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Deeply virtual Compton scattering beyond next-to-leading order: The flavor singlet case

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Abstract

We study radiative corrections to deeply virtual Compton scattering in the kinematics of HERA collider experiments to next-to-leading and next-to-next-to-leading order. In the latter case the radiative corrections are evaluated in a special scheme that allows us to employ the predictive power of conformal symmetry. As observed before, the size of next-to-leading order corrections strongly depends on the gluonic input, as gluons start to contribute at this order. Beyond next-to-leading order we find, in contrast, that the corrections for an input scale of few GeV^2 are small enough to justify the uses of perturbation theory. For $\xi \gtrsim 5 \times 10^{-3}$ the modification of the scale dependence is also small. However, with decreasing ξ it becomes moderate or even large.

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1. Deeply virtual Compton scattering (DVCS) is considered as the theoretically cleanest process to investigate generalized parton distributions (GPDs) [1,2]. These distributions are a hybrid of parton densities, form-factors, and distribution amplitudes and might be represented in terms of light-cone wave functions [3]. They are rather intricate functions depending on the longitudinal momentum fractions in the s- and t-channels, the momentum transfer squared, and the resolution scale. On the other hand they are phenomenologically very attractive, since they encode non-perturbative information that cannot be extracted from either inclusive or elastic measurements alone. Their second moments provide, e.g., the total angular momentum of partons in the nucleon [2] and the gravitational form factor of the nucleon. Moreover, in the impact parameter space they can be viewed as parton densities in dependence of the longitudinal momentum fraction and the transverse distance from the proton center [4]. The knowledge of transverse parton distribution does not only add substantially to our understanding of hadron structure but is also relevant for the prediction of cross sections in dependence of the impact parameter [5].

On one hand GPDs are thus a new window to study non-perturbative QCD and it has been already impressively demonstrated that they are experimentally accessible via the DVCS process [6,7]. On the other hand for several theoretical and experimental reasons the extraction of GPDs from measurements remains quite challenging because typically one is sensitive only to convolutions containing GPDs and one has to disentangle different contributions. The analysis simplifies substantially at large energies, where both photon [7] and vector–meson leptoproduction, e.g., Ref. [8], have been measured by H1 and ZEUS. In HERA kinematics the

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photon–proton interaction starts to be flavor blind and so one mainly accesses flavor singlet GPDs. Moreover, spin flip effects are suppressed, too, and only one set of GPDs is relevant, namely, the proton helicity conserved and parton helicity averaged GPDs $H(x,\xi,t,\mathcal{Q}^2)$.

In this Letter we study radiative corrections to DVCS at and beyond next-to-leading (NLO) order [15,16], as the non-singlet case has already been studied in [9]. First we present, after a short introduction to the conformal approach, the analytic result to NNLO accuracy. As usual, an analysis of the radiative corrections is only possible after adopting some parameterization for the dependence of GPDs on the s- and t-channel momentum fraction.[1] Relying on the pomeron pole as the dominant contribution at small momentum fraction, we numerically evaluate NNLO radiative corrections for the kinematics of HERA collider experiments. This analysis includes a comparison of the standard predictions in NLO with those of the conformal approach.

2. The DVCS amplitude is defined in terms of the hadronic tensor

$$T_{\mu\nu}(q,P,\Delta) = -\left(\tilde{g}_{\mu\nu} - \frac{\widetilde{P_\mu q_\nu}}{P\cdot q} - \frac{\widetilde{P_\nu q_\mu}}{P\cdot q}\right)\frac{q_\sigma V^\sigma}{P\cdot q} - i\tilde{\epsilon}_{\mu\nu q P}\frac{q_\sigma A^\sigma}{(P\cdot q)^2} + \cdots, \tag{1}$$

where $q=(q_1+q_2)/2$ (μ and q_2 refers to the outgoing real photon), $P=P_1+P_2$ and $\Delta=P_2-P_1$. The tilde-symbol denotes contraction $\tilde{X}_{\mu\nu}\equiv \mathcal{P}_{\mu\rho}X^{\rho\sigma}\mathcal{P}_{\sigma\nu}$ with appropriate projectors to ensure current conservation [17]. The ellipsis indicate terms that are finally power suppressed in the DVCS amplitude or are determined by the gluon transversity GPD, which is not considered here. In the parity even (odd) sector the (axial-)vector V^σ (A^σ) is decomposed into the target helicity conserving Compton form factor (CFF) $\mathcal{H}$ ($\tilde{\mathcal{H}}$) and the helicity flip one $\mathcal{E}$ ($\tilde{\mathcal{E}}$), see, e.g., Ref. [17]. The CFFs, denoted by the set $\mathcal{F}=\{\mathcal{H},\mathcal{E},\tilde{\mathcal{H}},\tilde{\mathcal{E}}\}$, depend on the scaling variable $\xi=\mathcal{Q}^2/2P\cdot q$, momentum transfer square Δ^2, and photon virtuality $\mathcal{Q}^2=-q_1^2$.

Before we proceed, let us decompose the CFFs in flavor non-singlet (NS) and singlet (S) ones:

$$\mathcal{F} = Q^2_{\mathrm{NS}}\,{}^{\mathrm{NS}}\mathcal{F} + Q^2_{\mathrm{S}}\,{}^{\mathrm{S}}\mathcal{F},\qquad {}^{\mathrm{S}}\mathcal{F} = {}^{\Sigma}\mathcal{F} + {}^{\mathrm{G}}\mathcal{F}, \tag{2}$$

where the singlet piece contains the quark flavor singlet ${}^{\Sigma}\mathcal{F}$ and gluon ${}^{\mathrm{G}}\mathcal{F}$ CFFs. The charge factors Q_i^2 with $i=\{\mathrm{NS},\mathrm{S}\}$ are given as linear combination of squared quark charges, e.g., the singlet one is $Q^2_{\mathrm{S}}=\sum_{i=u,d,\dots}Q_i^2/n_f$ for n_f active quarks. In the momentum fraction representation the CFFs are represented as convolution of the coefficient function with the corresponding GPD. In the singlet sector we might introduce the vector notation:

$${}^{\mathrm{S}}\mathcal{F}\big(\xi,\Delta^2,\mathcal{Q}^2\big) = \int_{-1}^{1}\frac{dx}{\xi}\boldsymbol{C}\big(x/\xi,\mathcal{Q}^2/\mu^2,\alpha_s(\mu)|\xi\big)\boldsymbol{F}\big(x,\eta=\xi,\Delta^2,\mu^2\big). \tag{3}$$

Here the column vector contains the GPDs,

$$\boldsymbol{F} = \begin{pmatrix} {}^{\Sigma}F \\ {}^{\mathrm{G}}F \end{pmatrix},\qquad F=\{H,E,\tilde{H},\tilde{E}\}, \tag{4}$$

and the row one $\boldsymbol{C}=({}^{\Sigma}C,(1/\xi)\,{}^{\mathrm{G}}C)$ consists of the hard scattering part that to LO reads

$$\frac{1}{\xi}\boldsymbol{C}\big(x/\xi,\mathcal{Q}^2/\mu^2,\alpha_s(\mu)|\xi\big) = \left(\frac{1}{\xi-x-i\epsilon},0\right) + \mathcal{O}(\alpha_s). \tag{5}$$

We remark that the ξ dependence in ${}^{\Sigma}C$ and ${}^{G}C$ enters only via the ratio x/ξ and that the u-channel contribution in the quark entry (5) is reabsorbed into the symmetrized quark distribution

$${}^{\Sigma}F\big(x,\eta,\Delta^2,\mu^2\big) = \sum_{q=u,d,\dots}\big[{}^{q}F\big(x,\eta,\Delta^2,\mu^2\big) \mp {}^{q}F\big(-x,\eta,\Delta^2,\mu^2\big)\big]. \tag{6}$$

Here the second term in the square brackets with $-(+)$-sign for H,E ($\tilde{H},\tilde{E}$)-type GPDs is for $x>\eta$ related to the s-channel exchange of an anti-quark. The gluon GPDs have definite symmetry property under the exchange of $x\to -x$: ${}^{\mathrm{G}}H$ (${}^{\mathrm{G}}\tilde{H}$) and ${}^{\mathrm{G}}E$ (${}^{\mathrm{G}}\tilde{E}$) are even (odd).

The convolution formula (3) has already at LO the disadvantage that it contains a singularity at the cross over point between the central region ($-\eta\leqslant x\leqslant\eta$) and the outer region ($\eta\leqslant x\leqslant 1$), i.e., for $x=\xi=\eta$. Its treatment is defined by the $i\epsilon$ prescription, coming from the Feynman propagator. The GPD is considered smooth at this point, but will generally not be holomorphic [18]. The fact that both regions are dual to each other, up to a so-called D-term contribution [19], makes the numerical treatment even more complicated. This motivated our development of a more suitable formalism [10]. The factorization scale μ in the GPDs is

[1] We believe that realistic models can be most easily constructed by means of the partial wave decomposition of GPDs and amplitudes [10,11], where the dominant contributions arise from the leading Regge trajectories [10,12,13]. Support for this conjecture arises also from lattice calculations [14].

ambiguous and at LO order this induces the main uncertainty. Beyond LO this factorization scale dependence will be cancelled in the considered order of perturbation theory. The NLO corrections to the coefficient functions [20] and evolution kernels [21] were predicted from conformal constraints. Note that the conformal symmetry in the standard $\overline{\mathrm{MS}}$ scheme is broken and that the predicted results, rotated to this scheme, coincide with the diagrammatic evaluation [22]. To this order and in this scheme a numerical code has been made accessible that includes evolution, see Ref. [16].

At present it seems hardly possible to study perturbative corrections beyond NLO accuracy in the $\overline{\mathrm{MS}}$ scheme, since the diagrammatical evaluation would require enormous effort. Fortunately, we can employ conformal symmetry to relate the perturbative corrections at NNLO to those for DIS [23,24], where the NNLO corrections in the vector case has been completed by the substantial effort of Vogt, Moch and Vermaseren [25]. From these calculations we get the normalization of the Wilson coefficients and anomalous dimensions. The conformal predictions arise from the application of the conformal operator product expansion (OPE) and are valid as long as the twist-two operators behave covariantly under conformal transformation [26,27]. This can be ensured for vanishing β-function in any order of perturbation theory within a special renormalization scheme [27]. To make contact with the conformal OPE, we expand the hard-scattering amplitude in terms of Gegenbauer polynomials with indices $3/2$ and $5/2$ for quarks and gluons, respectively, and introduce the conformal GPD moments, which formally yields

$$ {}^{\mathrm{S}}\mathcal{F}\big(\xi, \Delta^2, \mathcal{Q}^2\big) = 2\sum_{j=0}^{\infty} \xi^{-j-1} \boldsymbol{C}_j\big(\mathcal{Q}^2/\mu^2, \alpha_s(\mu)\big) \boldsymbol{F}_j\big(\xi, \Delta^2, \mu^2\big). \tag{7} $$

The expansion coefficients $\boldsymbol{C}_j$ can be calculated by the projection:

$$ \boldsymbol{C}_j = \frac{2^{j+1}\Gamma(j+5/2)}{\Gamma(3/2)\Gamma(j+3)} \frac{1}{2}\int_{-1}^{1} dx\, \boldsymbol{C}\big(x, \mathcal{Q}^2/\mu^2, \alpha_s(\mu)|1\big) \begin{pmatrix} [1-x^2]C_j^{3/2} & 0 \\ 0 & \frac{3[1-x^2]^2}{(j+3)} C_{j-1}^{5/2} \end{pmatrix} (x). \tag{8} $$

Note that we have here rescaled the integration variable with respect to ξ and that the integral runs only over the rescaled central region. The conformal GPD moments are defined as

$$ \boldsymbol{F}_j\big(\eta, \Delta^2, \mu^2\big) = \frac{\Gamma(3/2)\Gamma(j+1)}{2^j\Gamma(j+3/2)} \frac{1}{2}\int_{-1}^{1} dx\, \eta^{j-1} \begin{pmatrix} \eta C_j^{3/2} & 0 \\ 0 & (3/j)C_{j-1}^{5/2} \end{pmatrix} \frac{x}{\eta} \boldsymbol{F}\big(x, \eta, \Delta^2, \mu^2\big), \tag{9} $$

where j is an odd (even) non-negative integer for the (axial-)vector case.

In the forward kinematics ($\Delta \to 0$), our conventions are such that the helicity conserved GPDs coincide with the flavor singlet quark distribution and with x times the gluon distribution. Hence, for the moments we have agreement with the common Mellin moments of parton densities, e.g.,

$$ \begin{pmatrix} \Sigma \\ xG \end{pmatrix}(x) = \lim_{\Delta\to 0} \boldsymbol{H}\big(x, \eta, \Delta^2\big), \qquad \boldsymbol{q}_j \equiv \lim_{\Delta\to 0} \boldsymbol{H}_j\big(\eta, \Delta^2\big) = \int_0^1 dx\, x^j \begin{pmatrix} \Sigma \\ G \end{pmatrix}(x). \tag{10} $$

Unfortunately, the series (7) does not converge for DVCS kinematics, in particular not in the outer region, and one has to resum the OPE [9,10] or, equivalently, one can use a dispersion relation [28]. The result for ${}^{\mathrm{S}}\mathcal{H}$ in terms of a Mellin–Barnes integral reads

$$ {}^{\mathrm{S}}\mathcal{H}\big(\xi, \Delta^2, \mathcal{Q}^2\big) = \frac{1}{2i}\int_{c-i\infty}^{c+i\infty} dj\, \xi^{-j-1}\left[i + \tan\left(\frac{\pi j}{2}\right)\right] \boldsymbol{C}_j\big(\mathcal{Q}^2/\mu^2, \alpha_s(\mu)\big) \boldsymbol{H}_j\big(\xi, \Delta^2, \mu^2\big). \tag{11} $$

In the following we write the perturbative expansion as

$$ \boldsymbol{C}_j = \frac{2^{j+1}\Gamma(j+5/2)}{\Gamma(3/2)\Gamma(j+3)}\left[\boldsymbol{C}_j^{(0)} + \frac{\alpha_s(\mu_r)}{2\pi}\boldsymbol{C}_j^{(1)}\big(\mathcal{Q}^2/\mu^2\big) + \frac{\alpha_s^2(\mu_r)}{(2\pi)^2}\boldsymbol{C}_j^{(2)}\big(\mathcal{Q}^2/\mu^2, \mathcal{Q}^2/\mu_r^2\big) + \mathcal{O}\big(\alpha_s^3\big)\right], \tag{12} $$

where we choose to distinguish the renormalization (μ_r) and factorization (μ) scales. Corresponding to our conventions, the LO Wilson coefficients are normalized as $\boldsymbol{C}_j^{(0)} = (1, 0)$.

Let us first give here the DVCS NLO corrections in the $\overline{\mathrm{MS}}$ scheme. We restrict ourselves to the analysis of the kinematically dominant contribution, i.e., ${}^{\mathrm{S}}\mathcal{H}$, and so we provide here only the results for the vector case. The conformal moments (8) can be obtained from Refs. [20,22]. Using the representation of Ref. [15], the evaluation is straightforward for integer conformal spin, see Appendix C of Ref. [29] for the quark entries. The analytic continuation to complex j yields

$$ {}^{\Sigma}C_j^{(1)}\big(\mathcal{Q}/\mu^2\big) = C_F\left[2S_1^2(1+j) - \frac{9}{2} + \frac{5-4S_1(j+1)}{2(j+1)(j+2)} + \frac{1}{(j+1)^2(j+2)^2}\right] + \frac{{}^{\Sigma\Sigma}\gamma_j^{(0)}}{2}\ln\frac{\mu^2}{\mathcal{Q}^2}, \tag{13} $$

$$ {}^{\mathrm{G}}C_j^{(1)}\big(\mathcal{Q}/\mu^2\big) = -2n_f T_F \frac{(4+3j+j^2)[S_1(j)+S_1(j+2)]+2+3j+j^2}{(1+j)(2+j)(3+j)} + \frac{{}^{\Sigma G}\gamma_j^{(0)}}{2}\ln\frac{\mu^2}{\mathcal{Q}^2}, \tag{14} $$

where $C_F = 4/3$ and $T_F = 1/2$. The entries of the anomalous dimension matrix read at LO:

$$ {}^{\Sigma\Sigma}\gamma_j^{(0)} = -C_F\left(3+\frac{2}{(j+1)(j+2)} - 4S_1(j+1)\right), \qquad {}^{\Sigma G}\gamma_j^{(0)} = -\frac{4n_f T_F(4+3j+j^2)}{(j+1)(j+2)(j+3)}, $$
$$ {}^{G\Sigma}\gamma_j^{(0)} = -\frac{2C_F(4+3j+j^2)}{j(j+1)(j+2)}, \qquad {}^{GG}\gamma_j^{(0)} = -C_A\left(-\frac{4}{(j+1)(j+2)} + \frac{12}{j(j+3)} - 4S_1(j+1)\right) + \beta_0, \tag{15} $$

where $\beta_0 = 2n_f/3 - 11$, $C_A = 3$. In the $\overline{\mathrm{MS}}$ scheme also the complete anomalous dimension matrix is known to two-loop accuracy [30,31]. However, the conformal moments will mix with each other and the evolution equation has so far not been solved in terms of a Mellin–Barnes integral.

In a special conformal scheme ($\overline{\mathrm{CS}}$[2]) the structure of the Wilson coefficients up to NNLO is

$$ \boldsymbol{C}_j^{(1)}\big(\mathcal{Q}^2/\mu^2\big) = \boldsymbol{c}_j^{(1)} + \frac{s_j^{(1)}(\mathcal{Q}^2/\mu^2)}{2}\boldsymbol{c}_j^{(0)}\boldsymbol{\gamma}_j^{(0)}, \tag{16} $$

$$ \begin{aligned} \boldsymbol{C}_j^{(2)}\big(\mathcal{Q}^2/\mu^2, \mathcal{Q}^2/\mu_r^2\big) &= \boldsymbol{c}_j^{(2)} + \frac{s_j^{(1)}(\mathcal{Q}^2/\mu^2)}{2}\big[\boldsymbol{c}_j^{(0)}\boldsymbol{\gamma}_j^{(1)} + \boldsymbol{c}_j^{(1)}\boldsymbol{\gamma}_j^{(0)}\big] + \frac{s_j^{(2)}(\mathcal{Q}^2/\mu^2)}{8}\boldsymbol{c}_j^{(0)}\big(\boldsymbol{\gamma}_j^{(0)}\big)^2 \\ &\quad + \frac{\beta_0}{2}\left[\boldsymbol{C}_j^{(1)}\big(\mathcal{Q}^2/\mu^2\big)\ln\frac{\mathcal{Q}^2}{\mu_r^2} + \frac{1}{4}\boldsymbol{c}_j^{(0)}\boldsymbol{\gamma}_j^{(0)}\ln^2\frac{\mathcal{Q}^2}{\mu^2}\right], \end{aligned} \tag{17} $$

where $s_j^{(i)}(\mathcal{Q}^2/\mu^2)$ can be expressed in terms of harmonic sums $S_p(n) = \sum_{k=1}^{n} 1/k^p$ as

$$ s_j^{(1)}\big(\mathcal{Q}^2/\mu^2\big) = S_1(j+3/2) - S_1(j+2) + 2\ln(2) - \ln\frac{\mathcal{Q}^2}{\mu^2}, \tag{18} $$

$$ s_j^{(2)}\big(\mathcal{Q}^2/\mu^2\big) = \big(s_j^{(1)}\big(\mathcal{Q}^2/\mu^2\big)\big)^2 - S_2(j+3/2) + S_2(j+2), \tag{19} $$

and $\boldsymbol{c}_j^{(i)} = ({}^{\Sigma}c_j^{(i)}, {}^{G}c_j^{(i)})$ are the DIS Wilson coefficients. We have at LO $\boldsymbol{c}_j^{(0)} = (1, 0)$, at NLO

$$ {}^{\Sigma}c_j^{(1)} = C_F\left[S_1^2(1+j) + \frac{3}{2}S_1(j+2) - \frac{9}{2} + \frac{5-2S_1(j)}{2(j+1)(j+2)} - S_2(j+1)\right], \tag{20} $$

$$ {}^{G}c_j^{(1)} = -2n_f T_F \frac{(4+3j+j^2)S_1(j)+2+3j+j^2}{(1+j)(2+j)(3+j)}, \tag{21} $$

and at NNLO they are given by the Mellin moments of the DIS partonic structure functions [23]. To simplify their evaluation, we take for $\boldsymbol{c}_j^{(2)}$ a fit [24], rather than the exact expression.

The evolution of the singlet (integer) conformal moments in this $\overline{\mathrm{CS}}$ scheme is governed by

$$ \begin{aligned} \mu\frac{d}{d\mu}\boldsymbol{F}_j(\xi, \Delta^2, \mu^2) &= -\left[\frac{\alpha_s(\mu)}{2\pi}\boldsymbol{\gamma}_j^{(0)} + \frac{\alpha_s^2(\mu)}{(2\pi)^2}\boldsymbol{\gamma}_j^{(1)} + \frac{\alpha_s^3(\mu)}{(2\pi)^3}\boldsymbol{\gamma}_j^{(2)} + \mathcal{O}\big(\alpha_s^4\big)\right]\boldsymbol{F}_j\big(\xi, \Delta^2, \mu^2\big) \\ &\quad - \frac{\beta_0}{2}\frac{\alpha_s^3(\mu)}{(2\pi)^3}\sum_{k=0}^{j-2}\big[\Delta_{jk}^{\overline{\mathrm{CS}}} + \mathcal{O}(\alpha_s)\big]\boldsymbol{F}_k\big(\xi, \Delta^2, \mu^2\big), \end{aligned} \tag{22} $$

where the mixing matrix $\boldsymbol{\Delta}_{jk}^{\overline{\mathrm{CS}}}$ is not completely known. In the vector case the anomalous dimensions are known to NNLO [25]. In absence of the mixing term, the solution of the renormalization group equation $\boldsymbol{F}_j(\xi, \Delta^2, \mu^2) = \boldsymbol{\mathcal{E}}_j(\mu, \mu_0)\boldsymbol{F}_j(\xi, \Delta^2, \mu_0^2)$ is given by a path-ordered exponential. In the numerical analysis we will resum the leading logarithms and expand the non-leading ones

$$ \boldsymbol{\mathcal{E}}_j(\mu, \mu_0) = \sum_{a,b=\pm}\left[\delta_{ab}{}^{a}\boldsymbol{P}_j + \frac{\alpha_s(\mu)}{2\pi}{}^{ab}\boldsymbol{\mathcal{A}}_j^{(1)}(\mu, \mu_0) + \frac{\alpha_s^2(\mu)}{(2\pi)^2}{}^{ab}\boldsymbol{\mathcal{A}}_j^{(2)}(\mu, \mu_0) + \mathrm{O}\big(\alpha_s^3\big)\right]\left[\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right]^{-\frac{b_{\lambda_j}}{\beta_0}}. \tag{23} $$

[2] The treatment of β proportional terms, breaking conformal symmetry, is ambiguous. We employ the so-called $\overline{\mathrm{CS}}$ scheme in which the running of the coupling is implemented in the form of the conformal OPE that is valid for a hypothetical fixed point. In particular, conformal moments are multiplicatively renormalizable to NLO [9,29].

Here the projectors on the $\{+,-\}$ modes are

$$^{\pm}\boldsymbol{P}_j = \frac{\pm 1}{^{+}\lambda_j - {}^{-}\lambda_j}\big(\boldsymbol{\gamma}_j^{(0)} - {}^{\mp}\lambda_j \mathbf{1}\big), \tag{24}$$

where the eigenvalues of the LO anomalous dimension matrix are

$$^{\pm}\lambda_j = \frac{1}{2}\big({}^{\Sigma\Sigma}\gamma_j^{(0)} + {}^{GG}\gamma_j^{(0)}\big) \mp \frac{1}{2}\big({}^{\Sigma\Sigma}\gamma_j^{(0)} - {}^{GG}\gamma_j^{(0)}\big)\sqrt{1 + \frac{4\,{}^{\Sigma G}\gamma_j^{(0)}\,{}^{G\Sigma}\gamma_j^{(0)}}{\big({}^{\Sigma\Sigma}\gamma_j^{(0)} - {}^{GG}\gamma_j^{(0)}\big)^2}}. \tag{25}$$

A straightforward calculation leads to the matrix valued coefficients

$$^{ab}\boldsymbol{\mathcal{A}}_j^{(1)} = {}^{ab}R_j(\mu,\mu_0|1)\,{}^{a}\boldsymbol{P}_j\left[\frac{\beta_1}{2\beta_0}\boldsymbol{\gamma}_j^{(0)} - \boldsymbol{\gamma}_j^{(1)}\right]{}^{b}\boldsymbol{P}_j, \tag{26}$$

$$\begin{aligned}
^{ab}\boldsymbol{\mathcal{A}}_j^{(2)} = {} & \sum_{c=\pm}\frac{1}{\beta_0 + {}^{c}\lambda_j - {}^{b}\lambda_j}\left[{}^{ab}R_j(\mu,\mu_0|2) - {}^{ac}R_j(\mu,\mu_0|1)\left(\frac{\alpha_s(\mu_0)}{\alpha_s(\mu)}\right)^{\frac{\beta_0 + {}^{c}\lambda_j - {}^{b}\lambda_j}{\beta_0}}\right]{}^{a}\boldsymbol{P}_j\left[\frac{\beta_1}{2\beta_0}\boldsymbol{\gamma}_j^{(0)} - \boldsymbol{\gamma}_j^{(1)}\right] \\
& \times {}^{c}\boldsymbol{P}_j\left[\frac{\beta_1}{2\beta_0}\boldsymbol{\gamma}_j^{(0)} - \boldsymbol{\gamma}_j^{(1)}\right]{}^{b}\boldsymbol{P}_j - {}^{ab}R_j(\mu,\mu_0|2)\,{}^{a}\boldsymbol{P}_j\left[\frac{\beta_1^2 - \beta_2\beta_0}{4\beta_0^2}\boldsymbol{\gamma}_j^{(0)} - \frac{\beta_1}{2\beta_0}\boldsymbol{\gamma}_j^{(1)} + \boldsymbol{\gamma}_j^{(2)}\right]{}^{b}\boldsymbol{P}_j,
\end{aligned} \tag{27}$$

where the μ dependence is accumulated in the following functions:

$$^{ab}R_j(\mu,\mu_0|n) = \frac{1}{n\beta_0 + {}^{a}\lambda_j - {}^{b}\lambda_j}\left[1 - \left(\frac{\alpha_s(\mu_0)}{\alpha_s(\mu)}\right)^{\frac{n\beta_0 + {}^{a}\lambda_j - {}^{b}\lambda_j}{\beta_0}}\right]. \tag{28}$$

The expansion coefficients of the β function are defined as

$$\beta_0 = \frac{2}{3}n_f - 11, \qquad \beta_1 = \frac{38}{3}n_f - 102, \qquad \beta_2 = -\frac{325}{54}n_f^2 + \frac{5033}{18}n_f - \frac{2857}{2}. \tag{29}$$

3. To study the perturbative corrections in the small ξ-region, we adopt a simple ansatz for the conformal moments that is inspired by the dominance of the pomeron and by the assumption that $\mathcal{O}(\xi^2)$ terms in the conformal polynomials are insignificant:

$$\boldsymbol{H}_j(\xi,\Delta^2,\mathcal{Q}^2) = \begin{pmatrix} N_{\mathrm{sea}}\,{}^{\mathrm{sea}}F(\Delta^2)\mathrm{B}(1+j-\alpha_{\mathrm{sea}}(\Delta^2),8)/\mathrm{B}(2-\alpha_{\mathrm{sea}}(0),8) \\ N_G\,{}^{G}F(\Delta^2)\mathrm{B}(1+j-\alpha_G(\Delta^2),6)/\mathrm{B}(2-\alpha_G(0),6 \end{pmatrix} + \cdots. \tag{30}$$

Here $\mathrm{B}(x,n) = \Gamma(x)\Gamma(n)/\Gamma(x+n)$ and the ellipsis denotes the neglected $\mathcal{O}(\xi^2)$ terms, as well as valence components whose contributions are also small for small ξ. Here the normalization is $^{\mathrm{sea}}F(\Delta^2=0) ={}^{G} F(\Delta^2=0) = 1$ and for $\alpha_i(\Delta^2)$ we use the "effective" pomeron trajectory $\alpha(\Delta^2) = \alpha(0) + \alpha'\Delta^2$. We remind that in DIS the structure function $F_2 \sim (1/x_{\mathrm{Bj}})^{\lambda(\mathcal{Q}^2)}$ grows with increasing Q^2. Here the exponent is related to the intercept of the Regge trajectory $\lambda = \alpha(0) - 1$. The values for $\alpha(0)$ will be specified below. Although also the slope α' is scale dependent [12], we choose here the standard value of the soft pomeron $\alpha' = 0.25$. In the forward case the moments (30) arise from the parton densities for which we adopt a generic realistic parameterization:

$$\Sigma = \frac{N_{\mathrm{sea}}x^{-\alpha_\Sigma(0)}(1-x)^7}{\mathrm{B}(2-\alpha_{\mathrm{sea}}(0),8)} + u_{\mathrm{v}}(x) + d_{\mathrm{v}}(x), \qquad G = \frac{N_G x^{-\alpha_G(0)}(1-x)^5}{\mathrm{B}(2-\alpha_G(0),6)}. \tag{31}$$

Note that the valence component $u_{\mathrm{v}} + d_{\mathrm{v}}$ is not taken into account in (30), since it is a non-leading contribution for small x. The normalization factors are related by the momentum sum rule

$$\int_0^1 dx\, x\big[\Sigma(x) + G(x)\big] = 1 \quad \Rightarrow \quad N_{\mathrm{G}} + N_{\mathrm{sea}} + \int_0^1 dx\, x[u_{\mathrm{v}} + d_{\mathrm{v}}](x) = 1. \tag{32}$$

In the asymptotic limit $\mathcal{Q} \to \infty$, the evolution equation tells us that $N_G = 4C_F/(4C_F + n_f)$, i.e., that more than 50% of the longitudinal proton momentum is carried by gluons. At experimentally accessible large scales the gluons already carry about 40% of the momentum. For the momentum of the valence quarks we choose the generic value $1/3$ and so $N_{\mathrm{sea}} = 2/3 - N_G$.

Let us first study the changes of the CFF (11) in a given scheme and input scale that appear when one includes the next order. The changes to the modulus and phase are measured by

$$K_\lambda^P = \ln\big|{}^{\mathrm{S}}\mathcal{H}^{\mathrm{N}^P\mathrm{LO}}\big|/\ln\big|{}^{\mathrm{S}}\mathcal{H}^{\mathrm{N}^{P-1}\mathrm{LO}}\big|, \qquad K_{\arg}^P = \arg\big({}^{\mathrm{S}}\mathcal{H}^{\mathrm{N}^P\mathrm{LO}}\big)/\arg\big({}^{\mathrm{S}}\mathcal{H}^{\mathrm{N}^{P-1}\mathrm{LO}}\big). \tag{33}$$

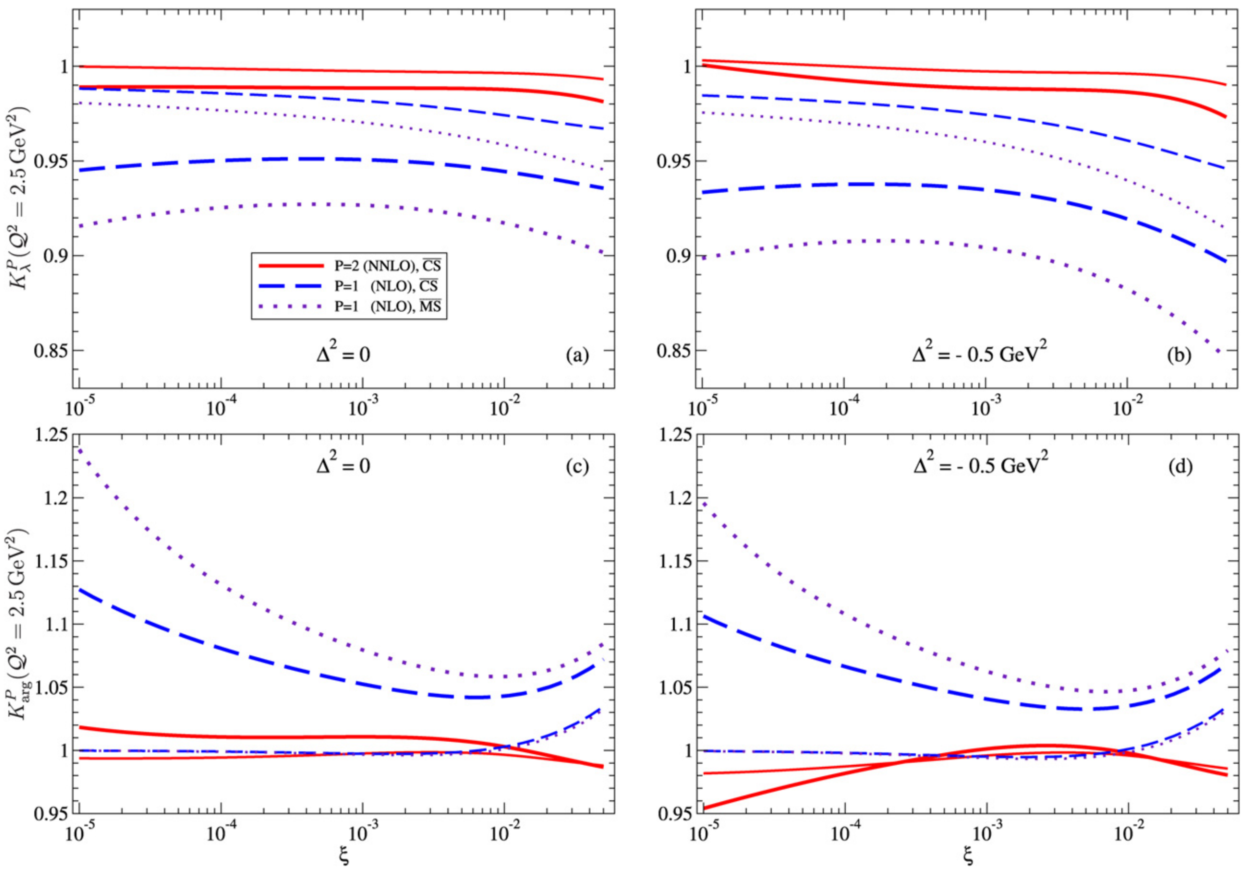

Fig. 1. The relative radiative corrections (33) are plotted versus ξ for the logarithm of the modulus [(a) and (b)] and phase [(c) and (d)] of ${}^{\mathrm{S}}\mathcal{H}$, see Eqs. (11) and (30), for $\Delta^2 = 0$ [(a) and (c)] and $\Delta^2 = -0.5\ \mathrm{GeV}^2$ [(b) and (d)]: NNLO (solid) as well as in NLO for the $\overline{\mathrm{CS}}$ (dashed) and $\overline{\mathrm{MS}}$ (dotted) scheme. Thick (thin) lines refer to the "hard" ("soft") gluon parameterization.

Here ${}^{\mathrm{S}}\mathcal{H}^{\mathrm{N}^P\mathrm{LO}}$ denotes the N^{P}LO approximation, e.g., $P = 0$ for LO. One should bear in mind that these K-factors actually measure the necessary reparameterization of the GPD to fit the given experimental data. We take the ansatz (30) with ${}^{\mathrm{sea}}F(\Delta^2) = {}^{G}F(\Delta^2)$. Hence the ratio of gluon GPD to quark one is controlled by the factor N_G/N_{sea} and, more importantly, by the differences of intercepts $\alpha_G(0) - \alpha_{\mathrm{sea}}(0)$. To study the influence of this ratio, we distinguish two cases:

$$\text{H) ``hard'' gluon:}\quad N_G = 0.4,\ \alpha_G(0) = \alpha_{\mathrm{sea}}(0) + 0.1, \tag{34}$$

$$\text{S) ``soft'' gluon:}\quad N_G = 0.3,\ \alpha_G(0) = \alpha_{\mathrm{sea}}(0). \tag{35}$$

We will use these parameters and $\alpha_{\mathrm{sea}}(0) = 1.1$ at the input scale $\mathcal{Q}^2 = 2.5\ \mathrm{GeV}^2$. Moreover, we set $\mu = \mathcal{Q}$ and independently of the considered approximation we choose $\alpha_s(\mu_r^2 = 2.5\ \mathrm{GeV}^2) = 0.1\pi$ and set the number of active flavors to three.

In Fig. 1 we depict for the typical kinematics of HERA collider experiments, i.e., $10^{-5} \lesssim \xi \lesssim 5 \times 10^{-2}$, the resulting K factors for the logarithm of the modulus [(a) for $\Delta^2 = 0$ and (b) for $\Delta^2 = -0.5\ \mathrm{GeV}^2$] and phase [(c) for $\Delta^2 = 0$ and (d) for $\Delta^2 = -0.5\ \mathrm{GeV}^2$].

Here the thick and thin lines correspond to the "hard" and "soft" gluon parameterizations, respectively. We observe an almost flat ξ dependence of the K_λ factors in panels (a) and (b). This is not surprising, since the essential contribution arises from the pomeron pole and the CFF behaves as:

$${}^{\mathrm{S}}\mathcal{H} \sim \left(\frac{1}{\xi}\right)^{\alpha(\Delta^2)}\left[i + \tan\left(\frac{\pi}{2}\left(\alpha\left(\Delta^2\right) - 1\right)\right)\right] \quad \Rightarrow \quad \ln\left|{}^{\mathrm{S}}\mathcal{H}\right| \cong \alpha\left(\Delta^2\right)\ln(1/\xi) + \mathrm{const}. \tag{36}$$

For small ξ this leads to the flatness we observe. The size of perturbative NLO corrections, see dashed ($\overline{\mathrm{CS}}$ scheme) and dotted ($\overline{\mathrm{MS}}$ scheme) lines, essentially depends on the ratio of gluon to quark GPDs. Since the gluons are a new entry, formally counted as NLO contribution, this finding is obvious and goes along with the observation that the perturbative corrections strongly vary within the used parameterization of parton densities in the Radyushkin GPD ansatz [32]. In the "hard" gluon parameterization the logarithm of the modulus reduces about 7–11% [5–8%] in the $\overline{\mathrm{MS}}$ [$\overline{\mathrm{CS}}$] scheme, corresponding to the reduction of the modulus itself in the range of 40–70% [30–55%], where the drastic upper values correspond to $\xi = 10^{-5}$.

The relative radiative corrections to the phase grow in the small ξ region with decreasing ξ and can be of the order of up to 24% [13%] in the $\overline{\mathrm{MS}}$ [$\overline{\mathrm{CS}}$] scheme. These effects are related to the signs for NLO Wilson coefficients, see Eqs. (13), (14), (20),

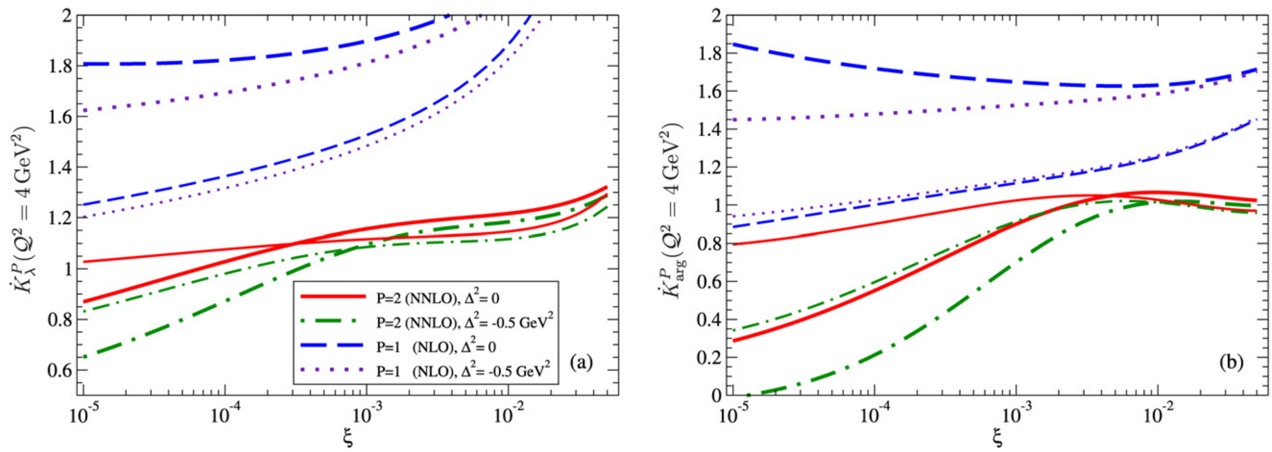

Fig. 2. The relative change of scale dependence (37) in the $\overline{\text{CS}}$ scheme at NLO (dashed, dotted) and NNLO (solid, dash-dotted) versus ξ is depicted for the logarithm of the modulus (a) and phase (b) of the CFF (11) with $\Delta^2 = 0$ (dashed, solid) and $\Delta^2 = -0.5\ \text{GeV}^2$ (dotted, dash-dotted) and $\mathcal{Q}^2 = 4\ \text{GeV}^2$. Thick and thin lines correspond again to "hard" and "soft" gluonic input.

and (21). In the "soft" gluon parameterization the NLO corrections are quite moderate for the modulus [(a) and (b)] and negligible for the phase [(c) and (d)]. From all four panels it can be realized that compared to $\overline{\text{MS}}$ scheme in the $\overline{\text{CS}}$ one the NLO corrections are typically reduced by 30–50%. This reduction has been also observed in the flavor non-singlet case [9]. The NNLO corrections (solid), compared to the NLO (dashed) ones, are drastically reduced. For the "soft" gluon parameterization they are practically negligible while for the "hard" gluon input they are reduced to the 1–2% level, except for the phase with $\Delta^2 = -0.5\ \text{GeV}^2$, where 5% are reached at $\xi = 10^{-5}$.

Let us finally address the modification of the scale dependence due to the higher order corrections. We only consider here the $\overline{\text{CS}}$ scheme and, analogously as in Eq. (33), we quantify the relative changes due to the evolution by the ratios

$$\dot{K}^P_\lambda = \frac{d\ln|{}^{\text{S}}\mathcal{H}^{\text{N}^P\text{LO}}|}{d\ln\mathcal{Q}^2}\Big/\frac{d\ln|{}^{\text{S}}\mathcal{H}^{\text{N}^{P-1}\text{LO}}|}{d\ln\mathcal{Q}^2}, \qquad \dot{K}^P_{\text{arg}} = \frac{d\arg({}^{\text{S}}\mathcal{H}^{\text{N}^P\text{LO}})}{d\ln\mathcal{Q}^2}\Big/\frac{d\arg({}^{\text{S}}\mathcal{H}^{\text{N}^{P-1}\text{LO}})}{d\ln\mathcal{Q}^2}. \tag{37}$$

For the (exact) evolution of $\alpha_s(\mathcal{Q})$ we take the same scale setting and initial condition as above. However, the conformal moments (30) are evolved in the $\overline{\text{CS}}$ scheme, starting at the input scale $\mathcal{Q}_0^2 = 1\ \text{GeV}^2$, to $\mathcal{Q}^2 = 4\ \text{GeV}^2$. The non-leading logs in the solution of the evolution equation (22) are expanded with respect to α_s and are consistently combined with the Wilson coefficients (12) in the considered order. The unknown NNLO mixing term $\Delta^{\overline{\text{CS}}}_{jk}$ in Eq. (22) is neglected. This mixing can be suppressed at the input scale and so we expect only a small numerical effect; see also Ref. [33].

The dashed and dotted lines in Fig. 2 show that in NLO the scale dependence changes can be rather large even of about 100% or more. The relative corrections to NNLO are getting smaller. For instance, the NNLO corrections in panel (a) are almost negligible for the "soft" gluonic input with $\Delta^2 = 0$ (thin solid), they increase, however, for $\Delta^2 = -0.5\ \text{GeV}^2$ and are becoming large for the "hard" gluonic input, e.g., about -35% at $\xi = 10^{-5}$ (thick dash-dotted). Note that these large corrections at very small ξ are essentially caused by those of the anomalous dimensions in the vicinity of $j = 0$, corresponding to the large corrections of the gluon splitting kernels at small x, reported in [25]. The same sources also cause the huge NNLO corrections to the phase in panel (b). We remark that the modulus of ${}^{\text{S}}\mathcal{H}$ is dominated by its imaginary part for which radiative corrections are milder than for the real part. The real part and so also the phase at very small ξ are rather strongly affected by the NNLO corrections to anomalous dimensions. On the other hand for $5\times10^{-4} \lesssim \xi$ and $5\times10^{-3} \lesssim \xi$ the NNLO corrections to the logarithm of the modulus (a) and phase (b), respectively, are rather mild (solid and dash-dotted).

4. In this Letter we have studied NLO and NNLO corrections to DVCS in the small ξ region. We confirmed that large radiative corrections at NLO can appear, reported before, and clarified their source which is entirely tied to the gluonic sector. In particular, if the gluon distribution starts to have a steeper increase at small ξ than the quark ones, the NLO corrections will be dominated by the negative NLO gluon contribution and so the modulus of ${}^{\text{S}}\mathcal{H}$ will drastically reduce. On the other hand, if the gluon contribution is relatively small, already the NLO corrections are moderate. In any case the NNLO corrections are becoming moderate or even small at a given input scale, even at a few GeV^2. This fact supports the perturbative framework of DVCS.

The situation with respect to the scale dependence is not so conclusive. Going from LO to NLO we observe in general a big enhancement that arises from the large corrections to the anomalous dimensions, cf. [25]. To NNLO they will be reduced and the relative changes for the logarithm of the modulus are getting reasonable but grows to be large with decreasing ξ. Note that in this region the NNLO gluonic evolution effects are comparable in size with the NLO ones [25]. Also the NLO radiative corrections to the scale dependence of the phase of ${}^{\text{S}}\mathcal{H}$ are rather large, in particular for $\Delta^2 = 0$, at the scale of $4\ \text{GeV}^2$. To NNLO accuracy the convergency improves for $5 \times 10^{-3} \lesssim \xi$. Unfortunately, at smaller values of ξ the convergency is lost. These large corrections due to evolution at small ξ are certainly related to those found in DIS [25].

If one is interested to access GPDs from the DVCS cross section measurement at small ξ, only the modulus of ${}^{\mathrm{S}}\mathcal{H}$ is essential. In that case perturbation theory seems to work in the sense that NNLO corrections of the Wilson coefficients are negligible. They are, however, important for the scale violating effects for $\xi \lesssim 5 \times 10^{-4}$ (at relatively low $\mathcal{Q}^2 \sim 4\ \mathrm{GeV}^2$). We also conclude that the photon and vector–meson leptoproduction data taken by the H1 and ZEUS collaborations should be perturbatively analyzed at NLO [34].

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