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LETTER TO THE EDITOR

ORBITING-CLUSTER MODEL WITH COMBINATORIAL LEVEL DENSITY

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Paper devoted to honour the memory of Professor Nikola Cindro

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The combinatorial nuclear level densities are used in the orbiting-cluster model calculations. Previous predictions of the model of resonance observability obtained for systems of mass $A \leq 56$ with nuclear statistical-model (Bethe formula) level densities are confirmed. The present calculation correctly predicts resonances in barium isotopes, in accordance with recent experimental results, while the statistical model does not.

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In the scientific opus of Nikola Cindro, the most cited reference is the theoretical work in Ref. [1] describing the conditions a colliding nuclear system has to fulfil to exhibit resonant behaviour. This phenomenological approach to the so-called *molecular resonances* named the *orbiting-cluster model* (OCM) was introduced in Cindro's earlier works [2]. The aim of the present paper is to check how the use of more realistic level densities, a crucial parameter of the model, influences the OCM predictions.

The existence of intermediate-width resonances is a well established feature of heavy-ion reactions [3]. Owing to the exceptionally small absorption (a property of several nuclei), two nuclei in a low-energy grazing collision stick together and orbit

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about their centre of mass. This simple transient di-nuclear (molecular) configuration is a doorway-like state [4] of an intermediate width. A necessary condition that a near-to-grazing entrance-channel partial wave of an angular momentum L should excite a resonance of spin $J = L$ is[2] that the above di-nuclear rotator should live long enough. Colliding nuclei should neither re-separate nor fuse into a compound nucleus before exhibiting an appreciable portion of a full rotation. A number of nuclear systems display resonances that for a given system in the E versus $J(J+1)$ plane fall close to a straight median line, as one expects for an energy spectrum of a rigid quantum rotator (see graphical presentations of resonant-system energy spectra, e.g., in Ref. [5])

$$E^*(J) = E_0 + \frac{\hbar^2}{2\mathcal{J}} J(J+1), \tag{1}$$

where E^* is the excitation energy of the resonance J (or, correspondingly, the system centre-of-mass energy) and $\hbar$ is the Planck constant divided by 2π. E_0 is the band-head corresponding to the touching configuration with no rotation ($J = 0$). Hence, E_0 may be seen as a sum of the binding energy $E_{\rm B}$ of the projectile and the target into the composite system and the respective Coulomb barrier $E_{\rm C}$, i.e. $E_0 \approx E_{\rm B} + E_{\rm C}$. The Coulomb repulsion energy of the two nuclei with atomic numbers Z_1 and Z_2 and mass numbers A_1 and A_2, respectively, is calculated according to Ref. [6]

$$E_C = \frac{1.44\ Z_1 Z_2}{1.12\,(A_1^{1/3} + A_2^{1/3}) - 0.94\,(A_1^{-1/3} + A_2^{-1/3}) + 3.0}\ \text{MeV}. \tag{2}$$

The slope of the band is proportional to the moment of inertia $\mathcal{J}$. The experimentally deduced effective moment of inertia $\mathcal{J}_{\rm eff}$ is in agreement with that calculated for the two osculating spherical nuclei A_1 and A_2 [1]

$$\mathcal{J} = 1.036 \left[\frac{2}{5}\,(A_1^{5/3} + A_2^{5/3}) + \frac{A_1 A_2}{A_1 + A_2}\,(A_1^{1/3} + A_2^{1/3})^2 \right] r_0^2 \times 10^{-44}, \tag{3}$$

where $\mathcal{J}$ is obtained in units MeV s^2 and $r_0 = 1.3$ fm.

In addition to the representation of orbiting clusters in a molecular-type configuration, OCM also relies on the concept of the so-called *molecular-resonance window* [7]. A molecular window is the domain of low (and relatively constant) level density in the (E^*, J) space of the corresponding compound nucleus within which the dissolution of the (simple) molecular configuration into more complex (compound nuclear) states of the system is expected to be strongly reduced. The probability of damping of a doorway state into compound-nucleus states is in the formal theory of intermediate structures given by the *spreading width* [4]

$$\Gamma^{\downarrow} = 2\pi\ |<\text{CN}|V|\text{el}>|^2\ \rho(E^*, J), \tag{4}$$

[2]Strictly valid only for collisions of spinless nuclei, but they are by far the most frequent cases among the observed resonating nuclear systems.

where V is the transition interaction which couples the reaction entrance (elastic) channel $|\mathrm{el}>$ and the compound-nucleus states $|\mathrm{CN}>$ of the nucleus corresponding to the complete system. $\rho(E^*, J)$ is the density of the final (compound-nucleus) levels. According to this picture, resonances will experimentally show up only when nuclear structure and/or other effects contribute to keep the value of $\Gamma^{\downarrow}$ within the limits of observability (of the order of a few hundred keV), while beyond this limit, the doorway state dumps down its strength into the surrounding continuum and the intermediate structure is washed out. Obviously, a small compound-nucleus level density means a small spreading width which ensures the weak absorption within the molecular window.

To estimate $\Gamma^{\downarrow}$ as a function of A, E^* and J in the OCM, it has been assumed that a matrix element decreases exponentially with A, without a detailed dependence on nuclear structure

$$|<\mathrm{CN}|V|\mathrm{el}>|^2 = const. \times \exp(-kA). \tag{5}$$

The matrix element is equal for all nuclear systems of the same mass number A and varies slowly and smoothly within the molecular window in such a way that its dependence on E^* and J may be neglected in the first approximation. Therefore, the reliability with which the $\rho(E^*, J)$ is calculated determines the reliability of the model predictions. The parameter k in Eq. (5) has been deduced from the fit to systems with well-established resonances. The constant of proportionality in Eq. (5) has been cancelled by considering the relative damping widths, $R_\Gamma = \Gamma^{\downarrow}/\Gamma^{\downarrow}_{\mathrm{C+C}}$. In this way, the above rough estimate of the matrix element via Eq. (5) uses the $\Gamma^{\downarrow}_{\mathrm{C+C}}$ of the $^{12}\mathrm{C} + {}^{12}\mathrm{C}$ system as a measure of damping in other systems. Accordingly, the value of R_Γ classifies the observability of resonances in nuclear systems. A projectile-target combination which, within its molecular window, displays the R_Γ value close to or smaller than unity is likely to exhibit resonating features in the excitation functions. The smaller the R_Γ, the higher the chance for resonant behaviour.

Predictions of resonance observation were given for nuclear systems with $A \leq 58$ [1] and with $A \leq 72$ [8]. These predictions agree remarkably with recent experimental results [5]. Encouraged by the success of the predictions, the OCM calculation with a small modification of the estimated decrease of the matrix element with A

$$|<\mathrm{CN}|V|\mathrm{el}>|^2 = const. \times \exp(-kA^{\kappa}) \tag{6}$$

($\kappa \leq 1$) was later systematically extended to medium- and high-mass systems with $A \leq 110$ [9]. Likelihood for resonant behaviour was evaluated for more than 400 target-projectile combinations and several new candidate-systems for resonant behaviour were identified [9].

After the above predictions were published, resonant behaviour was reported for even heavier systems, ^{116}Ba and ^{120}Ba excited in collisions of ^{58}Ni with ^{58}Ni and ^{62}Ni nuclei, respectively [10]. The values of angular momenta estimated from the experimental angular distributions, measured at energies for which intermediate structures were identified in the above collisions, were compared with the OCM

predictions. However, the value of the relative damping R_Γ, the main OCM criterion of resonance observability, was not reported for these Ba systems.

Until now, in all applications of OCM, the standard Bethe level density formula [11] for a given angular momentum J has been used

$$\rho(E^*, J) = 0.0295\ (2J+1)\ \exp\left[-\frac{(J+1/2)^2}{2\sigma^2}\right]\ \frac{\exp\left[2(aE^*)^{1/2}\right]}{a^{1/4}\ E^{*5/4}\ \sigma^{5/2}}, \qquad (7)$$

where a is the level-density parameter and σ the spin cut-off parameter. However, note that the angular momentum distribution underlying this formula is purely a classical result unreliable for the high-spin region. Nuclear structure effects in $\rho(E^*, J)$ are accounted for by taking a and the pairing energy correction Δ according to the parametrization in Ref. [12]. Values of ratios of the average level densities $<\rho>$ of the reaction systems used in fitting the expression (6) relative to the average level density $<\rho_{\mathrm{C+C}}>$ of the $^{12}\mathrm{C}+^{12}\mathrm{C}$ system plotted against the (shifted) system mass on κ are shown as open circles in Fig. 1. The systems and the corresponding

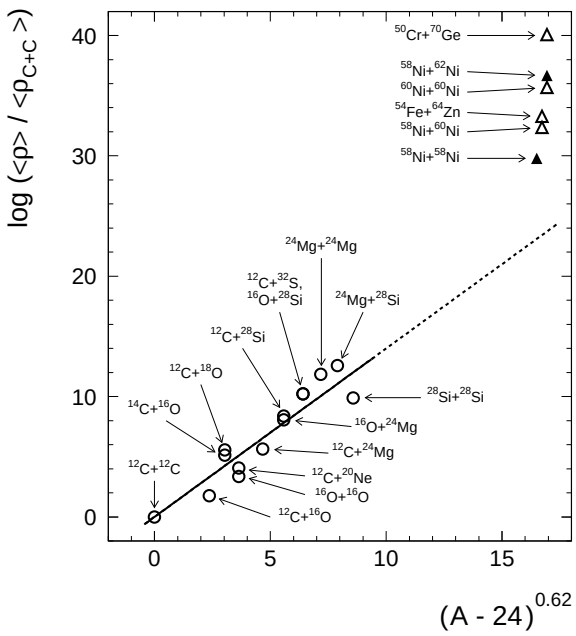

Fig. 1. Logarithm of the relative level densities calculated by the statistical-model Bethe formula (7) as a function of the (shifted) system mass with the best-fit exponent $\kappa = 0.62$. The heavy full line is a result of the linear best-fit following Eq. (6) of the systems with well-established resonant behaviour (open circles). Extrapolation of the best-fit line to heavier systems is represented by the dotted line and the several projectile-target combinations leading to the barium isotopes ^{116,118,120}Ba are marked with (open and filled) triangles. Filled triangles stand for the experimentally studied systems [10].

statistical-model parameters are taken from Ref. [9]. A more careful fitting of Eq. (6) has resulted in a slightly different fit result with $k=1.4$ and $\kappa=0.62$.

In the same figure, several projectile-target combinations which lead to the composite systems ^{116,118,120}Ba are displayed as open and filled triangles. The deduced relative damping width R_Γ of all studied systems is given in Table 1. Figure 1 and Table 1, show clearly that the criterion of resonance observability ($R_\Gamma \approx 1$) is not fulfilled for the systems leading to the Ba isotopes. In contrast to the OCM prediction, the experiment showed the existence of narrow structures in the peripheral collision of nickel isotopes [10]. This discrepancy might be attributed to the less reliable estimate of level density via the Bethe formula for the Ba isotopes, or to the crudity of the assumption on the matrix element behaviour (6) whose extension to these rather heavy systems may be unrealistic.

TABLE 1. Relative spreading width calculated with statistical-model and combinatorial level densities, for systems with observed resonant behaviour and systems leading to barium isotopes.

Composite system	Entrance channel	R_Γ^{stat}	R_Γ^{comb}
^{24}Mg	^{12}C+^{12}C	1.0	1.0
^{28}Si	^{12}C+^{16}O	0.2	0.2
^{30}Si	^{12}C+^{18}O	2.4	1.1
	^{14}C+^{16}O	3.7	1.9
^{32}S	^{12}C+^{20}Ne	0.4	2.4
	^{16}O+^{16}O	0.2	1.0
^{36}Ar	^{12}C+^{24}Mg	0.4	2.3
^{40}Ca	^{12}C+^{28}Si	1.8	0.4
	^{16}O+^{24}Mg	1.3	0.3
^{44}Ti	^{12}C+^{32}S	3.5	0.9
	^{16}O+^{28}Si	3.5	1.0
^{48}Cr	^{24}Mg+^{24}Mg	6.0	17
^{52}Fe	^{24}Mg+^{28}Si	4.6	1.9
^{56}Ni	^{28}Si+^{28}Si	0.1	0.1
^{116}Ba	^{58}Ni+^{58}Ni	800	0.002
^{118}Ba	^{58}Ni+^{60}Ni	7800	0.02
	^{54}Fe+^{64}Zn	2×10^4	0.05
^{120}Ba	^{60}Ni+^{60}Ni	2×10^5	0.05
	^{58}Ni+^{62}Ni	4×10^5	0.1
	^{50}Cr+^{70}Ge	1×10^7	1.7

At higher excitation energies, one expects failure of the Bethe formula (7) which predicts an exponential increase of the level density. In addition, this formula cannot reproduce the level density well over a large energy interval assuming a constant level density parameter a. In the derivation of level densities in Ref. [12] an approximation is made by replacing the exact single-particle-state density function g by a smooth one. This function is then expanded around the Fermi energy.

In order to verify the former hypothesis on the possible cause of the lack of a satisfactory OCM prediction in our calculation, for the same systems and in the same spirit, a simple approach based on the Bethe formula (7) was replaced by an exact combinatorial approach to the level-density calculation. As a first guess, the combinatorial calculations were performed using the realistic single-particle spectra [13] and a simplified pairing force as a residual interaction of the strength G. The basic algorithm used was developed in Ref. [14]. In this approach, non-collective excitations are treated by the combinatorial algorithm exploited by the code SPINDIS [15], while collective excitations are accounted for by the residual-interaction coupling-constant renormalization as described in Ref. [16]. As for the nuclei studied here, when there is no available data on the level densities, we have used the estimate $G = 0.5$ MeV that has been proved to account for the missing dynamics in many cases. In other aspects, the calculation is completely exact and prompts improvements using more realistic single-particle spectra. Total parity and angular-momentum dependent level densities are provided in tabular form up to a given excitation energy. The obtained results are shown in Fig. 2 and the last column of Table 1. For combinatorial level densities, the matrix ele-

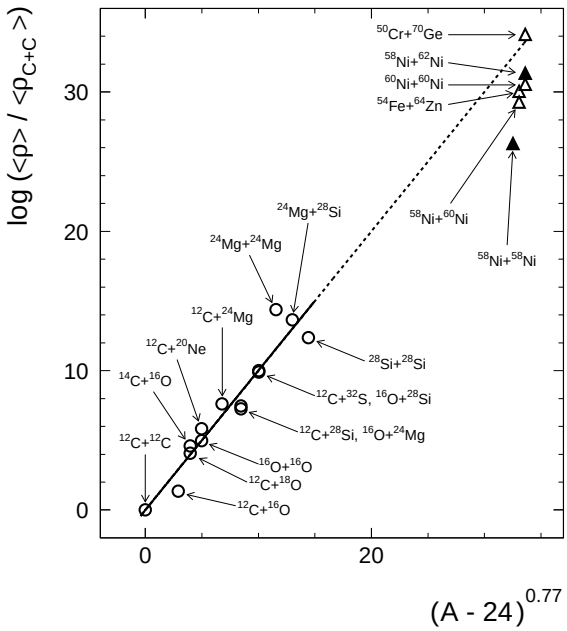

Fig. 2. Same as Fig. 1, but for combinatorial level densities. Parameter $\kappa = 0.77$.

ment dependence on A is closer to the original OCM pure exponential behaviour ($\kappa = 0.77$), while the parameter k is equal to 1.0. It turns out that the combinatorial level densities do not have an appreciable influence on the OCM estimate of the likelihood of resonant behaviour for systems with unambiguously observed resonances. The only exception is the ^{24}Mg+^{24}Mg system for which the relative damping R_Γ is as high as 17 (see the last column of Table 1). On the other hand, the relative combinatorial level densities of Ba isotopes are drastically lower, so even the ^{50}Cr+^{70}Ge system with a relatively high binding energy might exhibit resonant behaviour. The predicted likelihood for the observation of resonances in collisions of ^{58}Ni nuclei is extremely high — two orders of magnitude higher than for, e.g., collisions of ^{16}O nuclei. The results for the Ba isotopes deduced using the combinatorial-level-density calculation should, however, be taken with some precaution owing to the fact that the OCM estimate for resonant behaviour obtained for low- and medium-mass systems was extrapolated to as much as twice higher system masses.

In conclusion, the phenomenological orbiting-cluster model of molecular resonances with the combinatorial level densities confirms its predictions of resonance observability obtained with the statistical model (Bethe formula) level densities and which are in full agreement with experimental observations for $A \leq 56$ [5]. In contrast to the results obtained using the simple standard Bethe formula, the extrapolated OCM calculation with the combinatorial approach correctly predicts resonances in barium isotopes in accordance with the recent observations [10].

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MODEL KRUŽEĆIH GROZDOVA S KOMBINATORNOM GUSTOĆOM STANJA

Primijenili smo kombinatornu gustoću nuklearnih stanja u modelu orbitirajućih grozdova. Potvrdili smo ranija predviđanja vjerojatnosti opažanja rezonancija za sustave masa $A \leq 56$ koja su bila dobivena rabeći gustoće stanja izračunate nuklearnim statističkim modelom (Betheova formula). Kombinatorni račun ispravno predviđa rezonancije u izotopima barija, u skladu s ishodima nedavnih mjerenja, dok statistički model daje loše rezultate.