Conformal entropy as a consequence of the properties of stationary Killing horizons

M. Cvitan* and S. Pallua†
Department of Theoretical Physics, Faculty of Natural Sciences and Mathematics, University of Zagreb, Bijenička cesta 32, 10000 Zagreb, Croatia

P. Prester‡
Max-Planck-Institut für Gravitationsphysik, (Albert-Einstein-Institut), Am Mühlenberg 1, D-14476 Golm, Germany
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We show that the microscopic black hole entropy formula based on Virasoro algebra can be derived from the usual properties of stationary Killing horizons alone and the absence of singularities of curvature invariants on them. In such a way some usual additional assumptions are shown to be fulfilled. In addition, for all quantities power expansion near the horizon and thus explicit insight of the limiting procedure is given. More important the near horizon conformal symmetry proposed by Carlip together with its consequences on microscopic entropy is given a clear geometric origin.

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I. INTRODUCTION

One of the promising efforts to understand the microscopic origin of black hole entropy is due to the Solodukhin [1] and Carlip [2–5] approach which both try to exploit conformal symmetry and corresponding Virasoro algebra. In particular, Carlip in the case of Einstein gravity assumes a certain class of boundary conditions near the horizon which enable one to identify a subalgebra of algebra of diffeomorphisms which turns out to be Virasoro algebra. Calculations of central charge then enabled Carlip to calculate entropy via the Cardy formula. The refinements and open questions of this method have been discussed in various references [5–9]. These results have been subsequently generalized to Gauss-Bonnet gravity and for higher curvature Lagrangians [10–12]. This is the clear indication that these properties are properties of horizons and depend only on diffeomorphism invariance of gravity but not much of a particular form of interaction.

However, there have been several open questions in this method which have to be answered. For instance it is important to construct examples where the boundary conditions imposed by Carlip or their consequences are indeed realized. Also the derivation required additional assumptions. This refers, in particular, to the assumptions on behavior of the so-called "spatial derivatives" (assumed in Appendix A of Ref. [5]). In the subsequent generalizations [11,12] to higher order interactions this was even more important.

Also, from the conceptual point of view it would be desirable to add more understanding of the origin of the obtained properties. In fact one suggestion for its physical interpretation is given in the framework of induced gravity [13]. The other proposal is that these properties have geometrical origin. Indeed, recently in [14] it was shown that the existence of the stationary Killing horizon together with the absence of curvature singularities on the horizon implies very restricted geometry near the horizon and also leads to conformal properties of Einstein tensor. It was also suggested that these properties could be the realization of Carlip boundary conditions.

In this paper we assume the existence of such a stationary black hole horizon. We want to show that the boundary conditions proposed in [5] are then indeed realized. In addition we want to show that properties of stationary horizons enable one to calculate the necessary quantities for central charge and entropy. It will be possible to do the explicit calculation to leading order but also to next orders which vanish when we perform integration over the horizon. In such a way the conformal symmetry and Virasoro algebra have indeed geometrical interpretation in the sense that they are a consequence of the horizon properties, as suggested in [14].

II. BOUNDARY PROPERTIES AT KILLING HORIZONS

Axially symmetric black holes have two Killing vectors, e.g.,

$$t^a = \left(\frac{\partial}{\partial t}\right)^a, \qquad \phi^a = \left(\frac{\partial}{\partial \phi}\right)^a, \tag{1}$$

with corresponding coordinates t and ϕ. The other two coordinates n, z can be chosen so that in the equal time hypersurface one chooses the Gauss normal coordinate n ($n = 0$ on the horizon) and the remaining coordinate z so that the metric has the form

*Electronic address: mcvitan@phy.hr
†Electronic address: pallua@phy.hr
‡On leave of absence from the Department of Theoretical Physics, Faculty of Natural Sciences and Mathematics, Zagreb, Croatia.
Electronic address: pprester@phy.hr

$$ds^2 = -N(n,z)^2 dt^2 + g_{\phi\phi}(n,z)[d\phi - \omega(n,z)dt]^2 + dn^2 + g_{zz}(n,z)dz^2. \tag{2}$$

The horizon is defined with

$$N(n,z) = 0. \tag{3}$$

Now, well-known theorems imply

$$\kappa \equiv \lim_{n\to 0} \partial_n N = \text{const} > 0, \tag{4}$$

on the horizon (for nonextremal black holes), also

$$\Omega_H \equiv -\lim_{n\to 0} \frac{g_{\phi t}}{g_{\phi\phi}} = \lim_{n\to 0} \omega = \text{constant on horizon}, \tag{5}$$

and the property that the horizon is extrinsically flat with the consequence

$$\lim_{n\to 0} \frac{\partial g_{\mu\nu}}{\partial n} = 0 \text{ on the horizon.} \tag{6}$$

The absence of curvature singularities on the horizon implies [14] that metric coefficients have the following Taylor expansions:

$$\begin{aligned} N(n,z) &= \kappa n + \frac{1}{3!}\kappa_2(z)n^3 + O(n^4), \\ g_{\phi\phi}(n,z) &= g_{H\phi\phi}(z) + \frac{1}{2}g_{2\phi\phi}(z)n^2 + O(n^3), \\ g_{zz}(n,z) &= g_{Hzz}(z) + \frac{1}{2}g_{2zz}(z)n^2 + O(n^3), \\ \omega(n,z) &= \Omega_H + \frac{1}{2}\omega_2(z)n^2 + O(n^3). \end{aligned} \tag{7}$$

The Killing horizon has a Killing null vector

$$\chi^a = t^a + \Omega_H \phi^a. \tag{8}$$

On the horizon ($\chi^2 = 0$), this vector satisfies the well-known relation

$$\nabla^a \chi^2 = -2\kappa\chi^a. \tag{9}$$

For $\chi^2 \geq 0$, the left-hand side of (9) defines vector ρ_a

$$\nabla_a \chi^2 = -2\kappa\rho_a. \tag{10}$$

In the following we shall use the basis

$$\chi^a, \quad \phi^a, \quad \rho^a, \quad z^a. \tag{11}$$

Explicitly

$$\chi^a = \begin{pmatrix} 1 \\ \Omega_H \\ 0 \\ 0 \end{pmatrix}, \qquad \phi^a = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \tag{12}$$

$$\rho^a = \begin{pmatrix} 0 \\ 0 \\ \kappa n + O(n^3) \\ O(n^4) \end{pmatrix}, \qquad z^a = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}.$$

The leading terms of the nonvanishing products of the basis vectors are

$$\begin{aligned} \chi\cdot\chi &= -\kappa^2 n^2 + O(n^4), \\ \chi\cdot\phi &= -\tfrac{1}{2}g_{H\phi\phi}(z)\omega_2(z)n^2 + O(n^3), \\ \phi\cdot\phi &= g_{H\phi\phi}(z) + O(n^2), \\ \rho\cdot\rho &= \kappa^2 n^2 + O(n^4), \\ \rho\cdot z &= O(n^4), \\ z\cdot z &= g_{Hzz}(z) + O(n^2), \end{aligned} \tag{13}$$

and all other products are zero

$$\chi\cdot\rho = \chi\cdot z = \phi\cdot\rho = \phi\cdot z = 0. \tag{14}$$

Now we can, following Carlip, consider diffeomorphisms generated by the following vector fields:

$$\xi^a = T\chi^a + R\rho^a. \tag{15}$$

In principle diffeomorphisms could change the position of

$$\chi^2 = 0. \tag{16}$$

One requires therefore the condition that surface variations keep this surface fixed or

$$\delta\chi^2 = 0, \tag{17}$$

and even a stronger condition

$$\frac{\delta\chi^2}{\chi^2} = 0. \tag{18}$$

Straightforward calculation shows

$$\begin{aligned} \delta\chi^2 &= \chi^a\chi^b\delta g_{ab} = \chi^a\chi^b(\nabla_a\xi_b + \nabla_b\xi_a) \\ &= 2R\chi^b\nabla_\chi\rho_b + 2(\nabla_\chi T)\chi^2. \end{aligned} \tag{19}$$

Because of the exact relation [[5], Eq. A.4]

$$\chi^b\nabla_\chi\rho_b = -\kappa\rho^2, \tag{20}$$

it follows that

$$\delta\chi^2 = -2\kappa R\rho^2 + 2(\nabla_\chi T)\chi^2. \tag{21}$$

Thus requirement (17) will be satisfied automatically on the horizon, and the stronger requirement (18) will be fulfilled if

$$R = -\frac{1}{\kappa}\frac{\chi^2}{\rho^2}\nabla_\chi T. \tag{22}$$

Selecting one parameter group of diffeomorphisms one can calculate the commutator of two vector fields and provided we impose the additional condition on the diffeomorphism defining functions

$$\rho^a\nabla_a T = 0 \text{ at horizon}, \tag{23}$$

one obtains the closed algebra

$$\{\xi_1, \xi_2\}^a = (T_1\nabla_\chi T_2 - T_2\nabla_\chi T_1)\chi^a + \frac{1}{\kappa}\frac{\chi^2}{\rho^2}\nabla_\chi(T_1\nabla_\chi T_2 - T_2\nabla_\chi T_1)\rho^a. \quad (24)$$

As pointed out in [5], this is isomorphic to DiffS^1 or Diff$\mathbf{R}$. An additional natural requirement on fluctuations and thus on diffeomorphisms is made as usual

$$\delta\int_{\partial C}\hat{\boldsymbol{\epsilon}}\left(\tilde{\kappa} - \frac{\rho}{|\chi|}\kappa\right) = 0, \quad (25)$$

where $\tilde{\kappa}^2 = -a^2/\chi^2$, and $a^a = \chi^b\nabla_b\chi^a$ is the acceleration of an orbit of χ^a. As explained elsewhere it provides us with orthogonality relations for one parameter group of diffeomorphisms.

We are now in a position to calculate the near horizon expansion for fluctuations δg_{ab} using decompositions (7). The leading terms and next to leading order are

$$\begin{aligned}\delta g_{ab} = {}& 2\chi_{(a}\rho_{b)}\left[\frac{\ddot{T}}{\kappa^3 n^2} + \left(\frac{3\omega_2^2 g_{H\phi\phi}\ddot{T}}{4\kappa^5} - \frac{4\kappa_2\ddot{T}}{3\kappa^4} + \frac{\omega_2\partial_\phi\dot{T}}{2\kappa^3}\right) + O(n)\right] + \rho_a\rho_b\left[\frac{-2\dot{T}}{\kappa^2 n^2} + \left(\frac{-7\omega_2^2 g_{H\phi\phi}\dot{T}}{2\kappa^4} + \frac{14\kappa_2\dot{T}}{3\kappa^3}\right) + O(n)\right]\\ &+2\chi_{(a}\phi_{b)}\left[\left(\frac{-\omega_2\dot{T}}{2\kappa^2} + \frac{\partial_\phi T}{g_{H\phi\phi}}\right) - \frac{2\omega_3\dot{T}}{\kappa^2}n + O(n^2)\right] + 2\rho_{(a}\phi_{b)}\left[\left(\frac{\omega_2\ddot{T}}{2\kappa^3} - \frac{\partial_\phi\dot{T}}{g_{H\phi\phi}\kappa}\right) + \frac{\omega_3\ddot{T}}{\kappa^3}n + O(n^2)\right]\\ &+\chi_a\chi_b\left[\left(\frac{\omega_2^2 g_{H\phi\phi}\dot{T}}{2\kappa^4} - \frac{\omega_2\partial_\phi T}{\kappa^2}\right) + O(n)\right] + \phi_a\phi_b\left[\left(-\frac{g_{2H\phi\phi}\dot{T}}{g_{H\phi\phi}^2} - \frac{\omega_2^2\dot{T}}{2\kappa^2}\right)n^2 + O(n^3)\right]\\ &+z_a z_b\left[-\frac{g_{2Hzz}\dot{T}}{g_{Hzz}^2}n^2 + O(n^3)\right] + \text{terms of order} \geqq 3, \end{aligned} \quad (26)$$

where the expansion of the metric up to the n^4 terms was used [ω_3 is defined as $\omega(n,z) = \Omega_H + \frac{1}{2}\omega_2(z)n^2 + \omega_3 n^3 + \cdots$].

Taking into consideration Eqs. (13) we can ascribe to basis vectors χ, ρ order n^1 and to ϕ, z order n^0. Then the above expansion is the power series containing terms up to order n^2. The leading terms are

$$\delta g_{ab} = (\chi_a\rho_b + \rho_a\chi_b)\frac{\ddot{T}}{\kappa\rho^2} - \rho_a\rho_b\frac{2\dot{T}}{\rho^2}. \quad (27)$$

Because of the fact that our manifold has boundaries it is natural to look for central extensions of this algebra. The necessary formalism of covariant phase space was explained in [15–17] and exploited in [5,11,12]. For this reason we shall mention here just the main equations and refer details to the above mentioned references. For a given Lagrangian 4-form $\mathbf{L}$ we write the variation

$$\delta\mathbf{L}[\phi] = \mathbf{E}[\phi]\delta\phi + d\Theta[\phi, \delta\phi]. \quad (28)$$

The 3-form Θ or symplectic potential is implicitly defined in the above equation. To vector fields ξ^a we associate vector current 3-form

$$\mathbf{J}[\xi] = \Theta[\phi, \mathcal{L}_\xi\phi] - \xi\cdot\mathbf{L}, \quad (29)$$

and corresponding Noether charge 2-form

$$\mathbf{J} = d\mathbf{Q}. \quad (30)$$

It was shown in [15] that the Hamiltonian is a pure surface term for all diffeomorphism invariant theories and

$$\delta H[\xi] = \int_C(\delta\mathbf{J}[\xi] - d(\xi\cdot\Theta[\phi, \delta\phi])), \quad (31)$$

where C is a Cauchy surface.

The integrability condition requires that a 3-form $\mathbf{B}$ exists with the property

$$\delta\int_{\partial C}\xi\cdot\mathbf{B} = \int_{\partial C}\xi\cdot\Theta. \quad (32)$$

As explained elsewhere [5] one can, starting from the Hamiltonian $H[\xi]$ corresponding to some diffeomorphism ξ^a, write algebra of its surface terms $J[\xi]$

$$\{J[\xi_1], J[\xi_2]\}^* = J[\{\xi_1, \xi_2\}] + K[\xi_1, \xi_2], \quad (33)$$

with

$$J[\xi] = \int_{\partial C}\mathbf{Q}[\xi], \quad (34)$$

and the Dirac bracket

$$\{J[\xi_1], J[\xi_2]\}^* = \int_{\partial C}\{\xi_2\cdot\Theta[\phi, \mathcal{L}_{\xi_1}\phi] - \xi_1\cdot\Theta[\phi, \mathcal{L}_{\xi_2}\phi] - \xi_2\cdot(\xi_1\cdot\mathbf{L})\}. \quad (35)$$

From Eq. (33) one can determine central charge $K[\xi_1, \xi_2]$. Now the symplectic current is [15]

$$\Theta_{pef} = 2\epsilon_{apef}(E^{abcd}\nabla_d\delta g_{bc} - \nabla_d E^{abcd}\delta g_{bc}), \quad (36)$$

where

$$E^{abcd} = \frac{\partial L}{\partial R_{abcd}}. \quad (37)$$

This expression is valid for Lagrangians which do not contain derivatives of Riemann tensors.

In this paper we are interested primarily in the Einstein gravity case, but we shall include also the most general Lagrangian with quadratic terms in the Riemann tensor or

$$L = \frac{1}{16\pi} R + \alpha R^2 + \beta R_{\mu\nu} R^{\mu\nu} + \gamma R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}. \quad (38)$$

The integrals (34) and (35) are taken over 2-dimensional surface $\mathcal{H}$ which is the intersection of the Killing horizon $\chi^2 = 0$ with the Cauchy surface C. The volume element is

$$\epsilon_{abcd} = \hat{\epsilon}_{cd} \eta_{ab} + \cdots, \quad (39)$$

where only the first term contributes to the integral, and binormal η_{ab} is

$$\eta_{ab} = 2\chi_{[b} N_{c]} = \frac{2}{|\chi|\rho} \rho_{[a}\chi_{b]} + s_{[a}\chi_{b]}, \quad (40)$$

and $s^a = (0, D_1, 0, D_2)$ is tangent to $\mathcal{H}$. N^a is future directed null normal

$$N^a = k^a - \alpha\chi^a - s^a, \quad (41)$$

and

$$k^a \equiv -(\chi^a - \rho^a |\chi|/\rho)/\chi^2. \quad (42)$$

To find symplectic potential Θ and, in particular, to perform integration in (35) we need also the quantities $\nabla_d \delta g_{bc}$. We calculated this quantity including the $O(n^2)$ terms. Here we write the leading term

$$\nabla_d \delta g_{ab} = -2\chi_d \chi_a \chi_b \frac{\ddot{T}}{\chi^4} + 2\chi_d \chi_{(a}\rho_{b)} \left(\frac{\ddot{T}}{\kappa\chi^2\rho^2} + \frac{2\kappa\dot{T}}{\chi^4} \right). \quad (43)$$

In fact the expression (35) can be written more explicitly as

$$\{J[\xi_1], J[\xi_2]\}^* = -\int_{\partial C} \hat{\epsilon} \{2(X^{(12)}_{abcd} E^{abcd} - \tilde{X}^{(12)}_{abc} \nabla_d E^{abcd}) - \xi_2^a \xi_1^b \eta_{ab} L\}. \quad (44)$$

Here

$$X^{(12)}_{abcd} = \xi_1^p \eta_{ap} \nabla_d \delta_2 g_{bc} - (1 \leftrightarrow 2), \quad (45)$$

$$\tilde{X}^{(12)}_{abc} = \xi_1^p \eta_{ap} \delta_2 g_{bc} - (1 \leftrightarrow 2). \quad (46)$$

It is useful to note that tensors $X^{(12)}_{abcd}$ and $\tilde{X}^{(12)}_{abc}$ depend only on details of the black hole and its symmetry properties (diffeomorphism defining functions) but not on the form of the Lagrangian. We have evaluated the Taylor series near the horizon for $X^{(12)}_{abcd}$ and $\tilde{X}^{(12)}_{abc}$ and the Taylor series for interaction dependent tensors E^{abcd} and $\nabla_d E^{abcd}$. They allow us to establish the following properties on the horizon:

$$\lim_{n\to 0} (X^{(12)}_{abcd} E^{abcd}) = \lim_{n\to 0} \left(-\frac{1}{4} \eta_{ab} \eta_{cd} E^{abcd} \times \left[\left(\frac{1}{\kappa} T_1 \ddot{T}_2 - 2\kappa T_1 \dot{T}_2 \right) - (1 \leftrightarrow 2) \right] \right), \quad (47)$$

$$\lim_{n\to 0} (\tilde{X}^{(12)}_{abc} \nabla_d E^{abcd}) = 0. \quad (48)$$

Of course the last term in (44) vanishes on the horizon where we expect Lagrangian (38) to be regular (as a function of curvature invariants).

It is important to realize that contrary to previous procedures we now have explicitly under control the next to leading terms in the expansion parameter n (distance of horizon). We need also to calculate the Noether charge

$$Q_{ef} = -E^{abcd} \epsilon_{abef} \nabla_{[c} \xi_{d]}, \quad (49)$$

or

$$\mathbf{Q} = \hat{\epsilon} E^{abcd} Y_{abcd}, \quad (50)$$

where

$$Y_{abcd} = -\eta_{ab} \nabla_{[c} \xi_{d]}. \quad (51)$$

The tensor Y_{abcd} up to terms of order 2 can also be calculated but we omit the result here.

In that case from these definitions and (33) one obtains the expression for central charge

$$K = \int_{\mathcal{H}} \hat{\epsilon} E^{abcd} Z_{abcd}, \quad (52)$$

$$Z_{abcd} = 2X^{(21)}_{abcd} - Y_{abcd}. \quad (53)$$

Now tensors Z and E can be explicitly calculated due to previous expansions. The leading term is then the expression for central charge obtained in previous references

$$K[\xi_1, \xi_2] = -\frac{1}{2} \int_{\mathcal{H}} \hat{\epsilon} E^{abcd} \eta_{ab} \eta_{cd} \frac{1}{\kappa} (\dot{T}_1 \ddot{T}_2 - \ddot{T}_1 \dot{T}_2). \quad (54)$$

The explicit contributions in relations for X and Y are higher order and thus vanish at horizon $n = 0$. In the usual way one would then obtain the expression for entropy

$$S = -2\pi \int_{\mathcal{H}} \hat{\epsilon} E^{abcd} \eta_{ab} \eta_{cd}. \quad (55)$$

Here we want to add a remark. The original proposal of this approach assumed a set of boundary conditions. Despite the fact that we presented a straightforward calculation based on properties of the black hole it is of interest to check the above mentioned assumptions. In fact using expansion (26) one can check that the following conditions assumed in Refs. [5,11,12] are indeed valid

$$\chi^a t^a \delta g_{ab} \to 0, \qquad \rho^a \nabla_a (g_{bc} \delta g^{bc}) = 0, \tag{56}$$

$$\rho^a \nabla_a \left(\frac{\rho^b \delta \chi_b}{\chi^2} \right) = 0, \qquad \rho^a \nabla_a \left(\frac{\delta \rho^2}{\chi^2} \right) = 0.$$

It is important to note that relations (22), (23), and (25) are defining the diffeomorphisms and are thus also satisfied. The explicit form of diffeomorphisms is given in previous references.

The analysis of this paper has been done for $D = 4$ but there does not seem that there could be obstructions for higher dimensions. There is in fact one partial result in the case of spherical and static metric in D dimensions:

$$ds^2 = -f(x)dt^2 + \frac{dx^2}{f(x)} + r^2(x) d\Omega_{D-1}. \tag{57}$$

With explicit calculations we have checked that boundary conditions (56) and properties (47) and (48) are valid.

In this paper we have treated the axially symmetric black holes. As is well known axial symmetry follows from stationarity as shown by the uniqueness theorems [18] under some standard conditions. However one may be interested in situations where these conditions are not fulfilled and thus investigate horizons which are not axially symmetric. This question would require a separate analysis and would presumably be technically more complicated.

III. CONCLUSION

The well-known calculation of entropy via the Cardy formula is based on the calculation of central charge of a subalgebra of diffeomorphism algebra on the black hole horizon. The calculations have been based on additional plausible assumptions which then led to leading terms which gave contributions on the horizon and without evaluation of next to leading terms. The approach used here starts from usual properties of horizons of stationary black holes together with regularity of curvature invariants on them which then imply restrictive power series expansion for metric fluctuations near the horizon [14]. We are then able to obtain without previously mentioned assumptions the expansions for fluctuations of the metric and its covariant derivative and consequently for the tensor Z needed in the integrand of the central charge formula. The horizon limit was then possible to perform explicitly. In addition next to leading and next to next to leading terms are explicitly exhibited.

More important in such a way we have shown that near horizon geometry implies, as suggested by [14], near horizon conformal symmetry formulated by Carlip with its consequences for the black hole entropy.

APPENDIX

As mentioned in the text, the important ingredients in calculations are the Taylor series near the horizon for various quantities like δg_{ab}, $\nabla_d \delta g_{ab}$, $X^{(12)}_{abcd}$, $\tilde{X}^{(12)}_{abc}$, E^{abcd}, and $\nabla_d E^{abcd}$. In the text we have exhibited expansion for δg_{ab} (26). Here we present as another example the expansion of tensor $X^{(12)}_{abcd}$ including terms n^0, n^1, and n^2 (we also symmetrize $X^{(12)}_{abcd}$ such that $X^{(12)}_{abcd} = X^{(12)}_{cdab}$ and $X^{(12)}_{abcd} = X^{(12)}_{[ab][cd]}$, which does not change product $X^{(12)}_{abcd} E^{abcd}$):

$$\begin{aligned}
X^{(12)}_{abcd} = {} & \chi_a \rho_b \chi_c \rho_d \left[\frac{(2\kappa^2 \dot{T}_2 - \ddot{T}_2) T_1}{4\kappa^5 n^4} + C^{-2}_{0101} \frac{1}{n^2} + O\left(\frac{1}{n}\right) \right] + \chi_a \rho_b \chi_c \phi_d \left[C^{-2}_{0102} \frac{1}{n^2} + C^{-1}_{0102} \frac{1}{n} + O(n^0) \right] \\
& + \chi_a \rho_b \chi_c z_d \left[C^{-2}_{0103} \frac{1}{n^2} + O(n^0) \right] + \chi_a \rho_b \rho_c \phi_d \left[C^{-2}_{0112} \frac{1}{n^2} + C^{-1}_{0112} \frac{1}{n} + O(n^0) \right] + \chi_a \rho_b \phi_c z_d [C^0_{0123} + O(n)] \\
& + \chi_a \phi_b \chi_c \phi_d [C^0_{0202} + O(n)] + \chi_a \phi_b \chi_c z_d [C^0_{0203} + O(n)] + \chi_a \phi_b \rho_c \phi_d [C^0_{0212} + O(n)] \\
& + \chi_a \phi_b \rho_c z_d [C^0_{0213} + O(n)] + \chi_a z_b \chi_c z_d [C^0_{0303} + O(n)] + \chi_a z_b \rho_c \phi_d [C^0_{0312} + O(n)] + \chi_a z_b \rho_c z_d [C^0_{0313} + O(n)] \\
& + \rho_a \phi_b \rho_c \phi_d [C^0_{1212} + O(n)] + \rho_a \phi_b \rho_c z_d [C^0_{1213} + O(n)] \\
& + \text{terms related by permutation of indices according to symmetries of} X^{(12)}_{abcd} \\
& + \text{terms of order} \geqq 3, \\
& - (1 \leftrightarrow 2).
\end{aligned} \tag{A1}$$

Here the coefficients of nonleading terms are lengthy algebraic expressions given in terms of diffeomorphism defining functions T_1, T_2, and Taylor coefficients of metric functions and are not very informative. For these reasons we do not exhibit them here.

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