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Isgur-Wise function within a modified heavy-light chiral quark model

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We consider the Isgur-Wise function $\xi(\omega)$ within a new modified version of a heavy-light chiral quark model. While early versions of such models gave an absolute value of the slope that was too small, namely $\xi'(1) \simeq -0.4$ to -0.3, we show how extended version(s) may lead to values around -1, in better agreement with recent measurements. This is obtained by introducing a new mass parameter in the heavy-quark propagator. We also shortly comment on the consequences for the decay modes $B \to D\bar{D}$.

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I. INTRODUCTION

The Isgur-Wise (IW) function $\xi(\omega)$ [1], the universal function describing a class of $B(v_b) \to D(v_c)$ transitions, has been studied for many years. (Here, v_b and v_c are the four-velocities for the mesons containing a b, or a c quark, respectively, and $\omega \equiv v_b \cdot v_c$). While some general features of the function follow from heavy-quark symmetry [2], its more detailed shape has been studied in various model approaches [3–11].

It is well known that understanding the shape of the IW function, and, in particular, its slope $\xi'(1)$ at the zero recoil point $\omega = 1$ is a necessary prerequisite for determination of the V_{cb} element of the Cabibbo-Kobayashi-Maskawa quark mixing matrix. However, there is also another incentive to study this function in quark models—namely, it is an essential ingredient in the model description of amplitudes for important nonleptonic decays of the type $B \to D\bar{D}$, DK, $D\pi$. Thus, to approach such theoretically difficult processes, one first needs a reasonable description of the IW function.

This work will be based on the ideas of chiral quark models (χQM) [12], later extended to various versions of heavy-light chiral quark models (HLχQM) [3–7].

Some versions of HLχQM [3,4,6] gave a slope $\xi'(1) \simeq -0.4$ to -0.3, which is not in agreement with general theoretical expectations expressed in Bjorken [13] and Uraltsev [14] sum rules which together imply $-\xi'(1) \geq 3/4$. Also, a combined fit [15] to results of experimental measurements of $B \to D^* l\nu$ decays gives $-\xi'(1) = 1.16 \pm 0.05$, and although dispersion of experimental results is large leading to a small confidence level of this fit ($\approx 1\%$), it seems reasonable to assume that the absolute value of the slope cannot be significantly smaller than 1.

The purpose of the present paper is to show that the model in [6] might be modified in order to describe the IW function in a reasonable way. We construct a new version of the model, which still has the particular feature of explicit inclusion of the gluon condensate effects, enabling consistent estimation of nonfactorizable amplitudes [16–20]. Further, we demonstrate that this new model gives a satisfactory description of the IW function slope.

There are two slightly different philosophies within the family of heavy-light chiral quark models. In Refs. [3,4,6], the focus is on the *bosonization* procedure. The quark Lagrangian is bosonized by attaching meson fields to quark loops at zero external momentum, thereby integrating out the quarks. External momenta then correspond to derivatives of meson fields. On the other hand, in the approach of the Bari group [5] the external momenta are kept in the loop integral, and mass differences between heavy mesons and the heavy quark appear in the final result. This mostly works fine in [5], but a problem seems to arise when one tries to describe transitions between heavy mesons with different masses.

In this paper, we will work with zero external momenta (focus on bosonization), but introduce a parameter Δ in the heavy-quark propagator, which will, for positive values of Δ, correspond to an extra dynamical mass of the heavy quark.

In Sec. II, we describe this extended version of heavy-light chiral quark model. In Sec. III, we show how the model is bosonized and how the model parameters are related to physical quantities. In Sec. IV, we calculate the IW function, and then we conclude. The Appendix contains recursion formulas and expressions for the relevant heavy-light loop integrals.

II. HEAVY-LIGHT CHIRAL QUARK MODEL (HLχQM)

The total Lagrangian describing both quark and meson fields is [6]

$$\mathcal{L} = \mathcal{L}_{\mathrm{HQET}} + \mathcal{L}_{\chi\mathrm{QM}} + \mathcal{L}_{\mathrm{Int}}, \tag{1}$$

where [2]

$$\mathcal{L}_{\mathrm{HQET}} = \bar{Q}_v(iv \cdot D - \Delta)Q_v + \mathcal{O}(m_Q^{-1}) \tag{2}$$

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is the Lagrangian for heavy-quark effective field theory (HQEFT), with the mentioned extra mass added. The heavy-quark field Q_v annihilates a heavy quark with velocity v and mass m_Q. Moreover, D_μ is the covariant derivative containing the gluon field (eventually also the photon field). In [6], the $\mathcal{O}(m_Q^{-1})$ term was also considered, but it will not be needed in this paper, because a better description for the $B \to D$ current than in [3,4,6] is needed already for the IW function, i.e. to zeroth order in $1/m_Q$.

The light-quark sector is described by the χQM, having a standard QCD term and a term describing interactions between quarks and pseudoscalar light mesons:

$$\mathcal{L}_{\chi\mathrm{QM}} = \bar{q}(i\gamma^\mu D_\mu - \mathcal{M}_q)q - m(\bar{q}_R\Sigma^\dagger q_L + \bar{q}_L\Sigma q_R), \tag{3}$$

where $q^T = (u, d, s)$ is the light-quark field triplet. The left- and right-handed projections q_L and q_R are transforming after $SU(3)_L$ and $SU(3)_R$, respectively. $\mathcal{M}_q = \mathrm{diag}(m_u, m_d, m_s)$ is the current quark mass matrix, m is the [$SU(3)$ invariant] dynamical mass of light quarks, and $\Sigma = \exp(2i\Pi/f_\pi)$, where Π is a 3 by 3 matrix containing the pseudoscalar meson octet (π, K, η) in a standard way.

There is also a "rotated version" of the χQM with flavor-rotated quark fields χ given by

$$\chi_L = \xi^\dagger q_L; \qquad \chi_R = \xi q_R; \qquad \xi \cdot \xi = \Sigma. \tag{4}$$

[This field ξ containing the light mesons should be distinguished from the IW function $\xi(\omega)$.] In the rotated version, the chiral interactions are transformed into the kinetic term, while the interaction term proportional to m in (3) becomes a pure (constituent) mass term [12,16]:

$$\mathcal{L}_{\chi\mathrm{QM}} = \bar{\chi}[\gamma^\mu(iD_\mu + \mathcal{V}_\mu + \gamma_5\mathcal{A}_\mu) - m]\chi - \bar{\chi}\widetilde{M}_q\chi, \tag{5}$$

where the vector and axial vector fields $\mathcal{V}_\mu$ and $\mathcal{A}_\mu$ are given by

$$\left.\begin{matrix}\mathcal{V}_\mu \\ \mathcal{A}_\mu\end{matrix}\right\} = \pm\frac{i}{2}(\xi^\dagger\partial_\mu\xi \pm \xi\partial_\mu\xi^\dagger),$$

and

$$\widetilde{M}_q \equiv \xi^\dagger\mathcal{M}_q^\dagger\xi L + \xi\mathcal{M}_q^\dagger\xi R. \tag{6}$$

Here, L is the left-handed projector in Dirac space, $L = (1-\gamma_5)/2$, and R is the corresponding right-handed projector. The Lagrangian (5) is manifestly invariant under the unbroken $SU(3)_V$ symmetry. In the light sector, the various pieces of the Lagrangian describing strong interactions of mesons can be obtained by integrating out the constituent quark fields χ, and these pieces can be written in terms of the fields $\mathcal{A}_\mu$ and $\widetilde{M}_q$ which are manifestly invariant under local $SU(3)_V$ transformations.

In the heavy-light case, the generalization of the meson-quark interactions of the pure light sector χQM is given by the following $SU(3)$ invariant Lagrangian:

$$\mathcal{L}_{\mathrm{Int}} = -G_H[\bar{\chi}_a\bar{H}_v^aQ_v + \bar{Q}_vH_v^a\chi_a] + \frac{1}{2G_3}\mathrm{Tr}[\bar{H}_v^aH_v^a], \tag{7}$$

where G_H and G_3 are coupling constants, and H_v^a is the heavy-meson field containing a spin zero and spin one boson:

$$\begin{aligned} H_v^a &\equiv P_+(P_\mu^a\gamma^\mu - iP_5^a\gamma_5), \\ \bar{H}_v^a &= \gamma^0(H_v^a)^\dagger\gamma^0 = [(P_\mu^a)^\dagger\gamma^\mu - i(P_5^a)^\dagger\gamma_5]P_+, \end{aligned} \tag{8}$$

where

$$P_\pm \equiv \frac{1 \pm \gamma\cdot v}{2}, \qquad H_v\gamma\cdot v = -H_v, \qquad \gamma\cdot v\bar{H}_v = -\bar{H}_v. \tag{9}$$

The field $P_5^a(P_\mu^a)$ annihilates a heavy-light meson, $0^-(1^-)$, with velocity v. The index a runs over the light-quark flavors u, d, s, and the projection operators $P_\pm$ have the property

$$P_\pm\gamma^\mu P_\pm = \pm P_\pm v^\mu P_\pm. \tag{10}$$

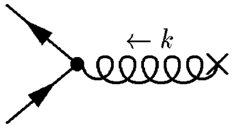
Note that in Refs. [3–5], $G_H = 1$ is used. However, in that case one uses a renormalization factor for the heavy-meson fields H_v, which is equivalent to the approach in [6] and here. The term $\propto 1/G_3$ is (partially) cancelled by a self-energy loop of order G_H^2, and it determines the mass difference between the heavy quark and the corresponding heavy meson.

In our model, the hard gluons are considered to be integrated out and we are left with soft gluonic degrees of freedom. These soft gluons can be described using the external field technique, and their effect will be parametrized by vacuum expectation values, such as the gluon condensate $\langle\frac{\alpha_s}{\pi}G^2\rangle$. Gluon condensates with higher dimensions could also be included, but we truncate the expansion by keeping only the one with lowest dimension. When calculating the soft gluon effects in terms of the gluon condensate, we follow the prescription given in [21]. The calculation is easily carried out in the Fock-Schwinger gauge, where one can expand the gluon field as

$$A_\mu^a(k) = -\frac{i(2\pi)^4}{2}G_{\rho\mu}(0)\frac{\partial}{\partial k_\rho}\delta^{(4)}(k) + \cdots. \tag{11}$$

Since each vertex in a Feynman diagram is accompanied by an integration, we get the Feynman rule given in Fig. 1. The gluon condensate is obtained by averaging in color space which yields the following replacement rule:

$$g_s^2G_{\mu\nu}^aG_{\alpha\beta}^b \to \frac{4\pi^2}{(N_c^2-1)}\delta^{ab}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\frac{1}{12} \times (g_{\mu\alpha}g_{\nu\beta} - g_{\mu\beta}g_{\nu\alpha}). \tag{12}$$



$$-\frac{1}{2} g_s t^a \gamma^\mu G_{\mu\rho}(0) \frac{\partial}{\partial k^\rho}\bigg|_{k^\rho=0} \quad + \cdots$$

FIG. 1. Feynman rule for the light quark–soft gluon vertex.

III. BOSONIZATION WITHIN THE HLχQM

The interaction term $\mathcal{L}_{\mathrm{Int}}$ in (7) can now be used to bosonize the model, i.e. to integrate out the quark fields. This can be done in the path integral formalism and the result is formally a functional determinant. This determinant can be expanded in terms of Feynman diagrams, by attaching the external fields H_v^a, $\bar{H}_v^a$, $\mathcal{V}^\mu$, $\mathcal{A}^\mu$, and $\tilde{M}_q$ of Sec. II to quark loops. Some of the loop integrals will be divergent, and analogously to the pure light sector case [12,16,22,23], they have to be related to physical parameters as will be described below. The resulting strong chiral Lagrangian for heavy-light chiral perturbation theory (χPT) has the following form [24–30]:

$$\mathcal{L}_{\mathrm{Str}} = -\mathrm{Tr}[\bar{H}_a(iv \cdot \mathcal{D})H_a] - g_{\mathcal{A}} \,\mathrm{Tr}[\bar{H}_a H_b \gamma_\mu \gamma_5 \mathcal{A}^\mu_{ba}] + \cdots, \tag{13}$$

where the dots indicate other terms of higher order in the chiral expansion, and the covariant derivative $\mathcal{D}$ contains both the photon field and the field $\mathcal{V}$. The $1/m_Q$ suppressed terms have been discarded in the present paper.

The Feynman diagrams responsible for the kinetic and axial vector terms in (13) are shown in Fig. 2. As mentioned in the Introduction, these two diagrams are calculated at zero external heavy-meson momentum. The nongluonic loop integral (first on Fig. 2) for the strong vector or axial vector current is

$$J_X^\mu = -N_c \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\{(-iG_H \bar{H}_v) iS_v(k) \times (-iG_H H_v) iS(k) \Gamma_X^\mu iS(k)\}, \tag{14}$$

where $\Gamma^\mu_{\mathcal{V}} = \gamma^\mu$ and $\Gamma^\mu_{\mathcal{A}} = \gamma^\mu\gamma_5$ for couplings to $X = \mathcal{V}$ and $X = \mathcal{A}$. Further, $S_v(k)$ and $S(k)$ are the heavy-quark propagator and the standard light-quark propagator, respectively:

$$S_v(k) = \frac{P_+(v)}{(v \cdot k - \Delta)}; \qquad S(k) = \frac{\gamma \cdot k + m}{(k^2 + m^2)}. \tag{15}$$

In previous papers [3,4,6], $\Delta = 0$ was assumed, but here we let $\Delta \neq 0$.

The gluonic part of the bosonized currents for one of the diagrams (lower left on Fig. 2) is

$$J^\mu_{X,G1} = -\int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\{(-iG_H \bar{H}_v) iS_v(k)(-iG_H H_v) iS(k) \times \Gamma^\mu_X iS(k)[ig_s \gamma^\alpha A_\alpha(q_2)] iS(k-q_2) \times [ig_s\gamma^\alpha A_\alpha(q_1)] iS(k-q_1-q_2)\}, \tag{16}$$

where gluon fields are represented by the expression from Eq. (11), and it is understood that the derivatives with respect to soft gluon momenta are to be applied to the whole integrand. There are two more diagrams with different ordering of gluon and (axial) vector vertices and after adding all four diagrams from Fig. 2, we obtain

$$J^\mu_{X,\mathrm{Tot}} = -g_X \,\mathrm{Tr}\{\bar{H}_v H_v \Gamma^\mu_X\}, \tag{17}$$

where

$$g_X = iG_H^2 N_c \bigg\{R_X - \frac{\pi^2}{24N_c} Z_X \bigg\langle \frac{\alpha_s}{\pi} G^2 \bigg\rangle\bigg\}, \tag{18}$$

$$R_{\mathcal{V}} = -2(m-\Delta)I_2 - I_{1,1} - 2\Delta(m-\Delta)I_{2,1}, \tag{19}$$

$$Z_{\mathcal{V}} = 144mI_4 + 192m^2(m-\Delta)I_5 + 24m(m+6\Delta)I_{4,1} + 192m^2\Delta(m-\Delta)I_{5,1}, \tag{20}$$

and for the axial case

$$R_{\mathcal{A}} = -\tfrac{2}{3}(3m-\Delta)I_2 + \tfrac{1}{3}I_{1,1} + \tfrac{2}{3}(m-\Delta)(2m-\Delta)I_{2,1}, \tag{21}$$

$$Z_{\mathcal{A}} = 48mI_4 + 64m^2(3m-\Delta)I_5 - 8m(13m-6\Delta)I_{4,1} - 64m^2(m-\Delta)(2m-\Delta)I_{5,1}. \tag{22}$$

The loop integrals I_n and $I_{n,r}$ occurring in the expressions above are defined in the Appendix, where also expressions for the finite ones are given. The integrals I_2, $I_{1,1}$ above and I_1 from Eq. (43) below are logarithmically, linearly, and quadratically divergent, respectively. Then, as explained in [6,12,16,22], these divergent integrals might be regularized, say, by ultraviolet cutoffs of the order of the chiral symmetry breaking scale Λ_χ [3,4,31]. The ultraviolet cutoff terms will be accompanied by various finite terms, which will be different for different cutoff procedures, such as those of Pauli-Villars–type [3,31] or proper time regularization [4,12,22]. We will, however, not go into details of this, but will simply identify the divergent inte-

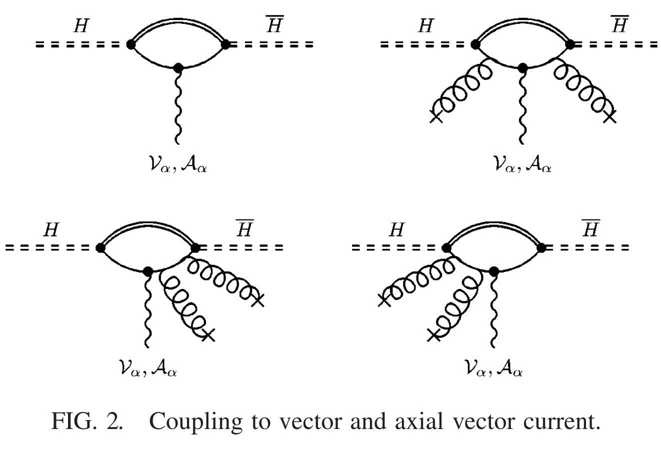


FIG. 2. Coupling to vector and axial vector current.

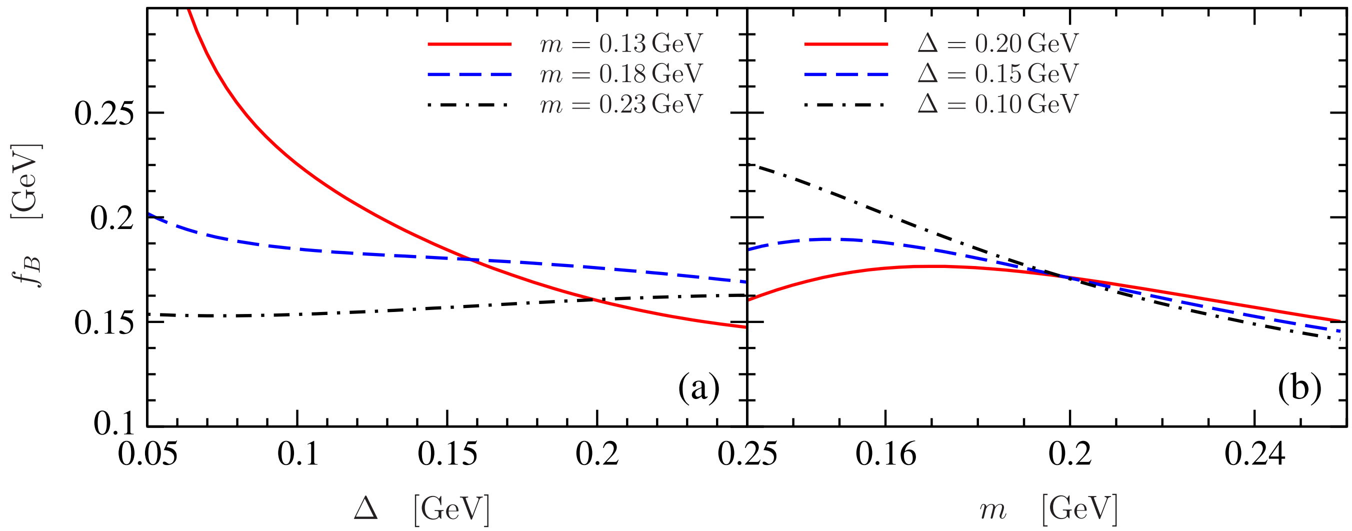


FIG. 3 (color online). The decay constant f_B in dependence on m and Δ. The condensates are taken to be $\langle\bar{q}q\rangle = (-0.27\ \text{GeV})^3$ and $\langle\frac{\alpha_s}{\pi}G^2\rangle = (0.32\ \text{GeV})^4$.

grals by appropriate quantities regarded as physical within our model.

Within the pure light sector, the logarithmically and quadratically divergent integrals are related to the pion decay constant f_π and the quark condensate $\langle\bar{q}q\rangle$ in the following way [16,22,23]:

$$f_\pi^2 = -i4m^2 N_c I_2 + \frac{1}{24m^2}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle, \tag{23}$$

$$\langle\bar{q}q\rangle = -4imN_c I_1 - \frac{1}{12m}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle. \tag{24}$$

This is obtained by relating loop diagrams to physical quantities analogously to Eqs. (29) and (30) below. (Here, the *a priori* divergent integrals I_1 and I_2 have to be interpreted as the regularized ones.) Since the pure light sector is a part of our model, we keep these relations in the heavy-light case studied here.

It should be noted that in principle the quark condensate $\langle\bar{q}q\rangle$, as well as the gluon condensate $\langle\frac{\alpha_s}{\pi}G^2\rangle$, depend on the renormalization scale μ. But when we are working in a model-dependent low-energy framework, the μ dependence of these quantities is lost. However, by construction, the chiral symmetry breaking scale Λ_χ is the effective ultraviolet cutoff for our description, including both HLχPT and HLχQM. Therefore, we use by assumption $\mu = \Lambda_\chi$ as the matching scale between perturbative, short distance description and the (model-dependent) long distance description.

In addition, in the heavy-light sector the (formally) linearly divergent integral $I_{1,1}$ will also appear. It will be related to the physical value of $g_\mathcal{V} - g_\mathcal{A}$ using Eq. (18) for $X = \mathcal{A}$ and $X = \mathcal{V}$. The negative parity axial coupling constant $g_\mathcal{A}$ is taken as model input parameter, $g_\mathcal{A} = 0.59$, while normalization of the kinetic term implies $g_\mathcal{V} = 1$.

Eliminating thus $I_{1,1}$ from the (18) and inserting the expression for I_2 obtained from (23), we find the following expression for G_H:

$$G_H^2 = \frac{2m}{f_\pi^2}\rho_\Delta, \tag{25}$$

where the quantity ρ_Δ is of order one and given by

$$\rho_\Delta \equiv \frac{1 + 3g_\mathcal{A}}{4(1 - \frac{\Delta}{2m} + \frac{N_c m^2}{8\pi f_\pi^2}\kappa_\Delta - \frac{\eta_1^\Delta}{m^2}\langle\frac{\alpha_s}{\pi}G^2\rangle)}, \tag{26}$$

where

$$\kappa_\Delta = i16\pi m\left(1 - \frac{\Delta}{m}\right)^2 I_{2,1}, \tag{27}$$

and

$$\eta_1^\Delta = \frac{1}{12}\left[1 - \frac{\Delta}{2m} + i\frac{\pi^2 m^3}{2}(Z_\mathcal{V} + 3Z_\mathcal{A})\right]. \tag{28}$$

Let us now consider these relations in two characteristic limiting cases: $\Delta \to 0$ and $\Delta \to m$.

In the limit $\Delta \to 0$, (18) reduces to

$$1 = -iG_H^2 N_c\left\{I_{3/2} + 2mI_2 + \frac{i\kappa_\mathcal{V}}{N_c m^3}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\right\}, \tag{29}$$

$$g_\mathcal{A} = iG_H^2 N_c\left\{\frac{1}{3}I_{3/2} - 2mI_2 - i\frac{m}{12\pi} - \frac{i\kappa_\mathcal{A}}{N_c m^3}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\right\}, \tag{30}$$

where

$$I_{3/2} \equiv (I_{1,1})_{\Delta\to 0}; \qquad \kappa_{\mathcal{V}} = -\kappa_{\mathcal{A}} = \frac{(8-3\pi)}{384}. \quad (31)$$

This is the result of [6], except for the sign of $\kappa_{\mathcal{A}}$ which is wrong there. Numerically, this change has no dramatic consequences. In the limit $\Delta \to 0$, we also have

$$\kappa_\Delta \to 1; \qquad \eta_1^\Delta \to \eta_1 = \frac{1}{12} - (\kappa_{\mathcal{V}} + 3\kappa_{\mathcal{A}}) = \frac{8-\pi}{64}. \quad (32)$$

Eliminating I_2, we obtain the relation for $I_{3/2}$:

$$-iN_c I_{3/2} = \frac{3(1-g_{\mathcal{A}})}{4G_H^2} - \frac{m}{16\pi} - \frac{3(\kappa_{\mathcal{A}} - \kappa_V)}{4m^3}\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle, \quad (33)$$

which will replace the Eq. (39) of [6].

In the limit $\Delta \to m$, which means that heavy and light quarks have the same *constituent* mass, we obtain

$$1 = G_H^2 \left\{ -iN_c I_{11} + \frac{1}{240m^3}\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \right\}, \quad (34)$$

$$g_{\mathcal{A}} = G_H^2 N_c \left\{ \frac{1}{3} iN_c I_{11} - \frac{4}{3} m I_2 - \frac{3}{240m^3}\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \right\}. \quad (35)$$

Then, for $\Delta \to m$, there is a simplification because $\kappa_\Delta \to 0$ and $\eta_1^\Delta \to 0$ and thereby

$$\rho_\Delta \to \tfrac{1}{2}(1 + 3g_{\mathcal{A}}). \quad (36)$$

The gluon condensate may be related to the matrix element of the chromomagnetic interaction [2]:

$$3\lambda_2 = \mu_G^2(H) = \frac{C_M(\mu)}{2M_H}\langle H|\bar{Q}_v \frac{1}{2}\sigma \cdot G Q_v|H\rangle = \frac{3}{2} m_Q (M_{H^*} - M_H). \quad (37)$$

Such a link was used in [6], but we will not use it because it formally belongs to $1/m_Q$ corrections, which are not considered here. Also, it turns out that such a choice does not lead to any dramatic differences, when numerical results in $\Delta \to 0$ limit are compared to those from [6].

Within the full theory (standard model) at quark level, the weak current is

$$J_f^\alpha = \bar{q}_f \gamma^\alpha (1-\gamma_5) Q, \quad (38)$$

where Q is the heavy-quark field in the full theory. Within HQEFT this current will, below the renormalization scale $\mu = m_Q (= m_b, m_c)$, be modified in the following way [2]:

$$J_f^\alpha = \bar{\chi}_a \xi_{af}^\dagger \Gamma^\alpha Q_v + \mathcal{O}(m_Q^{-1}), \quad (39)$$

where

$$\Gamma^\alpha = C_\gamma \gamma^\alpha L + C_v v^\alpha R. \quad (40)$$

Bosonizing this weak current, one obtains the standard expression:

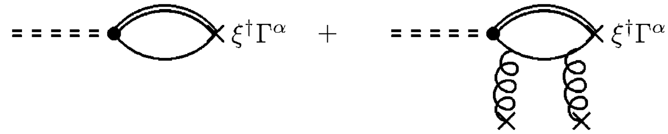


FIG. 4. Diagrams for bosonization of the left-handed quark current to leading order, determining α_H.

$$J_f^\alpha = \frac{\alpha_H}{2}\,\mathrm{Tr}[\xi_{hf}^\dagger \Gamma^\alpha H_{vh}], \quad (41)$$

where heavy-meson decay constant is given by

$$f_H = \frac{C_\gamma + C_v}{\sqrt{M_H}}\alpha_H, \quad (42)$$

with $C_\gamma = 1.077$ and $C_v = 0.0489$ being Wilson coefficients within HQEFT [2]. The decay constant f_B is plotted in Fig. 3. In addition, f_B and f_D have chiral corrections of order 20 MeV (see [6], and references therein), as well as $1/m_Q$ corrections not considered here. From the diagrams in Fig. 4, we obtain

$$\alpha_H = -2iG_H N_c \left\{ -I_1 + (m-\Delta) I_{1,1} + \frac{m\pi^2}{N_c}\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \times [-mI_4 + I_{3,1} + m(m-\Delta) I_{4,1}] \right\}, \quad (43)$$

where divergent integrals I_1 and $I_{1,1}$ will be expressed in terms of model parameters. Since this is the only predicted quantity depending on I_1, it is easy to accommodate physical values of f_H, using (24) and adjusting the quark condensate; see for example Eq. (45) below. In the limit $\Delta \to 0$,

$$\alpha_H \equiv -2iG_H N_c \left(-I_1 + mI_{3/2} + \frac{i\kappa_H}{N_c m^2}\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \right). \quad (44)$$

where $\kappa_H = (3\pi - 4)/384$, in agreement with [6]. Furthermore, eliminating divergent integrals, one obtains

$$\alpha_H = \frac{G_H}{2}\left(-\frac{\langle \bar{q}q\rangle}{m} - \frac{m^2}{4\pi} + \frac{3(1-g_A)}{2\rho} f_\pi^2 - \frac{(3\pi - 8)}{192m^2}\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \right). \quad (45)$$

In the limit $\Delta \to m$,

$$\alpha_H \equiv -2iG_H N_c \left(-I_1 + \frac{i}{96 N_c m^2}\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \right), \quad (46)$$

or, eliminating divergent integrals,

$$\alpha_H = \frac{G_H}{2}\left(-\frac{\langle \bar{q}q\rangle}{m} - \frac{\langle \frac{\alpha_s}{\pi} G^2 \rangle}{24m^2} \right). \quad (47)$$

This completes the specification and bosonization of the HLχQM. The remaining free parameters of the model are two condensates $\langle \bar{q}q\rangle$ and $\langle \frac{\alpha_s}{\pi} G^2 \rangle$ and two constituent masses m and Δ. We can now apply the model to calculation of phenomenological quantities, starting with the Isgur-Wise function.

IV. THE ISGUR-WISE FUNCTION

The Isgur-Wise function [1], $\xi(\omega)$, relates all the form factors describing the processes $B \to D(D^*)$ in the heavy-quark limit. In our framework, it can be defined via bosonization of heavy-heavy quark current responsible for $B \to D$ transition:

$$\bar{Q}^{(+)}_{v_b} \gamma^\mu L Q^{(+)}_{v_c} \to -\xi(\omega) \mathrm{Tr}[\bar{H}^{(+)}_c \gamma^\mu L H^{(+)}_b] \equiv J^\mu_{b\to c}. \tag{48}$$

Here, Q_{v_c} and Q_{v_c} are the c and b quark fields within HQEFT. The IW function can be determined by calculating the diagrams shown in Fig. 5.

One normally expects that emission of the soft gluons from a heavy quark does not occur at the zeroth order in $1/m_Q$. Namely, using the HQET Lagrangian (2), and differentiating the expressions involving heavy-quark propagators according (11), will naturally, for diagrams of same class as those on Fig. 4, lead to expressions proportional to $v_\mu v_\nu G^a_{\mu\nu} = 0$, where v_μ is either v_b or v_c. However, for the Isgur-Wise function there are two velocities (v_b and v_c) in play in the diagram. Therefore one may obtain contributions proportional to

$$v^\mu_b v^\nu_c G^a_{\mu\nu}, \tag{49}$$

which will generally, away from the strict heavy-quark limit $\omega \to 1$, be different from zero. Let us also mention that some care is required because momentum flow in the diagrams for the translationally noninvariant amplitudes in the Fock-Schwinger gauge is nontrivial; see Fig. 5.

The corresponding results for diagrams (a)–(d) are

$$\xi(\omega)_a = iG_H^2 N_c\left(\left(m - \frac{2}{\omega+1}\Delta\right)I_{1,1,1} - \frac{2}{\omega+1}I_{1,1}\right), \tag{50}$$

$$\xi(\omega)_b = iG_H^2\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle m\pi^2\left(m^2 I_{4,1,1} + I_{3,1,1} - \frac{2m}{\omega+1}(I_{4,1} + \Delta I_{4,1,1})\right), \tag{51}$$

$$\xi(\omega)_c = \frac{i}{12}G_H^2\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle m\pi^2(\omega-1)I_{2,2,2}, \tag{52}$$

$$\xi(\omega)_d = \frac{i}{12}G_H^2\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle \pi^2(\omega-1)(-I_{1,2,3} - I_{1,3,2} + (m(\omega+1) - 2\Delta)I_{1,3,3}). \tag{53}$$

Since loop integrals have at the least two heavy and one light-quark propagator, integrals of the type $I_{n,r,s}$ occur above, and they are defined in the Appendix. We find that the identity $\xi(\omega = 1) = 1$ follows from normalization of the vector current in Eq. (18):

$$\xi(\omega = 1) = (\xi_a + \xi_b + \xi_c + \xi_d)_{\omega=1} = (\xi_a + \xi_b)_{\omega=1} = 1. \tag{54}$$

Finally, for the slope of the IW function in the no-recoil limit $\omega \to 1$, we have

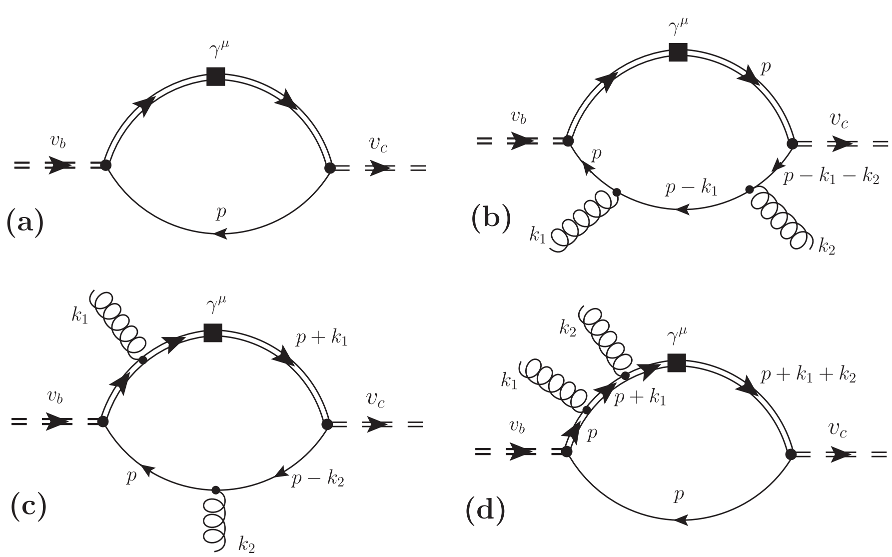


FIG. 5. Bosonization corresponding to the Isgur-Wise function.

$$\xi'(1)_a = iG_H^2 N_c(\tfrac{1}{2}I_{1,1} - (\tfrac{1}{3}m - \tfrac{5}{6}\Delta)I_{1,2} - \tfrac{2}{3}\Delta(m-\Delta)(I_{1,3} - 2m^2 I_{3,1})), \tag{55}$$

$$\xi'(1)_b = iG_H^2\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle m\pi^2\left(\frac{1}{4}I_{2,4} + \frac{m}{6}(m-\Delta)I_{3,4} + \frac{m}{2}(I_{4,1} + \Delta I_{4,2})\right), \tag{56}$$

$$\xi'(1)_c = iG_H^2\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\frac{m\pi^2}{12}I_{2,4} \tag{57}$$

$$\xi'(1)_d = -iG_H^2\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\frac{\pi^2}{6}(I_{1,5} - (m-\Delta)I_{1,6}). \tag{58}$$

All the $I_{n,r}$ integrals above can be evaluated using formulas from the Appendix.

V. RESULTS AND DISCUSSION

Numerical results are presented in Figs. 6 and 7. Figure 6 displays the slope $\xi'(1)$ of IW function as a function of the gluon condensate and the heavy-meson decay constant f_B as a function of the quark condensate, while Fig. 7 shows

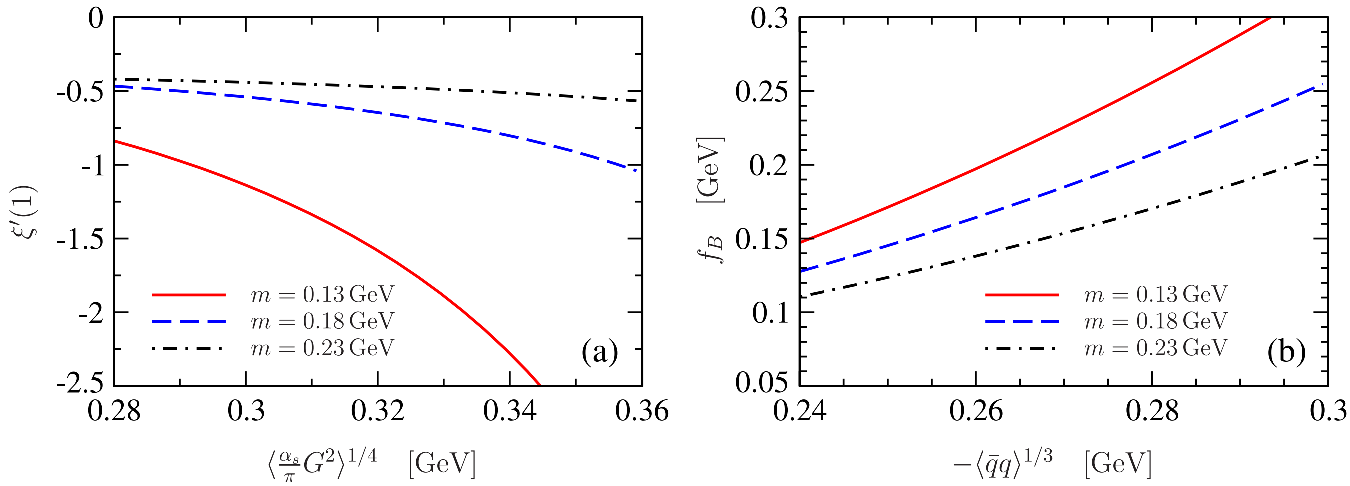


FIG. 6 (color online). The slope of Isgur-Wise function $\xi'(1)$ in dependence on the gluon condensate for various choices of the light-quark constituent mass m (a), and the decay constant f_B in dependence on the quark condensate (b), for $\Delta = 0.1$ GeV. Complementary plots are not so interesting because the dependence of $\xi'(1)$ on the *quark* condensate and the dependence of f_B on the *gluon* condensate are rather small.

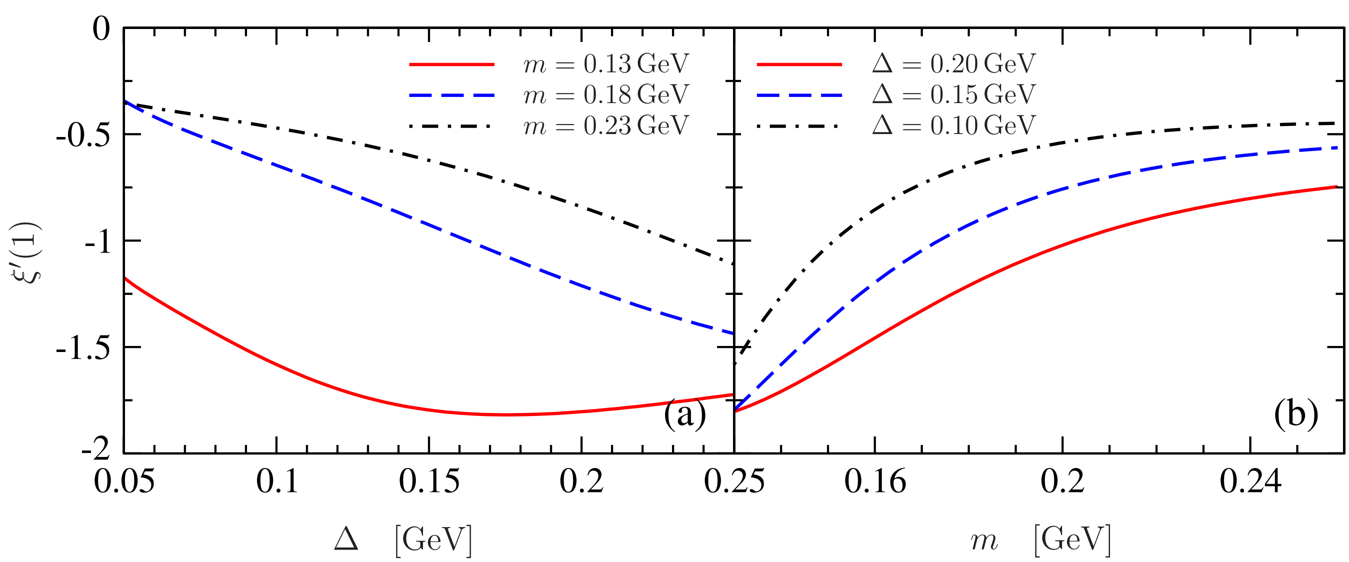


FIG. 7 (color online). The slope of Isgur-Wise function $\xi'(1)$ at no-recoil point as a function of Δ for three values of m (a), and as a function of m for three values of Δ (b). The condensates are taken to be $\langle\bar{q}q\rangle = (-0.27\ \text{GeV})^3$ and $\langle\frac{\alpha_s}{\pi}G^2\rangle = (0.32\ \text{GeV})^4$.

slope $\xi'(1)$ for some generic condensate values in dependence on the dynamical masses Δ and m. One observes that for reasonable intervals of masses[1] slope is of the order of -1 and beyond, which is in agreement with values mentioned in the Introduction, as well as with values obtained within other theoretical frameworks [32] and on the lattice [33]. It should be noticed that values for $\xi'(1)$ of order or bigger than one prefer smaller constituent mass than in [6]. For easier overview of analytical results, as well as for their numerical checks, it is convenient to again investigate limit $\Delta \to m$. After using constant values for integrals in this limit, as given in the Appendix one obtains simple expression

$$\xi'(1) = \frac{(3g_{\mathcal{A}} - 1)}{4} - \frac{(1 + 3g_{\mathcal{A}})}{4f_\pi^2} \times \left\{\frac{m^2 N_c}{2\pi^2} + \frac{11}{315m^2}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\right\}. \tag{59}$$

Numerical values in this limit are not unreasonable.

In conclusion, we have presented an improved heavy-light chiral quark model where introducing additional mass parameter in the heavy-quark propagator resulted in a flexible model capable of consistent description of heavy-meson decays, where we placed particular emphasis on a characterization of Isgur-Wise function. Further applications of the model, such as calculation of nonfactorizable amplitudes for nonleptonic heavy-meson decays could now be attempted. As the slope of the IW function is steeper than the one used in, say, Ref. [19], the partial amplitudes for $B \to D\bar{D}$ depending on the IW function, might be overestimated there. This will then have consequences for the size of the overall amplitude.

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APPENDIX: LOOP INTEGRALS

Three-, two-, and one-point loop integrals with one light-quark propagator occurring in the calculations are

$$I_{n,r,s}^{\alpha\beta\cdots} = \int \frac{d^4p}{(2\pi)^4} \frac{p^\alpha p^\beta \cdots}{(p^2 - m^2 + i\epsilon)^n (p\cdot v - \Delta + i\epsilon)^r (p\cdot v' - \Delta + i\epsilon)^s}, \tag{A1}$$

$$I_{n,r}^{\alpha\beta\cdots} = \int \frac{d^4p}{(2\pi)^4} \frac{p^\alpha p^\beta \cdots}{(p^2 - m^2 + i\epsilon)^n (p\cdot v - \Delta + i\epsilon)^r}, \tag{A2}$$

$$I_{n}^{\alpha\beta\cdots} = \int \frac{d^4p}{(2\pi)^4} \frac{p^\alpha p^\beta \cdots}{(p^2 - m^2 + i\epsilon)^n}. \tag{A3}$$

To evaluate such integrals, one first reduces tensor to scalar ones using relations ($\omega \equiv v\cdot v'$):

$$I_{n,s}^\alpha = (I_{n,s-1} + \Delta I_{n,s})v^\alpha, \tag{A4}$$

$$I_{n,1}^{\alpha\beta} = \tfrac{1}{3}[I_{n-1,1} + m^2 I_{n,1} - \Delta I_n - \Delta^2 I_{n,1}]g^{\alpha\beta} - \tfrac{1}{3}[I_{n-1,1} + m^2 I_{n,1} - 4\Delta I_n - 4\Delta^2 I_{n,1}]v^\alpha v^\beta, \tag{A5}$$

$$\begin{aligned} I_{n,1}^{\alpha\beta\gamma} = {} & \tfrac{1}{2}[-I_{n-1} - (m^2 - 4\Delta^2)I_n - 2\Delta I_{n-1,1} - 2\Delta(m^2 - 2\Delta^2)I_{n,1}]v^\alpha v^\beta v^\gamma \\ & + \tfrac{1}{3}[\tfrac{3}{4}I_{n-1} + (\tfrac{3}{4}m^2 - \Delta^2)I_n + \Delta I_{n-1,1} + \Delta(m^2 - \Delta^2)I_{n,1}](v^\alpha g^{\beta\gamma} + v^\beta g^{\alpha\gamma} + v^\gamma g^{\alpha\beta}) \end{aligned} \tag{A6}$$

$$I_{n,r,s}^\alpha = \frac{1}{\omega^2 - 1}\{(\omega I_{n,r,s-1} - I_{n,r-1,s})v^\alpha + (\omega I_{n,r-1,s} - I_{n,r,s-1})v'^\alpha + \Delta(\omega - 1)I_{n,r,s}(v^\alpha + v'^\alpha)\}. \tag{A7}$$

(Reduction of one-point $I_n^{\alpha\beta\cdots}$ tensor integrals is simple and well known.) Now all scalar two-point integrals $I_{n,s}$ can be reduced to linear combinations of $I_{n,1}$ integrals using general recursion formula

$$I_{n,s} = \frac{-4n}{s^2 - 3s + 2}\left\{(n + s - 3)I_{n+1,s-2} + m^2(n+1)I_{n+2,s-2} + \Delta\frac{s^2 - 3s + 2}{s - 1}I_{n+1,s-1}\right\}, \tag{A8}$$

valid for $s > 2$, whereas the special case $s = 2$ is

[1]Constituent mass of quarks in the presented model is smaller than in other similar quark models due to the explicit inclusion of gluon condensate in the dynamics of the model.

$$I_{n,2} = -2n(I_{n+1} + \Delta I_{n+1,1}). \tag{A9}$$

For the calculation of the slope $\xi'(1)$ of Isgur-Wise function, one additionally needs derivatives of three-point integrals at point $\omega = 1$. Integrals themselves at this point are trivially given by

$$I_{n,r,s}(1) = I_{n,r+s}. \tag{A10}$$

The derivatives are given by

$$\left.\frac{\partial I_{n,r,s}(\omega)}{\partial \omega}\right|_{\omega=1} = \frac{rs}{2(n-1)} I_{n-1,r+s+2} \quad \text{for } n > 1, \tag{A11}$$

while for $n = 1$ we have a special case:

$$\left.\frac{\partial I_{1,r,s}(\omega)}{\partial \omega}\right|_{\omega=1} = -\frac{rs}{r+s+1}\left(2\frac{r+s-1+\epsilon}{r+s} I_{1,r+s} + 2\Delta I_{1,r+s+1} + \frac{2m^2}{r+s} I_{2,r+s}\right), \tag{A12}$$

where dimensional regularization parameter ϵ is relevant only for divergent case $r = s = 1$, where using $\lim_{\epsilon\to 0}\epsilon I_{1,2} = -i/(8\pi)$ one gets

$$\left.\frac{\partial I_{1,1,1}(\omega)}{\partial \omega}\right|_{\omega=1} = -\frac{1}{3}(I_{1,2} + 2\Delta I_{1,3} - 4\Delta m^2 I_{3,1}). \tag{A13}$$

Then again, recursion formulas above can be used to reduce everything to $I_{n,1}$ integrals. These integrals can be explicitly evaluated, and they read

$$I_{n,1} \equiv \frac{i}{16\pi^2 m^{2n-3}} a_n\left(\frac{\Delta}{m}\right), \quad \text{where} \tag{A14}$$

$$a_2(x) = -\frac{1}{1-x^2}\mathcal{F}(x) \quad \overset{x\to 1}{\longrightarrow} -2, \tag{A15}$$

$$a_3(x) = \frac{1}{4}\frac{1}{(1-x^2)^2}(\mathcal{F}(x) - 2x + 2x^3) \quad \overset{x\to 1}{\longrightarrow} \frac{1}{3}, \tag{A16}$$

$$a_4(x) = -\frac{1}{24}\frac{1}{(1-x^2)^3}(3\mathcal{F}(x) - 10x + 14x^3 - 4x^5) \overset{x\to 1}{\longrightarrow} -\frac{2}{15}, \tag{A17}$$

$$a_5(x) = \frac{1}{192}\frac{1}{(1-x^2)^4}(15\mathcal{F}(x) - 66x + 118x^3 - 68x^5 + 16x^7) \overset{x\to 1}{\longrightarrow} \frac{1}{14}, \tag{A18}$$

$$a_6(x) = -\frac{1}{1920}\frac{1}{(1-x^2)^5}(105\mathcal{F}(x) - 558x + 1210x^3 - 1052x^5 + 496x^7 - 96x^9) \overset{x\to 1}{\longrightarrow} -\frac{2}{45}. \tag{A19}$$

Here, the function $\mathcal{F}(x)$ is [34]

$$\mathcal{F}(x) = \sqrt{x^2 - 1 + i\epsilon}(\log(x - \sqrt{x^2 - 1 + i\epsilon}) - \log(x + \sqrt{x^2 - 1 + i\epsilon})). \tag{A20}$$

Note that $\mathcal{F}(x) \overset{x\to 0}{\longrightarrow} \pi$, which gives the various $a_n(0)$. Furthermore, $\mathcal{F}(x) = -2xF(1/x)$, where $F(1/x)$ is function from Eq. (A2) of [35].

This reduction of integrals is easy to implement on a computer and the corresponding MATHEMATICA code is available from the authors upon request.

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