Lagrangian given by (39) and (44) can be used to calculate the amplitude for muonic hydrogen decaying to a photon and ordinary hydrogen, that is, the process $\mu^-\to e^- + \gamma$ for both leptons bound to a proton. This is a leptonic version of the celebrated B-meson decay $\bar{B}_d\to K^*\gamma$.

As a toy model, one might consider a process "μ" $\to$ "e" γ in an external Coulomb field, with "μ" and "e" rather close in mass such that the non-relativistic descriptions of the "leptons" might be used. The effective "μ" $\to$ "e" γ interaction is given in (44). If we assume that $(M-m)$ is of order $\alpha\, m$, we obtain off-shell effects of the order of α^2 due to $\mathcal{L}_F$, relative to the standard magnetic moment term $\mathcal{L}_\sigma$. Bigger mass differences give bigger effects, until the non-relativistic approximation breaks down.

Returning to the off-shell bound-state effects in the important $B_d \to K^*\gamma$ decay, they can be addressed in the framework of models [43–48], combining heavy-quark effective theories with the ideas of Nambu-Jona-Lasinio models and chiral quark models.

The ordinary, on-shell transition magnetic moment $\mathcal{L}_\sigma$-induced amplitude for the $\bar{B}_d \to K^*\gamma$ is shown in Fig. 5a.

Now, transforming away the term $\mathcal{L}_F$ by the field redefinitions produces the new contact term

$$\Delta\mathcal{L}'_\Phi = -B_F\, G_B\, \bar{s}'\sigma \cdot F\, R\, P_+(v)\gamma_5\, d'\, \Phi\,, \tag{69}$$

giving the amplitude displayed in Fig. 5b.

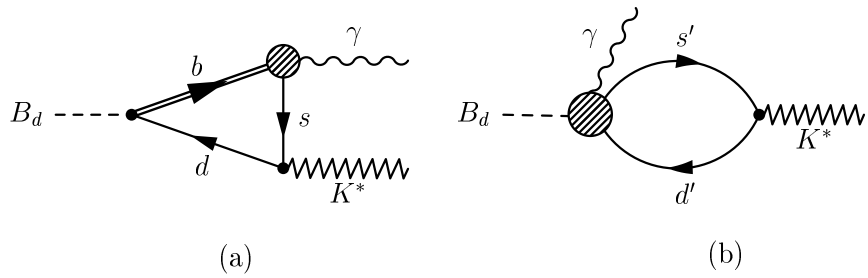

Fig. 5. Diagrams for $\bar{B}_d \to \gamma K^*$: (a) The magnetic moment transition amplitude. (b) The contact term (69) left over after field redefinitions.

The ratio of the off-shell and the on-shell amplitudes in the soft K^* limit can now be calculated to be

$$\frac{A(\bar{B}_d \to K^*\gamma)_{\mathrm{off-shell}}}{A(\bar{B}_d \to K^*\gamma)_{\mathrm{on-shell}}} \approx \frac{B_F}{B_\sigma}\frac{m_0}{2m_Q}\,, \tag{70}$$

where m_0 is the constituent mass of order of 200–300 MeV, and m_Q is the heavy (b quark) mass. Going away from the soft K^* limit, the amplitudes will change, but the result (70) will persist in the leading order.

A refined calculation would be desirable in view of the importance of this result. An earlier attempt (the preprint version of Ref. [49]) reported on large off-shell effect in the amplitude which happened to be reduced below 10% effect in the rate.

5. Discussion and conclusion

The present "atomic" approach enables us to see in a new light the off-shell effects studied first for the $K_L \to \gamma\gamma$ amplitude in the chiral quark model [13, 12], and subsequently in the bound-state model [15]. The observation that off-shell effects can be clearly isolated from the rest in the heavy-light quark atoms [14] was still plagued by the uncertainty in the QCD binding calculation [16]. Here, in the Coulomb-type QCD model, we are able to subsume the effect into an universal binding factor, in the same way as for the two-photon decay of muonium in the exactly solvable QED framework. It is a quite significant 10 percent effect in the $\bar{B}_s^0 \to \gamma\gamma$ case, whereas in the two-photon decay of muonium it is very small (of the order of 10^{-7}), but clearly identifiable.

As a byproduct, we obtain here also the on-shell amplitude already considered in the literature. There is an extensive list of calculations [39, 42, 50] relevant to the short-distance electroweak loop contributions to $b \to s\gamma\gamma$ which trigger $\bar{B}_s \to \gamma\gamma$. Comparing our results with the expression (22) of Ref. [39], we can express C_7^σ in our Eq. (54) in terms of their coefficient C,

$$C_7^\sigma = \frac{1}{4\sqrt{6}}\left(C + \frac{23}{3}\right), \qquad (71)$$

or, numerically, $C_7^\sigma = 0.4$ at the B meson scale. Still, there is another class of contributions, belonging to the LD regime. For example, Refs. [51, 52] present magnetic moment, O_7-type LD effects in $\bar{B}_s \to \gamma\gamma$ decays in the vector-meson dominance approach, whereas the other authors [53–55], though with controversial results, estimate the contribution of the charmed-meson intermediate states. These seem to be a natural representation for our short-distance loops when the loop momenta are below the b quark mass scale.

Our message is that such small SM effects might obscure possible new physics (BSM) signals that are of a comparable size. Without pretending on the completeness, we give some examples that the off-shell effects considered in the present paper might hinder possible BSM discovery.

Let us start with the famous magnetism of the muon, an ideal of the precision measurement. At some level, the binding effects might become relevant. Such $(g_{\rm Bound} - 2)$ effect due to the diagonal one-photon coupling would correspond to the $(g_{\rm Bound} - 2)$ calculated already for a bound electron [56]. This effect might be interesting in the light of a deviation from the SM expectation of the order of 10^{-9} recently measured for $(g-2)$ of the positively charged muon [57]. Actually, this measurement triggered various speculations ascribing this discrepancy to various BSM effects, the lepton compositeness [58] being one possibility. However, there are more direct ways to set a bound on the compositeness scale from the flavour-conserving processes. For example, there are flavour-diagonal $e^+e^-\gamma\gamma$ contact terms [59]

$$\mathcal{L}_{\rm contact} = {\rm i}\overline{\psi_e}\gamma_\mu(D_\nu\psi_e)\left(\frac{\sqrt{4\pi}}{\Lambda_6^2}F^{\mu\nu} + \frac{\sqrt{4\pi}}{\tilde{\Lambda}_6^2}\tilde{F}^{\mu\nu}\right), \qquad (72)$$

which would lead to a $(1+\delta_{\rm DEV})$ correction factor to the photon angular distribution $d\sigma/d\Omega$ in e^+e^- collisions. From

$$\delta_{\rm DEV} = s^2/(2\alpha)(1/\Lambda_6^4 + 1/\tilde{\Lambda}_6^4)(1-\cos^2\theta), \qquad (73)$$

LEP200 sets a bound $\Lambda > 1687$ GeV (for $\Lambda_6 = \tilde{\Lambda}_6 = \Lambda$) at the 95% CL. Thus, eventual non-standard BSM physics contribution at LEP energies are highly suppressed.

More promising route to reveal BSM contributions could be provided by flavour non-diagonal transitions. Recent evidence for neutrino oscillations has renewed interest in charged LFV searches. Among variety of probes reviewed in Ref. [60], $\mu \to e\gamma$ and $\mu - e$ conversion (invoking new high energy scale M_{12}) seem to be the most promising. Since the effects of new physics are expected to enter at the one-loop level, these transitions may be parameterized by

$$\mathcal{L}_{12} = e\frac{g^2}{16\pi^2}\frac{m_\mu}{M_{12}^2}\,\bar{\mu}\sigma^{\alpha\beta}e\,F_{\alpha\beta}\,, \tag{74}$$

in order to estimate the sensitivity of the current experimental facilities [61].

Although the BSM effects might be more pronounced for the flavour-changing quark transitions, their discovery might be hindered by the relatively more pronounced bound-state effects treated in the present paper. The off-shell contribution may affect the discovery potential in the radiative B meson decays (see, e.g., Refs. [62–64]). In particular, our result (70) indicates a hindrance of the BSM discovery potential in the otherwise promising $\bar{B}_d \to K^*\gamma$ decay.

References

[1] W. E. Lamb and C. R. Retherford, Phys. Rev. **72** (1947) 241.

[2] H. A. Bethe and E. Salpeter, *Quantum Mechanics of One and Two Electron Atoms*, Springer-Verlag, Berlin (1957).

[3] S. G. Karshenboim, (2000), physics/0008215.

[4] J. Ashkin and R. Marshak, Phys. Rev. **76** (1949) 58; Erratum, ibid, p. 989.

[5] H. W. Fearing and S. Scherer, Phys. Rev. C **62** (2000) 034003; nucl-th/9909076.

[6] H. W. Fearing, Few Body Syst. (Suppl.) **12** (2000) 263; nucl-th/0006040.

[7] S. Weinberg, *The Quantum Theory of Fields, Volume I: Foundations*, Cambridge University Press (1995) p. 331.

[8] M. J. Musolf and B. R. Holstein, Phys. Rev. D **43** (1991) 2956.

[9] J. O. Eeg and I. Picek, Fizika B (Zagreb) **3** (1994) 135.

[10] A. Rich, Rev. Mod. Phys. **53** (1981) 127.

[11] S. Mohorovičić, Astronomische Nachrichten **253** (1934) 93.

[12] J. O. Eeg and I. Picek, Phys. Lett. B **323** (1994) 193; hep-ph/9312320.

[13] J. O. Eeg and I. Picek, Phys. Lett. B **301** (1993) 423.

[14] J. O. Eeg and I. Picek, Phys. Lett. B **336** (1994) 549.

[15] D. Kekez, D. Klabučar, K. Kumerički and I. Picek, Phys. Lett. B **348** (1995) 239; hep-ph/9501350.

[16] E. Vanem and J. O. Eeg, Phys. Rev. D **58** (1998) 114010.

[17] J. O. Eeg, K. Kumerički and I. Picek, Eur. Phys. J. C **17** (2000) 163; hep-ph/0007183.

[18] F. Gross, *Relativistic Quantum Mechanics and Field Theory*, Wiley, New York (1993).

[19] W. E. Caswell and G. P. Lepage, Phys. Lett. B **167** (1986) 437.

[20] G. P. Lepage (1997), nucl-th/9706029.

[21] A. Czarnecki, G. P. Lepage and W. J. Marciano, Phys. Rev. D **61** (2000) 073001; hep-ph/9908439.

[22] D. E. Groom et al., Eur. Phys. J. C **15** (2000) 1.

[23] G. S. Adkins, R. N. Fell and J. Sapirstein, Phys. Rev. Lett. **84** (2000) 5086; hep-ph/0003028.

[24] P. Heiliger, B. McKellar and L. M. Sehgal, Phys. Lett. B **327** (1994) 145; hep-ph/9402216.

[25] D. Bouwmeester, J.-W. Pan, M. Daniell, H. Weinfurter and A. Zeilinger, Phys. Rev. Lett. **82** (1999) 1345; quant-ph/9810035.

[26] T.-P. Cheng and L.-F. Li, *Gauge Theory of Elementary Particle Physics*, Clarendon Press, Oxford (1984).

[27] J. J. Sakurai, *Advanced Quantum Mechanics*, Addison-Wesley, Reading (1967).

[28] C. Itzykson and J.-B. Zuber, *Quantum Field Theory*, McGraw-Hill, New York (1980).

[29] J. Wheeler, Ann. N. Y. Acad. Sci. **48** (1946) 219.

[30] O. Nachtmann, *Elementary Particle Physics: Concepts and Phenomena*, Springer, Berlin (1990).

[31] M. N. Z. Maki and S. Sakata, Prog. Theor. Phys. **28** (1962) 870.

[32] R. Mertig, M. Böhm and A. Denner, Comp. Phys. Comm. **64** (1991) 345.

[33] S. Wolfram, *Mathematica: A System for Doing Mathematics by Computer*, Addison-Wesley (1988).

[34] B. Grinstein, R. Springer and M. B. Wise, Nucl. Phys. B **339** (1990) 269.

[35] A. J. Buras, M. Misiak, M. Munz and S. Pokorski, Nucl. Phys. B **424** (1994) 374; hep-ph/9311345.

[36] R. Grigjanis, P. J. O'Donnell, M. Sutherland and H. Navelet, Phys. Lett. B **213** (1988) 355.

[37] G. Cella, G. Curci, G. Ricciardi and A. Vicere, Phys. Lett. B **248** (1990) 181.

[38] G. Cella, G. Curci, G. Ricciardi and A. Vicere, Phys. Lett. B **325** (1994) 227; hep-ph/9401254.

[39] G. G. Devidze, G. R. Dzhibuti and A. G. Liparteliani, Nucl. Phys. B **468** (1996) 241.

[40] T. Goldman (1996), hep-ph/9703222; presented at *2nd Int. Conf. on Quark Confinement and the Hadron Spectrum*, Como, Italy, 26-29 Jun 1996.

[41] H. D. Trottier and R. R. Mendel, *Proc. of 10th Annual Montreal-Rochester-Syracuse-Toronto Mtg. on High Energy Theory*, Toronto, Canada, May 9-10, 1988, pp. 134-155.

[42] S. Herrlich and J. Kalinowski, Nucl. Phys. B **381** (1992) 501.

[43] W. A. Bardeen and C. T. Hill, Phys. Rev. D **49** (1994) 409; hep-ph/9304265.

[44] D. Ebert, T. Feldmann, R. Friedrich and H. Reinhardt, Nucl. Phys. B **434** (1995) 619; hep-ph/9406220.

[45] A. Deandrea, N. D. Bartolomeo, R. Gatto, G. Nardulli and A. D. Polosa, Phys. Rev. D **58** (1998) 034004; hep-ph/9802308.

[46] A. D. Polosa, Riv. Nuovo Cim. **23** (2000) 1; hep-ph/0004183.

[47] J. O. Eeg, S. Fajfer and J. Zupan, Phys. Rev. D **64** (2001) 034010; hep-ph/0101215.

[48] A. Hiorth and J. O. Eeg (2001), in preparation.

[49] C. E. Carlson and J. Milana, Phys. Rev. D **51** (1995) 4950; hep-ph/9405344.

[50] L. Reina, G. Ricciardi and A. Soni, Phys. Rev. D **56** (1997) 5805; hep-ph/9706253.

[51] G. Hiller and E. O. Iltan, Phys. Lett. B **409** (1997) 425; hep-ph/9704385.

[52] G. Hiller, (1998), hep-ph/9809505.

[53] D. Choudhury and J. Ellis, Phys. Lett. B **433** (1998) 102; hep-ph/9804300.

[54] W. Liu, B. Zhang and H. qing Zheng, Phys. Lett. B **461** (1999) 295; hep-ph/9905504.

[55] A. N. Mitra, (1999), hep-ph/9902377.

[56] A. Czarnecki, K. Melnikov and A. Yelkhovsky, Phys. Rev. A **63** (2001) 012509; hep-ph/0007217.

[57] Muon g-2, H. N. Brown et al., Phys. Rev. Lett. **86** (2001) 2227; hep-ex/0102017.

[58] X. Calmet, H. Fritzsch and D. Holtmannspotter, Phys. Rev. D **64** (2001) 037701; hep-ph/0103012.

[59] A. Bajo et al., (2000), hep-ph/0012095.

[60] Y. Kuno and Y. Okada, Rev. Mod. Phys. **73** (2001) 151; hep-ph/9909265.

[61] J. L. Feng, (2000), hep-ph/0101122.

[62] A. K. Grant, Nucl. Instrum. Meth. A **408** (1998) 21; hep-ph/9801337.

[63] M. Misiak, (1999), hep-ph/0002007.

[64] S. Ahmad and Riazuddin, (2001), hep-ph/0101256.

PONIŠTENJE TEŠKO-LAKIH FERMIONSKIH VEZANIH STANJA DVOFOTONSKIM RASPADOM

Razmatramo dvofotonske raspade teško-lakih atoma kvantne elektrodinamike i kromodinamike (QED i QCD), $\mu^+e^- \to \gamma\gamma$ i $\bar{B}_s^0 \to \gamma\gamma$. Posebice, istražujemo doprinose operatora koji trnu na ljusci mase slobodnih kvarkova. Pokazujemo da se redefinicijom polja ovi operatori pretvaraju u kontaktne članove povezane s dinamikom vezanih stanja. Ukupan doprinos izvan ljuske mase je potisnut u odnosu na učinak dobro poznatog operatora magnetskog momenta zbog faktora vezanja vezanog stanja. Učinci izvan ljuske u slabo vezanim QED atomima su zanemarivi, međutim, oni postaju znatni u jako vezanim QCD atomima. Analiziramo te učinke izvan ljuske u modelskim pristupima QCD, od kojih nam jedan omogućuje blisku usporedbu s odnosnim učinkom u QED. Također navodimo učinak izvan ljuske u srodnom procesu $\bar{B}_d \to K^*\gamma$, kao prepreku za razotkrivanje najavljenih mogućih učinaka fizike izvan standardnog modela.