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## Nondecoupling of a terascale isosinglet quark and rare  $K$  and  $B$  decays

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We examine recent extensions of the standard model with an up-type vectorlike isosinglet  $T$  quark that mixes dominantly with the top quark. We take under scrutiny the nondecoupling effects which may reveal such a new heavy fermion through loop diagrams relevant for rare decays such as  $K \to \pi \nu \bar{\nu}$ ,  $B \to \pi (K)$ <br> $\pi^2 K^2$  and  $B \to \mu^+ \mu^-$ . After demonstrating in detail the concellation between the loading non- $\pi(K)\nu\bar{\nu}$ , and  $B_{s,d} \to \mu^+\mu^-$ . After demonstrating in detail the cancellation between the leading non-<br>decounting terms, we show that two residual forms  $\sigma r^2 \ln^2$  and  $\sigma r^4 m^2$  act in a complementary way, so decoupling terms, we show that two residual forms  $\sim s^2 \ln m_T^2$  and  $\sim s^4 m_T^2$  act in a complementary way, so that the maximal allowed values of the decay rates are practically independent of m. While they that the maximal allowed values of the decay rates are practically independent of  $m<sub>T</sub>$ . While they correspond to  $\sim$ 20% or  $\sim$ 30% corrections to the SM rates for  $K \to \pi \nu \bar{\nu}$  and  $B \to \pi (K) \nu \bar{\nu}$ , an increase by<br> $\sim$ 50% for  $B \to \mu^+ \nu^-$  decays offers a possibility to reveal an additional isosipalet state by measur ments of these decays at the Large Hadron Collider. ~50% for  $B_{s,d} \to \mu^+\mu^-$  decays offers a possibility to reveal an additional isosinglet state by measure-

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#### I. INTRODUCTION

Awaiting the forthcoming results from the Large Hadron Collider (LHC), we are trying to imagine ways to probe the conceivable new degrees of freedom belonging to the physics at the TeV scale. In case the mass of a new degree of freedom lies out of the reach of direct LHC production, one could still hope to infer on it from its virtual loop effects. The well-known examples from the past are charmand top-quark loop effects in the box diagrams contributing to  $K - \overline{K}$  and  $B - \overline{B}$  mixing, respectively. There has been a recent revival of the latter box diagrams by Vysotsky [1], in the context of a simple extension of the standard model (SM) with an additional up-quark state. The essential reason for examining such an additional degree of freedom has been the so-called nondecoupling of a heavy quark in the box diagrams. When the loop diagrams with exchanged Goldstone boson and heavy quark are evaluated in the renormalizable gauge, this nondecoupling is known to originate in the Yukawa coupling proportional to the heavy quark mass. Generally, in rare processes the box diagrams combine with the Z-penguin diagrams that exhibit similar nondecoupling. Therefore, we extend Vysotsky's investigation of the box diagrams to the Z-penguin diagrams, and explore their impact on some phenomenologically interesting rare decay processes.

Our present dwelling on the up-quark sector is motivated in part by a recent study by Alwall et al. [2] which showed that the Tevatron measurements leave ample space for an extra top state above 256 GeV [3]. An extra top state is expected to be part of some beyond the standard model (BSM), but the results of the present paper are quite general and do not rely on any specific BSM. Note that there is even a possibility that such an extra state originates on account of the nonperturbative effects within the SM

itself. There has been a proposal of such a bound state of 6 top and 5 antitop quarks [4] that could be probed at the LHC.

Extra vectorlike quark states of both up- and down-type were already subjected to detailed previous studies [5–8]. The more recent study that focused on the down-quark sector [9] is complementary to what we are exploring here in the up-quark sector. Moreover, being in favor of a rather heavy spectrum of isosinglet down states, the results by these authors indicate that an isosinglet up-quark could be well beyond the direct LHC production.

Our starting point is the model advocated by Vysotsky [1], where the new up-type quark is just above the LHC reach. This model has an appeal of repeating in the upquark sector a sort of seesaw mechanism, that is known to be operative in the up-lepton (neutrino) sector. By imposing the quark-lepton symmetric generalization of the SM, the existing modification of the SM in the leptonic upperhypercharge (neutrino) sector opts for the corresponding extra up-type isosinglet quark sector, rather than a full sequential fourth family. Then, a new heavy quark mass scale M ensures a smallness of the mixing of an additional top quark with the SM top and may explain why the SM top is much heavier than other SM fermions [1].

In the next section we first describe the predictive twoparameter model based on such an isosinglet extension. We first check the allowed parameter space against the flavorconserving electroweak precision observables (EWPO). The obtained constraints are drawn on account of the nondecoupling, both in the flavor specific parameter  $R_b$ and in the oblique  $T$  parameter. In the third section we focus on the flavor-nondiagonal Z-penguin and box diagram transitions that exhibit a subtle cancellation of the leading nondecoupling terms. Finally, we analyze the sensitivity of the golden  $K \to \pi \nu \bar{\nu}$ ,  $B \to \pi (K) \nu \bar{\nu}$ , and  $B \to \mu^+ \mu^-$  decays to such an extension of the SM and compare  $\mu^+ \mu^-$  decays to such an extension of the SM and compare it to the results of the littlest Higgs model (LHM) that provides a monitoring case.

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### <span id="page-2-1"></span>II. EWPO CONSTRAINTS ON MODEL WITH EXTRA ISOSINGLET QUARK

It is conceivable that possible new quark states can be out of the direct production reach also in the era of the LHC. Therefore, we take a closer look at the model proposed recently by the authors of Ref. [2] and check that the EWPO constraints allow for a wider parameter span than considered in [2]. This motivates us to further extend the approach by Vysotsky [1] toward a still unexplored region of the parameter space and to focus on rare decays which may be sensitive to this extended parameter region.

The model at hand represents an extension of the SM by the heavy isosinglet, which in the case of Ref. [2] is a new up-type T-quark with a mass just above the Tevatron bound of 256 GeV [3]. Vysotsky considers, instead, a state with the mass just above  $\simeq$  5 TeV, the limit of the direct production at the LHC, and in the present paper we further extend the considered mass region. This allows us to look at the loop effects for the whole allowed parameter space, irrespective of whether or not the new state can be produced directly. In the present paper we show that at  $m_T \sim$ <br>7 TeV the extended mass region splits into two regimes 7 TeV the extended mass region splits into two regimes that are dominated by two different types of nondecoupling. However, the new results of the present paper refer to the regime of the relatively heavy T state.

The adopted model, expressed in terms of the weak (primed) eigenstates, reads in the form of the Lagrangian that in addition to the usual SM piece has an additional BSM part consisting of two Dirac mass terms and one Yukawa term:

<span id="page-2-2"></span>
$$
\mathcal{L}_{\text{BSM}} = M \bar{T}'_L T'_R + \left[ \mu_R \bar{T}'_L t'_R + \frac{\mu_L}{v/\sqrt{2}} (\vec{r}, \vec{b}')_L \Phi^c T'_R \right] + \text{H.c.}
$$
\n(1)

Two new heavy  $SU(2)_L$  singlet states,  $T'_L$  and  $T'_R$ , have the Dirac mass term M that is nondiagonal because  $T'$  mixes Dirac mass term  $M$  that is nondiagonal because  $T<sup>'</sup>$  mixes with the  $t'$  state. This mixing is given by two terms in the square brackets: the  $\mu_R$  term describes the mixing of two  $SU(2)_L$  singlets,  $T'_L$  and  $t'_R$ , while the  $\mu_L$  term describes mixing of the SM weak isodoublet with the isosinglet state mixing of the SM weak isodoublet with the isosinglet state T'. Obviously, by switching off these  $\mu_{L,R}$  terms, the t' field would become the ordinary  $t$  quark, the mass eigenstate of the SM.

The presence of the  $T'-t'$  mixing [1,2] has several<br>fects. First, the mass matrix should be diagonalized effects. First, the mass matrix should be diagonalized, and for the charged current couplings, the SM unitary Cabibbo-Kobayashi-Maskawa (CKM) matrix  $V_{3\times 3}^{SM}$  has to be enlarged to  $V_{3\times 3}$  a generalized  $4 \times 3$  CKM matrix that be enlarged to  $V_{4\times 3}$ , a generalized  $4 \times 3$  CKM matrix that is not unitary. The adopted assumption that  $T<sup>'</sup>$  mixes only with the  $t'$  implies that the unitary transformation to mass eigenstates is a simple rotation parametrized by the single (real) angle  $\theta$ . Accordingly, the generalized CKM matrix entries in the charged current couplings are

<span id="page-2-0"></span>
$$
V_{ti} = V_{ti}^{\text{SM}} \cos \theta, \qquad V_{Ti} = V_{ti}^{\text{SM}} \sin \theta. \tag{2}
$$

Second, the  $T' - t'$  mixing modifies the neutral current<br>countings of  $U = (u, c, t, T)$  states couplings of  $U = (u, c, t, T)$  states

$$
\mathcal{L}_{NC} = -\frac{g}{2\cos\theta_W} Z_\mu (\bar{U}_L \mathbf{V} \mathbf{V}^\dagger \gamma^\mu U_L - 2\sin^2\theta_W J_{em}^\mu).
$$
\n(3)

This induces the flavor changing neutral current (FCNC) part, given by the nondiagonal terms in

$$
\mathbf{V}\mathbf{V}^{\dagger} = \begin{pmatrix} \mathbf{1}_{2\times 2} & 0 & 0 \\ 0 & \cos^2 \theta & \sin \theta \cos \theta \\ 0 & \sin \theta \cos \theta & \sin^2 \theta \end{pmatrix}.
$$
 (4)

However, the built-in restriction that the heavy isosinglet  $T'$  state mixes only with the SM  $t'$  quark, leads to the very predictive 2-parameter model expressed in terms of the  $T'-t'$  mixing angle  $\theta$ , and the mass  $m_T$  of the new<br>isosinglet quark isosinglet quark.

Let us turn to the constraints on these new parameters that can be drawn from EWPO tests. Both types of loop effects, the universal (oblique)  $T$  parameter, and nonuniversal (flavor specific)  $R_b$  parameter, are sensitive to nondecoupling of heavy quarks.

The effects of the heavy isosinglets in the oblique  $T$ parameter have been computed in Ref. [10]. In the model at hand, where a single isosinglet quark mixes only with the SM top quark, new contributions to the  $T$  parameter are summarized as

$$
T = \frac{3}{16\pi \sin^2 \theta_W \cos^2 \theta_W}
$$
  
 
$$
\times \{ [V_{tb}^{\text{SM}}]^2 \sin^2 \theta [\theta_+(y_T, y_b) - \theta_+(y_t, y_b)] - \sin^2 \theta \cos^2 \theta \theta_+(y_T, y_t) \}.
$$
 (5)

Here  $y_i = m_i^2/m_Z^2$  and

$$
\theta_+(y_1, y_2) = y_1 + y_2 - \frac{2y_1y_2}{y_1 - y_2} \ln \frac{y_1}{y_2}.
$$
 (6)

By comparing Eq.  $(5)$  to the most recent experimental value  $T = -0.03 \pm 0.09$  [11], we obtain our first constraint on the allowed part of parameter space  $(m_T, \theta)$ , lying below the lower curve displayed on Fig. [1](#page-3-0).

A similar constraint can also be drawn from the flavor specific parameter  $R_b = \Gamma_b/\Gamma_{\text{had}}$ . Namely, the SM loop diagrams involving the  $t$  quark (with the corresponding parameter  $x_t = m_t^2/m_W^2$  modify the Zbb coupling [12]. In<br>particular its left-handed part  $a_t^b$  is changed by particular its left-handed part  $g_L^b$  is changed by

$$
\delta g_L^b = \left(\frac{\alpha}{2\pi}\right) |V_{tb}^{\text{SM}}|^2 F(x_t). \tag{7}
$$

In the BSM at hand, the presence of the  $T' - t'$  mixing<br>modifies this loop correction further leading to [13] modifies this loop correction further, leading to [13]

$$
|V_{tb}^{\text{SM}}|^2 F(x_t) \rightarrow \sum_{j=t,T} |V_{jb}|^2 (F(x_j) + \tilde{F}(x_j))
$$
  
+  $V_{tb}^* V_{Tb} \tilde{F}(x_t, x_T)$ . (8)

<span id="page-3-0"></span>

FIG. 1 (color online). Allowed parameter space from EWPO, lying below the upper curve determined from  $R_b$ , or the lower curve determined from the T parameter.

The functions  $\tilde{F}(x_j)$  and  $\tilde{F}(x_t, x_T)$  in Eq. (8) appear also in the flavor changing rare decays, and they will be presented in more detail in this context in Eqs. (17) and ([18\)](#page-4-0). From the expression given in Ref. [13],

$$
R_b = R_b^{\text{SM}} (1 - 3.56 \delta g_L^b + 0.645 \delta g_R^b + 0.00066S - 0.0004T),
$$
\n(9)

it is clear that the oblique  $T$  parameter term can be neglected in the ratio  $R_b$ . Using the experimental value  $R_b$  = 0.21629(66) and the SM prediction  $R_b^{\text{SM}} = 0.21578(10)$ <br>[11] we obtain our second constraint on the allowed part of [11], we obtain our second constraint on the allowed part of the parametric space  $(m_T, \theta)$ . Then, the constraints obtained from both EW precision parameters can be displayed in the  $(m<sub>T</sub>, sin $\theta$ )$  plane, as shown in Fig. 1. Note that the excluded parameter space from the universal T parameter suffers from the uncertainties from the Higgs mass, so that in further considerations we rely on the nonuniversal parameter  $R<sub>b</sub>$ , which provides more reliable constraints.

The curves on Fig. 1 can be compared to previous plots for low  $m<sub>T</sub>$  values, given in [2,8]. Our plots are extended to higher values of  $m<sub>T</sub>$  [1], where the new state escapes the direct production at the LHC. In the next section we consider possible effects for the whole allowed parameter space. Thereby we will show how these effects originate in a nondecoupling of the heavy isosinglet quark in the loop diagrams for selected rare decays.

#### III. THE STRUCTURE OF NONDECOUPLING AND ITS MANIFESTATION IN SELECTED RARE DECAYS

The model introduced in the previous section has been applied by Vysotsky [1] to  $\Delta F = 2$ ,  $B - \bar{B}$  mixing, represented by the quark-box diagrams. Now we extend such a study to the quark-lepton box diagrams and Z-penguin diagrams that govern the present day golden decays of flavor physics,  $K^+ \to \pi^+ \nu \bar{\nu}$ ,  $K_L \to \pi^0 \nu \bar{\nu}$ ,  $B \to \pi \nu \bar{\nu}$ ,<br>  $R \to K \nu \bar{\nu}$  and  $R \to \mu^- \mu^+$  These EC transitions are  $B \to K \nu \bar{\nu}$ , and  $B \to \mu^- \mu^+$ . These FC transitions are<br>dominated by a nondecounling existing in the box and dominated by a nondecoupling existing in the box and the Z-penguin diagrams. Huge effects are a priori possible, if there were no cancellations among the leading nondecoupling terms. We are left by effective suppression of nondecoupling that will appear in the SM Inami-Lim functions [14] describing the rare decays  $K \to \pi \nu \bar{\nu}$ ,  $B \to \pi (K) \nu \bar{\nu}$  and  $B \to \mu^- \mu^+$ . These short distance functions  $\pi(K)\nu\bar{\nu}$ , and  $B \to \mu^- \mu^+$ . These short distance functions,<br>by now, have a standard notation  $X(r)$  and  $Y(r)$  [15] by now, have a standard notation,  $X(x_t)$  and  $Y(x_t)$  [15].

For the adopted BSM with the extra isosinglet  $T$  quark, these functions are modified by contributions from loops involving this new quark. In addition to contributions from loops involving only a  $t$  or only a  $T$  quark, there are additional contributions from loops involving simultaneously  $t$  and  $T$  quarks. In the first case there is a change in the relevant CKM elements for the charged current coupling, Eq. ([2](#page-2-0)), and a change in flavor diagonal NC coupling Eq. ([4\)](#page-2-1), but in the second case also the FCNC coupling in Eq. ([4](#page-2-1)) occurs. In general, the powers of small factor  $s \equiv \sin \theta$  will suppress the nondecoupling of the new heavy T quark. We are going to demonstrate in detail the structure of modulation of  $m<sub>T</sub>$ -dependent terms by the powers of s. The evaluation of relevant Feynman diagrams involving only a  $t$  or only a  $T$  quark in the  $'t$  Hooft-Feynman gauge has been presented in Ref. [14]. All divergent and  $sin\theta_W$ -dependent terms cancel mutually in the sum.

Let us first focus on diagrams involving only the T quark. Here, from the full set of diagrams in Inami and Lim [14], we display in Fig. [2](#page-4-1) only the most characteristic: The subgroup (a) to (c) in which the potentially leading nondecoupling  $\sim x_T$  cancels out, and the diagram (d) in<br>which such decoupling is suppressed by an additional which such decoupling is suppressed by an additional factor of  $s^2$ . In the 't Hooft-Feynman gauge, the leading nondecoupling of the  $T$  quark arises in conjunction with the Goldstone exchange in off-diagonal self-energy diagrams (a) and (b), and in diagram (c). These diagrams, involving only the T quark, will have a factor of  $s^2$  due to the CKM factors in Eq. [\(2](#page-2-0)):

<span id="page-3-1"></span>
$$
G_{a+b} \approx \frac{s^2}{8} \left( -\frac{1}{2} x_T \ln x_T + \frac{3}{4} x_T + \mathcal{O}(\ln x_T) \right),
$$
  
\n
$$
G_c \approx \frac{s^2}{8} \left( \frac{1}{2} x_T \ln x_T - \frac{3}{4} x_T + \mathcal{O}(\ln x_T) \right).
$$
\n(10)

The diagram (d) in Fig. [2](#page-4-1), involving NC coupling from

<span id="page-4-1"></span>

FIG. 2. Loop diagrams: (a) to (c), in which the potentially leading nondecoupling  $\sim s^2 x_T$  reduces to merely  $\sim s^2 \ln x_T$  as demonstrated in Eq. (10) and the diagram (d) giving the second remaining form  $\sim s^4 r_c$ demonstrated in Eq. ([10](#page-3-1)), and the diagram (d) giving the second remaining form  $-s^4x_T$ .

Eq. ([4\)](#page-2-1), will get another factor of  $s^2$ :

$$
G_d \approx \frac{s^4}{8} (x_T - \ln x_T). \tag{11}
$$

The leading nondecoupling terms  $\sim s^2 x_T$  and  $\sim s^2 x_T \ln x_T$ <br>in Eq. (10) that are only suppressed by a factor of  $s^2$  could in Eq. [\(10\)](#page-3-1), that are only suppressed by a factor of  $s^2$ , could a priori lead to huge effects in FC rare decays. That is not the case because they cancel mutually in the sum, and only the terms of the form  $\sim s^4 x_T$  and  $\sim s^2 \ln x_T$  survive in the sum. This cancellation is not accidental, but is known to be sum. This cancellation is not accidental, but is known to be a consequence of the  $SU(2)$  gauge symmetry. For an underlying Lagrangian [\(1\)](#page-2-2), the effective sdZ vertex is generated by the  $SU(2)$  breaking in the loops, corresponding to  $\mu_L$ . The explicated cancellation ensures that in the limit of large  $m<sub>T</sub>$  with fixed Yukawa coupling, the T contribution decouples. Then, after adding together the contributions from all diagrams, we are left with a relic of nondecoupling of the T quark that has the form:

 $G(x_T) = s^2 P(x_T) + s^4 R(x_T).$  (12)

<span id="page-4-2"></span>**Here** 

$$
P(x) = \frac{x}{8(x-1)} \left( 3 + \frac{5x-8}{x-1} \ln x \right),\tag{13}
$$

$$
R(x) = \frac{1}{8} \left( x - 2 \frac{x \ln x}{x - 1} \right).
$$
 (14)

From here, the contributions from loops involving only the t quark can be obtained by simple replacements  $s \rightarrow c$  $(c \equiv \cos \theta)$  and  $x_T \rightarrow x_t$  in Eq. [\(12\)](#page-4-2):

$$
G(x_t) = c^2 P(x_t) + c^4 R(x_t).
$$
 (15)

Additional Feynman diagrams involving both  $t$  and  $T$ quarks give a contribution that also has only terms of the form  $\sim s^2 \ln x_T$ :

$$
c^{2} s^{2} C(x_{t}, x_{T}) = c^{2} s^{2} \frac{1}{4} \left[ \frac{-1}{x_{T} - x_{t}} \left( \frac{x_{T}^{2} \ln x_{T}}{x_{T} - 1} - \frac{x_{t}^{2} \ln x_{t}}{x_{t} - 1} \right) + \frac{x_{T} x_{t}}{x_{T} - x_{t}} \left( \frac{x_{T} \ln x_{T}}{x_{T} - 1} - \frac{x_{t} \ln x_{t}}{x_{t} - 1} \right) \right].
$$
 (16)

<span id="page-4-0"></span>The functions  $R(x)$  and  $C(x_t, x_T)$  are related to the functions  $\tilde{F}(x_i)$  and  $\tilde{F}(x_t, x_T)$  in Eq. ([8](#page-3-0)) by

$$
\tilde{F}(x_t) = -\frac{1}{\sin^2 \theta_W} s^2 R(x_t),
$$
\n
$$
\tilde{F}(x_T) = -\frac{1}{\sin^2 \theta_W} c^2 R(x_T),
$$
\n(17)

$$
\tilde{F}(x_t, x_T) = \frac{1}{\sin^2 \theta_W} scC(x_t, x_T). \tag{18}
$$

After obtaining the upper loop functions for the BSM at hand, we are ready to present the generalized Inami-Lim functions for specific rare processes. The function  $X^{SM}(x_t)$ that is relevant for  $K \to \pi \nu \bar{\nu}$  and  $B \to \pi (K) \nu \bar{\nu}$  decays in the SM in the BSM at hand is replaced by the SM, in the BSM at hand is replaced by

$$
X^{SM}(x_t) \to X^{BSM} = G(x_t) + G(x_T) + c^2 s^2 C(x_t, x_T).
$$
\n(19)

By collecting the contributions of the same order in  $s^2$  we

<span id="page-5-2"></span><span id="page-5-1"></span>find

$$
X^{\text{BSM}} = X^{\text{SM}}(x_t) + s^2[-X^{\text{SM}}(x_t) - R(x_t) + X(x_T) - R(x_T) + C(x_t, x_T)] + s^4[R(x_t) + R(x_T) - C(x_t, x_T)].
$$
\n(20)

<span id="page-5-0"></span>Similarly, we generalize the function  $Y^{SM}(x_t)$  that is relevant for the  $B \to \mu^- \mu^+$  decay:

$$
Y^{\text{BSM}} = Y^{\text{SM}}(x_t) + s^2[-Y^{\text{SM}}(x_t) - R(x_t) + Y(x_T) - R(x_T) + C(x_t, x_T)] + s^4[R(x_t) + R(x_T) - C(x_t, x_T)].
$$
\n(21)

For both cases,  $X^{\text{BSM}}$  and  $Y^{\text{BSM}}$ , the parts that are represented by terms of order  $s^2$  are of the form  $s^2$  lnx<sub>T</sub> and will be dominant for a relatively light  $T$  quark. We will show that for a heavy T quark, exceeding  $m<sub>T</sub> \ge 5$  TeV, the terms of the form  $s^4x_T$  will also be important.

We will explicate it on the branching ratios for rare decays, which we define like in Ref. [16]:

$$
R_{+} = \frac{\text{Br}(K^{+} \to \pi^{+} \nu \bar{\nu})_{\text{BSM}}}{\text{Br}(K^{+} \to \pi^{+} \nu \bar{\nu})_{\text{SM}}},\tag{22}
$$

$$
R_L = \frac{\text{Br}(K_L \to \pi^0 \nu \bar{\nu})_{\text{BSM}}}{\text{Br}(K_L \to \pi^0 \nu \bar{\nu})_{\text{SM}}} = \frac{\text{Br}(B \to \pi(K) \nu \bar{\nu})_{\text{BSM}}}{\text{Br}(B \to \pi(K) \nu \bar{\nu})_{\text{SM}}}
$$

$$
= \left[\frac{X^{\text{BSM}}}{X^{\text{SM}}}\right]^2, \tag{23}
$$

$$
R_{s,d} \equiv \frac{\text{Br}(B_{s,d} \to \mu^+ \mu^-)_{\text{BSM}}}{\text{Br}(B_{s,d} \to \mu^+ \mu^-)_{\text{SM}}} = \left[\frac{Y^{\text{BSM}}}{Y^{\text{SM}}}\right]^2. \tag{24}
$$

By using constraints from EWPO displayed in Fig. [1](#page-3-0) we are able to find maximal allowed values for the ratios  $R_+$ ,  $R_L$ , and  $R_{s,d}$  as functions of  $m<sub>T</sub>$ . These plots are drawn on Fig. 3. For comparison, in the same figure we reproduce also the ratio for the quark-box contribution to the  $B - B$ mixing considered by Vysotsky [1]. In contrast to his box diagram case, all our ratios are slowly decreasing functions of  $m<sub>T</sub>$ . Typically, for say  $m<sub>T</sub> = 5$  TeV, these maximal ratios  $R_+, R_L$ , and  $R_{s,d}$  reach the values of ~1.2, 1.3, and<br>1.5 respectively Among them R, has the smallest en-1.5, respectively. Among them,  $R_+$  has the smallest enhancement due to a nonnegligible charm contribution that is unaffected by  $t - T$  mixing. On the other hand,  $R_{s,d}$  can reach values of  $\sim$  1.5 almost independently of  $m<sub>T</sub>$ . The reason is the complementary behavior of the two relics of reason is the complementary behavior of the two relics of nondecoupling forms,  $s^2 \ln x_T$  and  $s^4 x_T$ , building Eq. [\(21\)](#page-5-0) and displayed in Fig. 4 separately, and then as a sum. For  $m_T \ge 7$  TeV the  $m_T$  dependence is effectively changed from a  $\sim \ln m_T^2$  to a  $\sim m_T^2$  form, but with an extra factor<br>of s<sup>2</sup>. The same pattern applies to the decays  $K \to \pi \nu \bar{\nu}$  and of  $s^2$ . The same pattern applies to the decays  $K \to \pi \nu \bar{\nu}$  and  $R \to \pi (K) \nu \bar{\nu}$  and we do not repeat here the plots for them  $B \to \pi(K)\nu\bar{\nu}$ , and we do not repeat here the plots for them.<br>The obvious favorite  $B \rightarrow \mu^- \mu^+$  decays are ideally

The obvious favorite  $B_{s,d} \to \mu^- \mu^+$  decays are ideally suited for an investigation at the LHC ([17] and references



FIG. 3 (color online). Maximal possible enhancements with respect to the SM for the BSM branching ratios  $R_{s,d}$ ,  $R_L$ , and  $R_+$ , going from top down. For comparison we show the corresponding enhancement for the  $B - \bar{B}$  mixing, as the lowest curve.

therein). The current upper bounds from the CDF Collaboration read

$$
Br(Bs \to \mu^{-} \mu^{+}) < 5.8 \times 10^{-8},
$$
  
\n
$$
Br(Bd \to \mu^{-} \mu^{+}) < 1.8 \times 10^{-8},
$$
\n(25)

and LHCb will soon reach the possibility to test an increase in  $B_s \to \mu^- \mu^+$  with respect to the SM prediction [18]:

$$
Br(Bs \to \mu^- \mu^+) = (3.35 \pm 0.32) \times 10^{-9},
$$
  
\n
$$
Br(Bd \to \mu^- \mu^+) = (1.03 \pm 0.09) \times 10^{-10}.
$$
 (26)

Let us therefore display in Fig. [5](#page-6-0) the  $B \to \mu^- \mu^+$  decay rates as a 3D plot, constrained by Fig. [1](#page-3-0) with respect to the allowed parameter space. Figure [5](#page-6-0) illustrates the possibility to infer on the value of  $m<sub>T</sub>$  from the measurements of the



FIG. 4 (color online). The complementary behavior with respect to  $m<sub>T</sub>$  of the two relics of nondecouplings, entering  $\hat{Y}^{\text{BSM}} = Y^{\text{SM}} + \Delta Y$  in Eq. ([21](#page-5-0)) relevant for  $R_{s,d}$ : the decreasing  $m_T$ -independent sum on the upper curve.  $s^2$  and the increasing  $\sim s^4$  term in Eq. [\(21\)](#page-5-0), and their almost<br>
symplement sum on the upper curve

<span id="page-6-0"></span>

FIG. 5 (color online). Full plot for possible enhancement of the Br  $(B \to \mu^- \mu^+)$ , given by the functional dependence of the  $R_{s,d}$  on the parameters of the model. Let us note that by projecting the top of the displayed surface on the  $(R_{s,d}, m_T)$ plane, one obtains the uppermost curve in Fig. [3.](#page-5-1)

 $B \rightarrow \mu^{-} \mu^{+}$  decays, provided that the mixing sin $\theta$  is fixed from some other observable.

#### IV. DISCUSSION AND CONCLUSIONS

In this paper we examine the loop-induced effects of a possible terascale up-type isosinglet quark. The quarklepton symmetry and some additional arguments listed in the introduction are in favor of such an economical extension of the SM. We adopt the BSM defined by Eq. ([1\)](#page-2-2), where the new  $T$  quark modifies the weak and scalar interactions involving  $Q = 2/3$  quarks, while the strong and electromagnetic interactions are not affected. Such a minimal intervention in the gauge sector satisfies the requirements of minimal flavour violation, that was recently the focus of Ref. [9] for the down-quark variant of the Lagrangian [\(1](#page-2-2)). The same study points to a spectrum with rather heavy extra down-quark states, that are generally beyond the direct LHC reach. Likewise, the new up-type quark investigated here is expected to be rather heavy, and is supposed to be a part of some more complete BSM. A quest for the complete BSM may require a more general principle, in a spirit of the Freedberg-Lee symmetry [19] discussed recently by Jarlskog [20].

The adopted model represents a predictive scenario with only two new parameters: the mass of the new  $Q = 2/3$ isosinglet quark  $(m_T)$  and its mixing angle to the neighboring standard model t quark state. The loop effects of heavy top quarks are governed by the characteristic forms expressed in terms of these two parameters. The ''nondecoupling" of the T contributions holds for increasing  $m<sub>T</sub>$  with fixed  $s$ , which in the context of the Lagrangian  $(1)$  $(1)$  implies an increasing Yukawa coupling  $\mu_L$ . However, our priority is to deduce an allowed enhancement of selected SM processes for T quarks beyond the LHC reach, from the mixing angle satisfying phenomenological bounds, rather than to explore in detail in which region of the parameter space the Yukawa coupling  $\mu_L/(v/\sqrt{2})$  remains perturbative. Thus the existing bounds on the parameters from the tive. Thus, the existing bounds on the parameters from the electroweak precision measurements allow us to predict the maximal possible enhancement of selected golden rare decays, respecting the parameter space allowed by the EWPO. Our novel results refer to a relatively heavy isosinglet quark that is beyond the direct production reach of the LHC.

Possible enhancements for the  $\Delta F = 1$  decay rates considered here turn out to be more significant than for previously considered [1]  $B - \overline{B}$  mixing. The effects shown in Fig. [3](#page-5-1) are specific for the quark-lepton box and Z-penguin loop amplitudes, which are exposed in detail in the present paper. Eventual mild dependence on  $m<sub>T</sub>$  is determined by the complementary character of the two relics of nondecoupling. Namely, after the leading nondecoupling terms for the rare decays such as  $K \to \pi \nu \bar{\nu}$ ,  $B \to \pi (K) \nu \bar{\nu}$ , or  $B \to \mu \bar{\mu}$  cancel out we are left with two residual terms  $B \rightarrow \mu \bar{\mu}$  cancel out, we are left with two residual terms that give complementary contributions along the whole range of the  $m<sub>T</sub>$  values. We illustrate this in detail in Fig. [4](#page-5-1) for the example of  $R_{s,d}$  which, owing to a particular loop structure of the  $B \to \mu \bar{\mu}$  decays, acquires the largest relative contribution from the new heavy quark state.

We can compare our results with those of some existing elaborated framework possessing an extra top quark, like the littlest Higgs model [21]. However, this framework contains in addition to an extra heavy top also new vector bosons and an additional weak-triplet scalar field. The latter two produce already the tree-level corrections to EWPO and create the tensions that can be cured by introducing T-parity. Since this requires extra mirror fermions, we limit ourselves to the original LHM without Tparity, elaborated in Ref. [22] and applied by Buras and collaborators  $[16]$ , to rare decays which we are studying here. Truncated further to the sector of the top quarks, this LHM version matches our model and serves as a monitoring case for our calculations.

Let us note that in the framework with single real mixing angle, CP conserving and CP violating amplitudes are equally enhanced. Extra CP phases enter by allowing the mixing of  $T$  to lighter quarks by introducing further nondiagonal Yukawa terms, as attempted recently by Ref. [23] in the context of the LHM. Previously, by such mixing to extra states, Ref. [8] achieved large enhancement of  $K_L \rightarrow$  $\pi^0 \nu \nu$  decays.

Two parameters of our model,  $m<sub>T</sub>$  and sin $\theta$ , have two corresponding LHM parameters,  $f$  and  $x_L$ , and can be expressed in terms of them:

$$
m_T = \frac{f}{v} \frac{m_t}{\sqrt{x_L(1 - x_L)}}, \qquad \sin \theta = x_L \frac{v}{f}.
$$
 (27)

Here,  $f$  is the new scale of the LHM and  $x<sub>L</sub>$  is given by a specific ratio of the Yukawa couplings in the LHM sector of top quarks. New contributions to the functions  $X$  and  $Y$ 



FIG. 6 (color online). LHM parameter region, as restricted by Buras et al., touches the EWPO curve from Fig. [1](#page-3-0) in the point  $(s = 0.16, m<sub>T</sub> = 2 \text{ TeV}).$ 

of the order  $s^2$  given in Eqs. [\(20\)](#page-5-2) and ([21](#page-5-0)) are identical to the contributions of the Feynman diagrams in Fig. 3 in Ref. [16] at the order  $x_L^2 \frac{v^2}{f^2}$ , as given in Eqs. (4.19) and (4.25) in [16]. Note that the parameters of the LHM are constrained in Ref. [16] to the range  $0.2 \le x_L \le 0.8$ , and

- [1] M. I. Vysotsky, Phys. Lett. B 644, 352 (2007).
- [2] J. Alwall, R. Frederix, J.-M. Gerard, A. Giammanco, M. Herquet, S. Kalinin, E. Kou, V. Lemaitre, and F. Maltoni, Eur. Phys. J. C 49, 791 (2007).
- [3] CDF Collaboration, http://www-cdf.fnal.gov/physics/new/ top/2005/ljets/tprime/gen6/public.html.
- [4] C. D. Froggatt, H. B. Nielsen, and L. V. Laperashvili, Int. J. Mod. Phys. A 20, 1268 (2005).
- [5] F. del Aguila and M. J. Bowick, Nucl. Phys. B224, 107 (1983).
- [6] P. Langacker and D. London, Phys. Rev. D 38, 886 (1988).
- [7] V. D. Barger, M. S. Berger, and R. J. N. Phillips, Phys. Rev. D 52, 1663 (1995).
- [8] J. A. Aguilar-Saavedra, Phys. Rev. D 67, 035003 (2003); 69, 099901(E)(2004).
- [9] Y. Grossman, Y. Nir, J. Thaler, T. Volansky, and J. Zupan, Phys. Rev. D 76, 096006 (2007).
- [10] L. Lavoura and J.P. Silva, Phys. Rev. D 47, 2046 (1993).
- [11] W. M. Yao et al. (PDG), J. Phys. G 33, 1 (2006).
- [12] J. Bernabeu, A. Pich, and A. Santamaria, Nucl. Phys. B363, 326 (1991).

 $5 \le f/\nu \le 10$ . Mapped to the  $(m_T, \sin\theta)$  plane, these constraints result in the shaded region shown in Fig. 6. straints result in the shaded region shown in Fig. 6.

Let us stress that the point  $(f = 5, x_L = 0.8)$ , equivalent to ( $s = 0.16$ ,  $m_T = 2 \text{ TeV}$ ) saturates the EWPO bound and corresponds to the extreme values for the ratios  $R_+$ ,  $R_L$ ,  $R_{s,d}$  that can be identified in Fig. 7 in Ref. [16]. This touching in a single point with the EWPO curve in Fig. [1](#page-3-0) compares to the maximal enhancement which is obtained in our model for a much wider region of the parameter space shown on Fig. [5.](#page-6-0) Accordingly, our analysis shows that a rather heavy  $m_T$  ( $m_T \ge 5$  TeV) has the best chance to be recovered via an increase in  $B_s \to \mu^+ \mu^-$  decay rate measured at the LHC.

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Note added.—After this work had been completed, we noticed paper [24] in which the studies of Ref. [1] were extended to rare decays (but not  $B_{s,d} \to \mu^+ \mu^-$  decays). In the case of decays governed by the  $X^{\text{BSM}}$  function, on which Ref. [24] restricts, we agree with their results.

- [13] P. Bamert, C.P. Burgess, J.M. Cline, D. London, and E. Nardi, Phys. Rev. D 54, 4275 (1996).
- [14] T. Inami and C. S. Lim, Prog. Theor. Phys. **65**, 297 (1981); 65, 1772(E) (1981).
- [15] G. Buchalla, A. J. Buras, and M. K. Harlander, Nucl. Phys. B349, 1 (1991).
- [16] A. J. Buras, A. Poschenrieder, S. Uhlig, and W. A. Bardeen, J. High Energy Phys. 11 (2006) 062.
- [17] R. Fleischer, arXiv:0802.2882.
- [18] M. Blanke, A. Buras, D. Guadagnoli, and C. Tarantino, J. High Energy Phys. 10 (2006) 003.
- [19] R. Friedberg and T. D. Lee, Ann. Phys. (N.Y.) 323, 1087 (2008).
- [20] C. Jarlskog, Phys. Rev. D 77, 073002 (2008).
- [21] N. Arkani-Hamed, A.G. Cohen, E. Katz, and A.E. Nelson, J. High Energy Phys. 07 (2002) 034.
- [22] T. Han, H. E. Logan, B. McElrath, and L.-T. Wang, Phys. Rev. D 67, 095004 (2003).
- [23] S. Fajfer and J.F. Kamenik, J. High Energy Phys. 12 (2007) 074.
- [24] P. N. Kopnin and M. I. Vysotsky, arXiv:0804.0912.