

# Exotic seesaw-motivated heavy leptons at the LHC

---

**Kumerički, Krešimir; Picek, Ivica; Radovčić, Branimir**

*Source / Izvornik:* **Physical Review D - Particles, Fields, Gravitation and Cosmology, 2011, 84**

**Journal article, Published version**

**Rad u časopisu, Objavljena verzija rada (izdavačev PDF)**

<https://doi.org/10.1103/PhysRevD.84.093002>

*Permanent link / Trajna poveznica:* <https://um.nsk.hr/um:nbn:hr:217:829877>

*Rights / Prava:* [In copyright](#)/[Zaštićeno autorskim pravom.](#)

*Download date / Datum preuzimanja:* **2025-03-31**



*Repository / Repozitorij:*

[Repository of the Faculty of Science - University of Zagreb](#)



**Exotic seesaw-motivated heavy leptons at the LHC**

Krešimir Kumerički, Ivica Picek, and Branimir Radovčić

*Department of Physics, Faculty of Science, University of Zagreb, P.O.B. 331, HR-10002 Zagreb, Croatia*

(Received 10 June 2011; published 9 November 2011; publisher error corrected 6 August 2012)

We study the LHC potential for discovering TeV-scale  $SU(2)_L$  5-plet fermions introduced recently to explain small neutrino masses. We show that the Drell-Yan production and the decays of new exotic  $\Sigma$  leptons are testable at the LHC. Their production is abundant due to nontrivial electroweak gauge charges. For  $1 \text{ fb}^{-1}$  of integrated luminosity at the present LHC  $\sqrt{s} = 7 \text{ TeV}$ , there can be 270  $\Sigma - \bar{\Sigma}$  pairs produced for  $M_\Sigma = 400 \text{ GeV}$ . Besides producing same-sign dilepton events, they could lead, due to a chosen small mixing between heavy and light leptons, to  $\sim 10$  golden decays  $\Sigma^{+++} (\bar{\Sigma}^{+++}) \rightarrow W^\pm W^\pm l^\pm$  with a specific decay signature.

DOI: [10.1103/PhysRevD.84.093002](https://doi.org/10.1103/PhysRevD.84.093002)

PACS numbers: 14.60.Hi, 14.60.Pq, 14.60.St

**I. INTRODUCTION**

The start-up of the Large Hadron Collider (LHC) has enabled the exploration of previously inaccessible domains of heavy particles, which are at the core of various extensions of the standard model (SM). A distinguished class of modifications of the particle content of the SM attributes the smallness of the masses of observed neutrinos to the presence of a seesaw mechanism. In such a setup the neutrino masses can be realized through an effective dimension-five operator  $LLHH$  [1], which can be generated both at the tree and at the loop level. At the tree level there are three realizations of the dimension-five operator [2], all of them based on an addition of a single new multiplet: Type-I [3], type-II [4] and type-III [5] seesaw mechanisms, mediated by heavy fermion singlet, scalar triplet, and a fermion triplet, respectively. Thereby, the fermionic type-I and III models involve hypercharge zero heavy Majorana mediators. Originally, the seesaw scale was linked to high energy GUT scale, while a recent reincarnation of  $SU(5)$  GUT model which within the adjoint 24 representation contains both a singlet and hypercharge zero fermion triplet, providing a low scale hybrid type-I and III seesaw model [6]. In a multiple seesaw approach first put forward in [7] and later extended in [8], the involved additional fields and additional discrete symmetries are instrumental in bringing the seesaw mechanism to the TeV scale [9] accessible at the LHC.

In the recently proposed approach of [10], which we are scrutinizing here, the lowering of the seesaw scale has been achieved without use of discrete symmetries. By relying only on the gauge symmetry and the renormalizability of the SM, we arrived at novel seesaw mechanism supported by nonzero hypercharge fermions in higher isomultiplets.

Such a line of research follows from our examination [11] of the vectorlike top partner, which by a sort of Dirac seesaw mechanism increases the mass of the top [12]. We then extended our quark-sector study to the leptons, to

explore a possible role of TeV-scale vectorlike Dirac fermions as seesaw mediators.

Indeed, the heaviness of the top quark and the lightness of the Higgs boson have been the landmarks for several extensions of the SM which rely on vectorlike rather than on sequential fourth generation fermions. We have observed that there are two options that such states, in conjunction with appropriate new scalar fields, act as seesaw mediators: via the triplet fermions already introduced in [13] and via the fermion 5-plet proposed in [10]. The dimension nine (dim 9) operator produced at the tree level in the latter case leads to the seesaw formula  $m_\nu \sim v^6/\Lambda_{\text{NP}}^5$ , which reproduces the empirical neutrino masses  $m_\nu \sim 10^{-1} \text{ eV}$  with sub-TeV 5-plet mass  $M_\Sigma$ . This calls to investigate the phenomenology of new Dirac-type heavy leptons as the subject of our present study. Let us note that the focus of a study in [13] with their exotic nonzero hypercharge triplet fermions has been on the accompanied triply-charged scalars, whereas a more recent study [14] considered the effects of these triplet fermions on the LFV decays of charged SM leptons. In addition, some further constraints on exotic triplets have been placed very recently in [15,16].

The present study of exotic 5-plet fermions can be compared to previous studies of hypercharge-zero triplets belonging to type-III seesaw mechanism: The earlier one performed by Franceschini *et al.* [17] and a subsequent by Li and He [18], which has some overlap with more recent study [19] of type-III triplets appearing also in already mentioned hybrid type-I and III seesaw model [6].

This article is organized as follows. In Sec. II we describe a tree-level seesaw model based on nonzero hypercharge fermions and point out the nonminimal (conjunct) appearance of exotic fermions and scalars needed for dim 7 and dim 9 seesaw mechanisms. Sec. III deals with the production rates, and Sec. IV with the subsequent decays of exotic 5-plet states. We summarize our results in Sec. V. The details on the mixing between light and heavy leptons are exposed in the Appendix.

## II. TEV-SCALE SEESAW MODEL WITH HIGHER ISOMULTIPLETS

As a distinctive feature of the SM, the observed fundamental matter fields (quarks and leptons) are distributed in the lowest possible representations of the SM group,  $SU(3)_C \times SU(2)_L \times U(1)_Y$ . In addition to the leptonic left-handed doublets and the right-handed singlets, there is also the SM Higgs doublet,  $H = (H^+, H^0)^T$ . The values  $(2T + 1, Y)$  of the weak isospin  $T$  and hypercharge  $Y$ , indicated in the parentheses,

$$L_L \sim (2, -1), \quad l_R \sim (1, -2), \quad H \sim (2, 1), \quad (1)$$

reproduce the electric charges  $Q = T_3 + Y/2$  of the respective multiplet components.

Higher, isotriplet multiplets  $\sim(3, 2)$  and  $\sim(3, 0)$  are already familiar from type-II and III seesaw models, and it is obvious that we may encounter a 5-plet of zero hypercharge as the simplest generalization of type-III seesaw Majorana triplet to a higher-isospin multiplet. Incidentally, such a  $\sim(5, 0)$  fermion multiplet has been singled out in the literature as a viable minimal dark matter (MDM) candidate [20]. It provides a completely neutral ( $T_3 = 0$  and  $Y = 0$ ) component of an additional  $SU(2)_L$  multiplet, and the minimality refers to its isolated appearance: MDM is realized as a single additional exotic matter particle coupled to the SM fields.

However, our concern here is with the seesaw models where exotic fermionic seesaw mediators appear together with appropriate exotic scalar isomultiplets. Thereby, in order to add something new to already existing seesaw mechanisms, we employ vectorlike fermionic multiplets with nonzero hypercharge. In contrast to chiral SM fermions, the masses of such vectorlike fermions are not restricted to the electroweak scale, so that they can be naturally adjusted to the new physics scale. If in a novel seesaw mechanism the new physics appears already around the TeV scale, then this scenario would not suffer from an additional, seesaw-induced hierarchy problem. In order to produce a tree-level seesaw, the newly introduced fermion multiplets have to have a neutral component which will mix with light neutrinos. Accompanying new scalar multiplets also have to have a neutral component which can acquire the vacuum expectation value ( $v_{ev}$ ). As exposed to more detail in the next subsection, these requirements together with the SM gauge symmetry lead to rather restricted viable quantum number assignments for the new multiplets: the one with vectorlike triplet fermions introduced in [13] leading to dim 7 seesaw operator, and the vectorlike fermion 5-plet introduced by [10] leading to dim 9 seesaw operator.

### A. Dim 9 seesaw model with fermionic 5-plets

The effective operators of the form  $(LLHH)(H^\dagger H)^n$  have already been studied in [21,22], and more recently in [23], where it has been pointed out that the mass

operator at each higher dimension is unique, independent of the mechanism leading to it.

The model pursued here starts from three generations of SM leptons represented by  $L_L$  and  $l_R$  in Eq. (1), and adds to them  $n_\Sigma$  isospin  $T = 2$  vectorlike 5-plets of leptons with hypercharge two, where both left and right components transform as  $(5, 2)$  under the electroweak part of the SM group,

$$\Sigma_{L,R} = \begin{pmatrix} \Sigma^{+++} \\ \Sigma^{++} \\ \Sigma^+ \\ \Sigma^0 \\ \Sigma^- \end{pmatrix}_{L,R} \sim (5, 2). \quad (2)$$

Besides the SM Higgs doublet  $H$  in Eq. (1) there are two additional isospin  $T = 3/2$  scalar multiplets  $\Phi_1$  and  $\Phi_2$  transforming as  $(4, -3)$  and  $(4, -1)$ , respectively:

$$\Phi_1 = \begin{pmatrix} \phi_1^0 \\ \phi_1^- \\ \phi_1^{--} \\ \phi_1^{---} \end{pmatrix} \sim (4, -3), \quad \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \\ \phi_2^- \\ \phi_2^{--} \end{pmatrix} \sim (4, -1). \quad (3)$$

Explaining the neutrino masses by dim 9 operator leads to the exotic matter shown in Eqs. (2) and (3). Concerning the fermion fields  $\Sigma$ , they should be vectorlike in order to avoid the chiral anomaly. The extra scalar fields are constrained by a requirement of the renormalizability of the extended SM, restricting the Lagrangian of the scalar sector to dimension 4 terms. Accordingly, the weak isospin of the exotic scalar fields ( $\Phi_{1,2}$ ) can not be higher than three half, in order that they couple linearly with the Higgs field and develop an induced  $v_{ev}$  after the spontaneous symmetry breaking of the SM gauge group. For this reason, the scalar fields should be in multiplets which contain electrically neutral components.

Since the sought-after dim 9 operator can be relevant only in the absence of possible dim 5 operators, we forbid the appearance of those states which generate conventional seesaw mechanisms. In first place, we should avoid a scalar triplet  $\Delta \sim (3, 2)$  generating type-II seesaw, and a fermion singlet  $N_R \sim (1, 0)$  or triplet  $N_R \sim (3, 0)$  generating type-I and III seesaw, respectively, (see the Table I). Note that type-I, II, and III mechanisms correspond to already mentioned minimal (single) appearance of exotic (EX) scalar/fermionic particles in the Yukawa term

$$\begin{aligned} \mathcal{L}_Y^{\text{minimal}} &\sim (\text{SM-fermion})(\text{SM-scalar/fermion}) \\ &\times (\text{EX-fermion/scalar}) \end{aligned} \quad (4)$$

Also, a recent study of exotic triplets in [15,16] restricts to such minimal Yukawa term. We generalize Eq. (4) to new Yukawa terms with double appearance of exotic particles,

TABLE I. The assignments of electroweak charges for exotic particles leading to the tree-level seesaw operator up to dim 9.

Seesaw Type	Exotic Fermion	Exotic Scalar	Scalar Coupling	$m_\nu$ at
Type-I	$N_R \sim (1, 0)$	-	-	dim 5
Type-II	-	$\Delta \sim (3, 2)$	$\mu\Delta HH$	dim 5
Type-III	$N_R \sim (3, 0)$	-	-	dim 5
Conjunct Mediator	Exotic Fermion Pair	Exotic Scalars $\Phi_1, \Phi_2$	Scalar-Higgs Couplings	$m_\nu$ at
doublet	$\Sigma_{L,R}(2, 1)$	$(3, -2), (3, 0)$	$\mu_{1,2}\Phi_{1,2}HH$	dim 5
triplet	$\Sigma_{L,R}(3, 2)$	$(4, -3), (2, -1)$	$\lambda_1\Phi_1 HHH$	dim 7
4-plet	$\Sigma_{L,R}(4, 1)$	$(3, -2), (3, 0)$	$\mu_{1,2}\Phi_{1,2}HH$	dim 5
5-plet	$\Sigma_{L,R}(5, 2)$	$(4, -3), (4, -1)$	$\lambda_{1,2}\Phi_{1,2} HHH$	dim 9

$$\mathcal{L}_Y^{\text{conjunct}} \sim (\text{SM-fermion})(\text{EX-scalar})(\text{EX-fermion}), \quad (5)$$

where a conjunct exotic scalar-fermion pair can lead to the neutrino mass operators of dimension larger than 5, as shown in Table I.

Consequently, the focus here is on nonzero hypercharge Dirac fermions  $\Sigma_{L,R}$  from the last row of Table I, playing the role of the seesaw mediators which produce the mechanism displayed on Fig. 1. The seesaw diagram on this figure arises due to the Yukawa couplings explicated in Eq. (6) and the scalar-field couplings  $\lambda_1$  and  $\lambda_2$  explicated in Eq. (13). Note that it involves a decomposition of the appropriate couplings in terms of the Clebsch-Gordan coefficients.

In order to avoid already known seesaw mechanisms, there is a gap between the quantum numbers of the SM fields and the EX fields. To overbridge this gap and to obtain the gauge invariant interaction in Eq. (5), the conjunct EX scalar-fermion pair should form an object with the weak isospin half and the hypercharge one. One possible way to achieve this is shown in the doublet and 4-plet ‘‘Seesaw Type’’ rows in Table I, where new fermion fields of isospin one half (doublet) or three half (4-plet) hold together with a scalar of isospin one (triplet). However, the latter appears already in type-II seesaw and would lead to dim 5 operator for neutrino masses. Then we are left with new fermion fields, which should be of either isospin one (triplet) or isospin two (5-plet). Whereas the first option corresponds to dim 7 operator introduced by Babu *et al.* [13], we adopt the latter case corresponding to

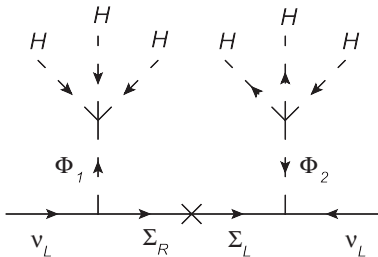


FIG. 1. Dim 9 tree-level seesaw diagram produced by exotic fields in Eqs. (2) and (3).

dim 9 operator which might be more accessible for tests at the LHC. The requirement of employing the Dirac fermions as tree-level seesaw mediators restricts their hypercharge to  $Y = 2$  for both the triplet and the 5-plet case.

### B. Mass matrices of the 5-plet states

For our choice of EX fields in Eqs. (2) and (3), the conjunct Yukawa interaction in Eq. (5) is explicated by the second row in the following gauge invariant Lagrangian:

$$\begin{aligned} \mathcal{L} = & \overline{\Sigma}_L i \not{D} \Sigma_L + \overline{\Sigma}_R i \not{D} \Sigma_R - \overline{\Sigma}_R M_\Sigma \Sigma_L - \overline{\Sigma}_L M_\Sigma^\dagger \Sigma_R \\ & + (\overline{\Sigma}_R Y_1 L_L \Phi_1^* + \overline{\Sigma}_L^c Y_2 L_L \Phi_2 + \text{H.c.}). \end{aligned} \quad (6)$$

This contains the bare Dirac mass terms for the introduced vectorlike lepton 5-plet fields

$$\begin{aligned} -\mathcal{L}_D = & \overline{\Sigma}_R^{+++} M_\Sigma \Sigma_L^{+++} + \overline{\Sigma}_R^{++} M_\Sigma \Sigma_L^{++} + \overline{\Sigma}_R^+ M_\Sigma \Sigma_L^+ \\ & + \overline{\Sigma}_R^0 M_\Sigma \Sigma_L^0 + \overline{\Sigma}_R^- M_\Sigma \Sigma_L^- + \text{H.c.} \end{aligned} \quad (7)$$

Additional mass terms arise from the Yukawa part in Eq. (6). As discussed shortly, an analysis of the scalar potential shows that the neutral components of two scalar quadruplets  $\Phi_1^0$  and  $\Phi_2^0$  develop the induced  $\nu\nu s$ ,  $\nu_{\Phi_1}$  and  $\nu_{\Phi_2}$ , respectively. This means that Yukawa terms in Eq. (6) lead to the mass terms connecting the SM lepton doublet with new vectorlike lepton 5-plet. Thereby, the triply- and doubly-charged states do not mix with SM leptons whereas the singly-charged and neutral states have peculiar mixing with SM leptons which we consider in detail in the Appendix.

When extracting the mass matrices produced by the mass and Yukawa terms, we suppress the generation indices of the fields. Three neutral left-handed fields  $\nu_L$ ,  $\Sigma_L^0$  and  $(\Sigma_R^0)^c$  span the symmetric neutral mass matrix as follows:

$$\begin{aligned} \mathcal{L}_{\nu\Sigma^0} = & -\frac{1}{2} \overline{(\nu_L)^c} \overline{(\Sigma_L^0)^c} \overline{\Sigma_R^0} \begin{pmatrix} 0 & m_2^T & m_1^T \\ m_2 & 0 & M_\Sigma^T \\ m_1 & M_\Sigma & 0 \end{pmatrix} \begin{pmatrix} \nu_L \\ \Sigma_L^0 \\ (\Sigma_R^0)^c \end{pmatrix} \\ & + \text{H.c.} \end{aligned} \quad (8)$$

For singly-charged fermions we arrive at the nonsymmetric mass matrix, which explicates the mixing between the SM leptons and new singly-charged states

$$\mathcal{L}_{l\Sigma} = -(\overline{l}_R \overline{\Sigma}_R^-(\Sigma_L^+)^c) \begin{pmatrix} m_1 & 0 & 0 \\ m_3 & M_\Sigma & 0 \\ m_4 & 0 & M_\Sigma^T \end{pmatrix} \begin{pmatrix} l_L \\ \Sigma_L^- \\ (\Sigma_R^+)^c \end{pmatrix} + \text{H.c.} \quad (9)$$

Here  $M_\Sigma$  is given in Eq. (7), while  $m_3$  and  $m_4$  entries in Eq. (9), and  $m_1$  and  $m_2$  in the neutral sector in Eq. (8), come from the respective terms in Eq. (6):

$$\begin{aligned} m_1 &= \sqrt{\frac{1}{10}} Y_1 v_{\Phi_1}^*, & m_2 &= -\sqrt{\frac{3}{20}} Y_2 v_{\Phi_2}, \\ m_3 &= \sqrt{\frac{2}{5}} Y_1 v_{\Phi_1}^*, & m_4 &= \sqrt{\frac{1}{10}} Y_2 v_{\Phi_2}. \end{aligned} \quad (10)$$

Accordingly, the masses  $m_i$  ( $i = 1, 2, 3, 4$ ) are determined by the  $vev$   $v_{\Phi_1}$  and  $vev$   $v_{\Phi_2}$  of the neutral components of the scalar quadruplets, whereas  $M_\Sigma$  is on the new physics scale  $\Lambda_{\text{NP}}$ , which is larger than the electroweak scale.

The diagonalization of the mass matrices proceeds by unitary transformations presented in the Appendix. They are the result of a procedure developed in [24], whereby we write the leading order expressions up to  $M_\Sigma^{-2}$  in the basis where the matrices  $m_i$  and  $M_\Sigma$  are already diagonalized. Block diagonalization of the neutral mass matrix in Eq. (8) gives to this order

$$\tilde{m}_\nu \simeq -m_2^T M_\Sigma^{-1} m_1 - m_1^T M_\Sigma^{-1} m_2, \quad (11)$$

where a familiar unitary  $V(3 \times 3)_{\text{PMNS}}$  matrix diagonalizes the obtained effective light neutrino mass matrix:

$$V_{\text{PMNS}}^T \tilde{m}_\nu V_{\text{PMNS}} = m_\nu. \quad (12)$$

The same formalism has been applied in [18,25] and the results in our Appendix agree with theirs when taking the appropriate limits.

### C. DIM 9 WRT DIM 5 LOOP-LEVEL SEESAW MECHANISMS

Turning to the scalar potential, we restrict ourselves to already mentioned renormalizable terms relevant for our mechanism:

$$\begin{aligned} V(H, \Phi_1, \Phi_2) &\sim \mu_H^2 H^\dagger H + \mu_{\Phi_1}^2 \Phi_1^\dagger \Phi_1 + \mu_{\Phi_2}^2 \Phi_2^\dagger \Phi_2 \\ &+ \lambda_H (H^\dagger H)^2 + \{\lambda_1 \Phi_1^* H^* H^* H^* + \text{H.c.}\} \\ &+ \{\lambda_2 \Phi_2^* H H^* H^* + \text{H.c.}\} \\ &+ \{\lambda_3 \Phi_1^* \Phi_2 H^* H^* + \text{H.c.}\}. \end{aligned} \quad (13)$$

The electroweak symmetry breaking proceeds in the usual way from the  $vev$   $v = 174$  GeV of the Higgs doublet, implying  $\mu_H^2 < 0$ . On the other hand, the electroweak  $\rho$  parameter dictates the  $vevs$   $v_{\Phi_1}$  and  $v_{\Phi_2}$  to be small,

implying  $\mu_{\Phi_1}^2, \mu_{\Phi_2}^2 > 0$ . However, the  $\lambda_1$  and  $\lambda_2$  terms in Eq. (13) result in the induced  $vevs$  for the isospin 3/2 scalar multiplets,

$$v_{\Phi_1} \simeq -\lambda_1 \frac{v^3}{\mu_{\Phi_1}^2}, \quad v_{\Phi_2} \simeq -\lambda_2 \frac{v^3}{\mu_{\Phi_2}^2}. \quad (14)$$

Let us stress that we allow only the SM scalar doublet to wear a Mexican hat, and that the terms linear in the extra scalar fields, which induce their  $vevs$ , only slightly deform the total scalar potential without affecting the  $vev$  of the SM Higgs. Generically, the potential in Eq. (13) is at large field values dominated by the terms quartic in fields, and with a number of free parameters in the potential there is no problem to ensure the stability of the SM vacuum.

The nonvanishing  $vevs$  in Eq. (14) change the electroweak  $\rho$  parameter to  $\rho(\Phi_1) \simeq 1 - 6v_{\Phi_1}^2/v^2$  and  $\rho(\Phi_2) \simeq 1 + 6v_{\Phi_2}^2/v^2$ , respectively. If these  $vevs$  are taken separately, the experimental value  $\rho = 1.0000_{-0.0007}^{+0.0011}$  [26] leads to the upper bounds  $v_{\Phi_1} \leq 1.9$  GeV and  $v_{\Phi_2} \leq 2.4$  GeV, respectively. Thus, the value of a few GeV can be considered as an upper bound on both  $v_{\Phi_1}$  and  $v_{\Phi_2}$ , if there is no fine-tuning between them. By merging Eqs. (10), (11), and (14), we obtain for the light neutrino mass

$$m_\nu \sim \frac{Y_1 Y_2 \lambda_1 \lambda_2 v^6}{M_\Sigma \mu_{\Phi_1}^2 \mu_{\Phi_2}^2}, \quad (15)$$

which leads to dim 9 seesaw formula result,  $m_\nu \sim v^6/\Lambda_{\text{NP}}^5$ .

Neutrino masses can arise from dim 5 operators generated at the loop level. The loop factors provide a sufficient suppression as exploited by one- [27], two- [28,29] and three-loop level [30] mechanisms. Such a loop level generated dim 5 operator arises also in our extension of the SM. As explained in [10], there is a restricted range of the parameter space where the dim 5 loop-contributions shown on Fig. 2 are smaller than the dim 9 tree-level contribution shown on Fig. 1. With an extra assumption on the scalar field coupling strengths in (13),  $\lambda_3 \simeq \lambda_1 \cdot \lambda_2$ , the dim 9 mechanism turns out to be the leading one for  $\Lambda_{\text{NP}} \sim \text{few}100$  GeV. The loop contributions prevail for  $\Lambda_{\text{NP}} \sim \text{several TeV}$ , which are beyond the direct reach of the LHC, and may also mark a virtual physics in the LHC era.

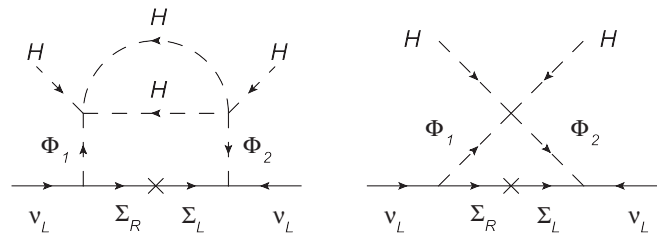


FIG. 2. Dim 5 operator arising at the loop level: the two-loop seesaw diagram given by the  $\lambda_1$  and  $\lambda_2$  couplings in Eq. (13), and the one-loop seesaw diagram given by the  $\lambda_3$  coupling in Eq. (13).

### III. PRODUCTION OF 5-PLET LEPTONS AT THE LHC

The production channels of the heavy 5-plet leptons in proton-(anti)proton collisions are dominated by the quark-antiquark annihilation via neutral and charged gauge bosons

$$q + \bar{q} \rightarrow A \rightarrow \Sigma + \bar{\Sigma}, \quad A = \gamma, Z, W^\pm,$$

where the gauge Lagrangian relevant for the production

$$\begin{aligned} \mathcal{L}_{\text{gauge}}^{\Sigma\bar{\Sigma}} = & +e(3\overline{\Sigma}^{+++}\gamma^\mu\Sigma^{+++} + 2\overline{\Sigma}^{++}\gamma^\mu\Sigma^{++} + \overline{\Sigma}^+\gamma^\mu\Sigma^+ - \overline{\Sigma}^-\gamma^\mu\Sigma^-)A_\mu + \frac{g}{c_W}((2 - 3s_W^2)\overline{\Sigma}^{+++}\gamma^\mu\Sigma^{+++} \\ & + (1 - 2s_W^2)\overline{\Sigma}^{++}\gamma^\mu\Sigma^{++})Z_\mu + \frac{g}{c_W}((-s_W^2)\overline{\Sigma}^+\gamma^\mu\Sigma^+ + (-1)\overline{\Sigma}^0\gamma^\mu\Sigma^0 + (-2 + s_W^2)\overline{\Sigma}^-\gamma^\mu\Sigma^-)Z_\mu \\ & + g(\sqrt{2}\overline{\Sigma}^{++}\gamma^\mu\Sigma^{+++} + \sqrt{3}\overline{\Sigma}^+\gamma^\mu\Sigma^{++} + \sqrt{3}\overline{\Sigma}^0\gamma^\mu\Sigma^+ + \sqrt{2}\overline{\Sigma}^-\gamma^\mu\Sigma^0)W_\mu^- + \text{H.c.} \end{aligned} \quad (16)$$

is contained in Eq. (A17) in the Appendix. The cross section for the partonic process is determined entirely by gauge couplings and is given by

$$\hat{\sigma}(q\bar{q} \rightarrow \Sigma\bar{\Sigma}) = \frac{\beta(3 - \beta^2)}{48\pi} \hat{s}(V_L^2 + V_R^2), \quad (17)$$

where  $\hat{s} \equiv (p_q + p_{\bar{q}})^2$  is the Mandelstam variable  $s$  for the quark-antiquark system, the parameter  $\beta \equiv \sqrt{1 - 4M_\Sigma^2/\hat{s}}$  denotes the heavy lepton velocity, and the left- and right-handed couplings in Eq. (17) are given by

$$V_{L,R}^{(\gamma+Z)} = \frac{Q_\Sigma Q_q e^2}{\hat{s}} + \frac{g^{Z\Sigma} g_{L,R}^q g^2}{c_W^2(\hat{s} - M_\Sigma^2)}, \quad (18)$$

$$V_L^{(W^-)} = \frac{g^{W\Sigma} g^2 V_{ud}}{\sqrt{2}(\hat{s} - M_\Sigma^2)} = V_L^{(W^+)*}, \quad (19)$$

$$V_R^{(W^\pm)} = 0. \quad (20)$$

Here,  $g_L^q = T_3 - s_W^2 Q_q$  and  $g_R^q = -s_W^2 Q_q$  are the SM chiral quark couplings to the  $Z$  boson. The vector couplings of heavy leptons to gauge bosons are

$$g^{Z\Sigma} = T_3 - s_W^2 Q_\Sigma, \quad g^{W\Sigma} = \sqrt{2}, \sqrt{3}, \sqrt{3} \text{ and } \sqrt{2}, \quad (21)$$

where  $g^{W\Sigma}$  can be read of the last row in Eq. (16) relevant for the production of  $\Sigma^{+++}\overline{\Sigma}^{+++}$ ,  $\Sigma^{++}\overline{\Sigma}^{++}$ ,  $\Sigma^+\overline{\Sigma}^0$  and  $\Sigma^0\overline{\Sigma}^-$  pairs, respectively. We note, in passing, that the choices  $g^{Z\Sigma} = c_W^2$  and  $g^{W\Sigma} = 1$ , appropriate for the type-III seesaw model, reproduce the expressions given in [17].

To obtain the cross sections for a hadron collider, the partonic cross-section (17) has to be convoluted with the appropriate parton distribution functions (PDF)  $q(x, \mu^2)$ . For the case of the proton-proton collisions we have

$$\begin{aligned} & \frac{d^2\sigma(pp \rightarrow \Sigma\bar{\Sigma})}{dx_1 dx_2} \\ & = \frac{1}{N_c} \sum_{f_1=u,d,\dots} \sum_{f_2=\bar{u},\bar{d},\dots} \hat{\sigma}(q_{f_1} q_{f_2} \rightarrow \Sigma\bar{\Sigma}) \\ & \quad \times (q_{f_1}(x_1, \mu^2) q_{f_2}(x_2, \mu^2) + q_{f_2}(x_1, \mu^2) q_{f_1}(x_2, \mu^2)), \end{aligned} \quad (22)$$

leading to the total cross section

$$\begin{aligned} \sigma(pp \rightarrow \Sigma\bar{\Sigma}) = & \frac{1}{N_c} \sum_{f_1=u,d,\dots} \sum_{f_2=\bar{u},\bar{d},\dots} \int_{4M^2/s}^1 d\tau \\ & \times \int_\tau^1 \frac{dx_1}{x_1} \hat{\sigma}(q_{f_1} q_{f_2} \rightarrow \Sigma\bar{\Sigma}) \\ & \times \left( q_{f_1}(x_1, \mu^2) q_{f_2}\left(\frac{\tau}{x_1}, \mu^2\right) \right. \\ & \left. + q_{f_2}(x_1, \mu^2) q_{f_1}\left(\frac{\tau}{x_1}, \mu^2\right) \right). \end{aligned} \quad (23)$$

Here, the inverse of the  $N_c = 3$  factor is due to the color averaging in the initial state, corresponding to the summation over colors implicit in the standard published PDF functions. To evaluate the cross sections we have used CTEQ6.6 PDFs [31] via LHAPDF software library [32], and have chosen  $\mu = M_\Sigma$  as a factorization scale. Comparison with other PDF sets has shown that uncertainty in cross sections induced by the choice of PDFs is less than 3%. We find an excellent agreement when checking our calculations with an independent evaluation of cross sections performed by using MadGraph [33], where the present model has been implemented with the help of FeynRules package [34].

The cross sections for proton-proton collisions are presented at  $\sqrt{s} = 7$  TeV, appropriate for the current LHC run, on Fig. 3, and for designed  $\sqrt{s} = 14$  TeV on Fig. 4. On Fig. 5 cross sections for proton-antiproton collisions, relevant for the Tevatron at  $\sqrt{s} = 1.96$  TeV are presented. Thereby, we distinguish separately the production via neutral currents shown on LHS, and via charged currents shown on RHS of Figs. 3 to 5.

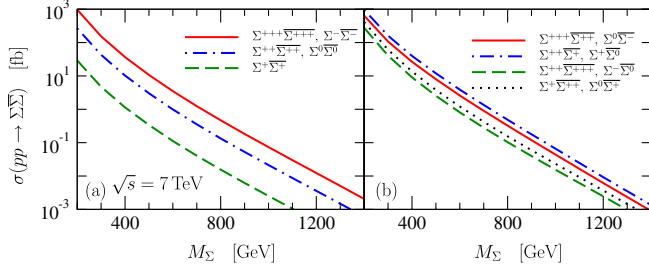


FIG. 3 (color online). The cross sections for production of 5-plet lepton pairs on LHC proton-proton collisions at  $\sqrt{s} = 7$  TeV via neutral  $\gamma$ ,  $Z$  (a) and charged  $W^\pm$  currents (b), in dependence on the heavy 5-plet mass  $M$ .

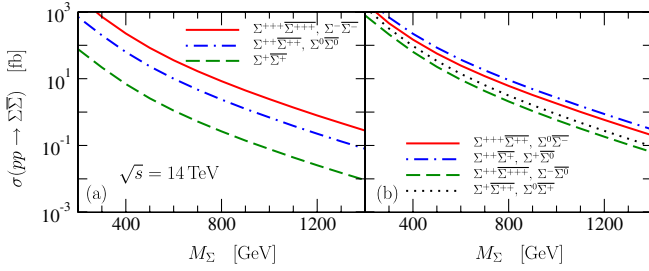


FIG. 4 (color online). Same as Fig. 3, but for designed  $\sqrt{s} = 14$  TeV at the LHC.

We extract the pair production cross sections shown in Fig. 3 for three selected values of  $M_\Sigma$  which we show in Table II. From this table we find that the triply-charged  $\Sigma^{+++}$  has the biggest production cross section  $\sigma(\Sigma^{+++})|_{M_\Sigma=400 \text{ GeV}} = 63.1 \text{ fb}$  and singly-charged  $\Sigma^-$  has the smallest, but comparable, production cross section  $\sigma(\Sigma^-)|_{M_\Sigma=400 \text{ GeV}} = 43.7 \text{ fb}$ . Production cross sections for all other particles and antiparticles from a 5-plet are in between. In particular, for  $1 \text{ fb}^{-1}$  of integrated luminosity at the present LHC  $\sqrt{s} = 7 \text{ TeV}$ , there would be  $270 \Sigma - \bar{\Sigma}$  pairs in total produced for  $M_\Sigma = 400 \text{ GeV}$ , among which are  $\sim 100$  triply-charged  $\Sigma^{+++}$  or  $\bar{\Sigma}^{+++}$  fermions. Note that the production at the LHC and Tevatron, which is comparable in the case of light  $\Sigma$ -states, changes dramatically in favor of the LHC for higher values of the  $\Sigma$ -lepton masses. Therefore, only the restricted mass region is covered on Fig. 5 for the Tevatron.

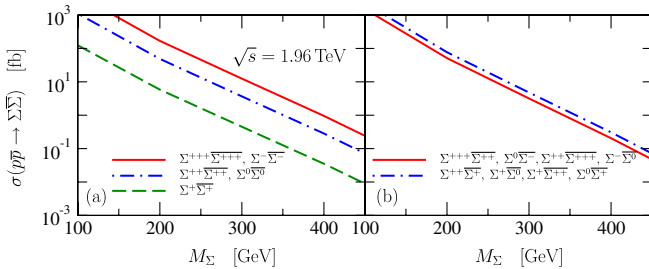


FIG. 5 (color online). Same as Fig. 3, but for  $\sqrt{s} = 1.96$  TeV at the Tevatron.

TABLE II. Production cross sections for  $\Sigma - \bar{\Sigma}$  pairs for the current LHC run at  $\sqrt{s} = 7 \text{ TeV}$ , for three selected values of  $M_\Sigma$ .

Produced pair	Cross section (fb)		
	$M_\Sigma = 200 \text{ GeV}$	$M_\Sigma = 400 \text{ GeV}$	$M_\Sigma = 800 \text{ GeV}$
$\Sigma^{+++}\bar{\Sigma}^{+++}$	967	36.6	0.47
$\Sigma^{++}\bar{\Sigma}^{++}$	267	10.3	0.13
$\Sigma^+\bar{\Sigma}^+$	29	1.1	0.02
$\Sigma^0\bar{\Sigma}^0$	253	9.3	0.11
$\Sigma^-\bar{\Sigma}^-$	939	34.6	0.43
$\Sigma^{+++}\bar{\Sigma}^{++}$	641	26.5	0.32
$\Sigma^{++}\bar{\Sigma}^+$	961	39.8	0.49
$\Sigma^+\bar{\Sigma}^0$	961	39.8	0.49
$\Sigma^0\bar{\Sigma}^-$	641	26.5	0.32
$\Sigma^{++}\bar{\Sigma}^{+++}$	276	9.1	0.10
$\Sigma^+\bar{\Sigma}^{++}$	414	13.6	0.16
$\Sigma^0\bar{\Sigma}^+$	414	13.6	0.16
$\Sigma^-\bar{\Sigma}^0$	276	9.1	0.10
Total	7040	269.9	3.30

By testing the heavy lepton production cross sections one can hope to identify the quantum numbers of the 5-plet particles, but in order to confirm their relation to neutrinos one has to study their decays.

#### IV. DECAYS OF 5-PLET STATES

Provided that in our scenario the exotic scalar states in Eq. (3) are slightly heavier than the exotic leptons in Eq. (2), the exotic scalars will not appear in the final states in heavy lepton decays. Also, the most phenomenologically interesting triply-charged scalar state is common to our model and the model by Babu *et al.* [13], where it has been in the focus of their study, so that there is no need to repeat their study here. Thus we focus entirely on the decay modes of the heavy lepton states in Eq. (2). Let us note that the ordering of the states in this formula corresponds to the spectrum of these states, running from the lightest  $\Sigma^-$  to the heaviest  $\Sigma^{+++}$  state.

The mass difference induced by loops of SM gauge bosons between two components of  $\Sigma$  5-plet with electric charges  $Q$  and  $Q'$  is explicitly calculated in [20]

$$\begin{aligned}
 M_Q - M_{Q'} &= \frac{\alpha_2 M}{4\pi} \left\{ (Q^2 - Q'^2) s_W^2 f\left(\frac{M_Z}{M}\right) \right. \\
 &\quad \left. + (Q - Q')(Q + Q' - Y) \right. \\
 &\quad \left. \times \left[ f\left(\frac{M_W}{M}\right) - f\left(\frac{M_Z}{M}\right) \right] \right\}, \quad (24)
 \end{aligned}$$

where

$$\begin{aligned}
 f(r) &= \frac{r}{2} \left[ 2r^3 \ln r - 2r + (r^2 - 4)^{1/2} (r^2 + 2) \right. \\
 &\quad \left. \times \ln\left(\frac{r^2 - 2 - r\sqrt{r^2 - 4}}{2}\right) \right]. \quad (25)
 \end{aligned}$$

For  $M_\Sigma = 400$  GeV we get

$$\begin{aligned} M_{\Sigma^{+++}} - M_{\Sigma^{++}} &\simeq 1130 \text{ MeV}, \\ M_{\Sigma^{++}} - M_{\Sigma^+} &\simeq 804 \text{ MeV}, \\ M_{\Sigma^+} - M_{\Sigma^0} &\simeq 477 \text{ MeV}, \\ M_{\Sigma^0} - M_{\Sigma^-} &\simeq 150 \text{ MeV}. \end{aligned} \quad (26)$$

The neutral  $\Sigma^0$  and singly-charged  $\Sigma^+$  states, coming in Eq. (26), receive additional corrections given in Eqs. (A12) and (A16), respectively. However, being of the order of magnitude of the light neutrino masses, these corrections are completely negligible.

The splitting  $\sim 1$  GeV between triply and doubly charged 5-plet states opens additional decay channels, in particular, the  $\rho$  resonance enhanced decay channel to the two pions in the final state.

### A. Pointlike $M_\Sigma$ -dependent decays

The Lagrangian in the mass-eigenstate basis relevant for the decays of the heavy leptons is the result of a detailed calculation exposed in the Appendix. Let us repeat here the neutral and charged current terms,

$$\begin{aligned} \mathcal{L}_{\text{NCZ}} = \frac{g}{c_W} &\left[ \bar{\nu} \left( \frac{3}{2} V_{\text{PMNS}}^\dagger V_1 \gamma^\mu P_L - \frac{\sqrt{3}}{2\sqrt{2}} V_{\text{PMNS}}^T V_2^* \gamma^\mu P_R \right) \Sigma^0 \right. \\ &\left. + \bar{l} (3V_1 \gamma^\mu P_L) \Sigma^- + \bar{l}^c \left( \frac{1}{2} V_2^* \gamma^\mu P_R \right) \Sigma^+ \right] Z_\mu^0 + \text{H.c.}, \end{aligned} \quad (27)$$

These three decays, for two chosen set of couplings  $V_1$  and  $V_2$ , are displayed on Figs. 6 and 7 as a counterpart of Fig. 2 in [17] for type-III triplets. Thereby, the product of  $V_1$  and  $V_2$  is constrained by Eq. (11). Decays of  $\Sigma^0$  to leptons (antileptons) are governed by  $V_1$  ( $V_2$ ), so that the branching ratios to leptons or antileptons are strongly dependent on the hierarchy between  $V_1$  and  $V_2$ , as illustrated in Figs. 6 and 7.

Besides the representative  $\Sigma^0$  decays above, let us present also the rest of the pointlike decays. For a negative singly-charged heavy lepton  $\Sigma^-$  the partial decay widths are given by:

and

$$\begin{aligned} \mathcal{L}_{\text{CC}} = g &\left[ \bar{\nu} (-\sqrt{3} V_{\text{PMNS}}^\dagger V_1 \gamma^\mu P_L + \sqrt{2} V_{\text{PMNS}}^T V_2^* \gamma^\mu P_R) \Sigma^+ \right. \\ &+ \bar{\Sigma}^- (\sqrt{3} V_2^T V_{\text{PMNS}}^* \gamma^\mu P_R) \nu + \bar{l} \left( -\frac{3}{\sqrt{2}} V_1 \gamma^\mu P_L \right) \Sigma^0 \\ &+ \bar{\Sigma}^0 \left( -\frac{\sqrt{3}}{2} V_2^T \gamma^\mu P_R \right) l^c \\ &\left. + \bar{l}^c (-\sqrt{3} V_2^* \gamma^\mu P_R) \Sigma^{++} \right] W_\mu^- + \text{H.c.} \end{aligned} \quad (28)$$

Here, all the couplings can be expressed in terms of the matrix-valued quantities  $V_1$  and  $V_2$  explicated in the Appendix, corresponding to the mass matrices in Eq. (10)

$$V_1 = \sqrt{\frac{1}{10}} Y_1^\dagger v_{\Phi_1} M_\Sigma^{-1}, \quad V_2 = \sqrt{\frac{1}{10}} Y_2^\dagger v_{\Phi_2}^* M_\Sigma^{-1}. \quad (29)$$

Out of the five  $\Sigma$ -states in Eq. (2), the four lying lowest have the pointlike decays to the gauge bosons and SM leptons. Here, in contrast to type-III triplet states, the decays of 5-plet states into the SM Higgs boson are suppressed. When appropriate, we will comment on further differences with respect to the decays of type-III triplet states.

Let us start our list of the partial decay widths by the decays of neutral  $\Sigma^0$  state:

$$\begin{aligned} \Gamma(\Sigma^0 \rightarrow \ell^- W^+) &= \frac{g^2}{32\pi} \left| \frac{3}{\sqrt{2}} V_1^{\ell\Sigma} \right|^2 \frac{M_\Sigma^3}{M_W^2} \left( 1 - \frac{M_W^2}{M_\Sigma^2} \right)^2 \left( 1 + 2 \frac{M_W^2}{M_\Sigma^2} \right), \\ \Gamma(\Sigma^0 \rightarrow \ell^+ W^-) &= \frac{g^2}{32\pi} \left| \frac{\sqrt{3}}{2} V_2^{\ell\Sigma} \right|^2 \frac{M_\Sigma^3}{M_W^2} \left( 1 - \frac{M_W^2}{M_\Sigma^2} \right)^2 \left( 1 + 2 \frac{M_W^2}{M_\Sigma^2} \right), \\ \sum_{m=1}^3 \Gamma(\Sigma^0 \rightarrow \nu_m Z^0) &= \frac{g^2}{32\pi c_W^2} \sum_{\ell=\{e,\mu,\tau\}} \left( \left| \frac{3}{2} V_1^{\ell\Sigma} \right|^2 + \left| \frac{\sqrt{3}}{2\sqrt{2}} V_2^{\ell\Sigma} \right|^2 \right) \frac{M_\Sigma^3}{M_Z^2} \left( 1 - \frac{M_Z^2}{M_\Sigma^2} \right)^2 \left( 1 + 2 \frac{M_Z^2}{M_\Sigma^2} \right). \end{aligned} \quad (30)$$

$$\begin{aligned} \Gamma(\Sigma^- \rightarrow \ell^- Z^0) &= \frac{g^2}{32\pi c_W^2} |3V_1^{\ell\Sigma}|^2 \frac{M_\Sigma^3}{M_Z^2} \left( 1 - \frac{M_Z^2}{M_\Sigma^2} \right)^2 \left( 1 + 2 \frac{M_Z^2}{M_\Sigma^2} \right), \\ \sum_{m=1}^3 \Gamma(\Sigma^- \rightarrow \nu_m W^-) &= \frac{g^2}{32\pi} \sum_{\ell=\{e,\mu,\tau\}} |\sqrt{3} V_2^{\ell\Sigma}|^2 \frac{M_\Sigma^3}{M_W^2} \left( 1 - \frac{M_W^2}{M_\Sigma^2} \right)^2 \left( 1 + 2 \frac{M_W^2}{M_\Sigma^2} \right). \end{aligned} \quad (31)$$



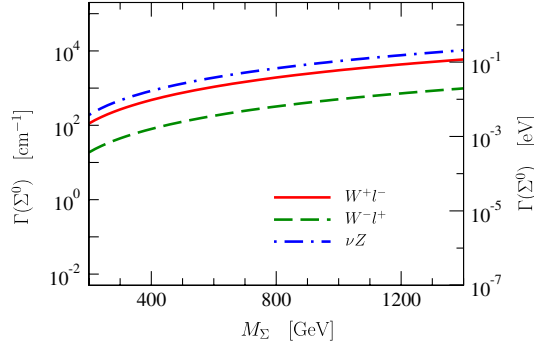


FIG. 6 (color online). Partial decay widths of  $\Sigma^0$  5-plet lepton for  $|V_1^{\ell\Sigma}| = |V_2^{\ell\Sigma}| = 10^{-6} \sqrt{\frac{20}{M_\Sigma(\text{GeV})}}$ , in dependence on heavy 5-plet mass  $M_\Sigma$ .

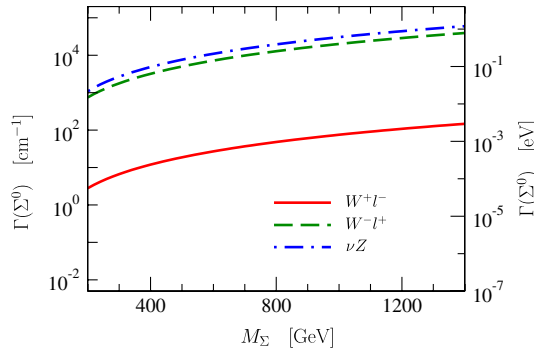


FIG. 7 (color online). Partial decay widths of  $\Sigma^0$  5-plet lepton for  $|V_1^{\ell\Sigma}| = 10^{-6} \sqrt{\frac{0.5}{M_\Sigma(\text{GeV})}}$  different from  $|V_2^{\ell\Sigma}| = 10^{-6} \sqrt{\frac{800}{M_\Sigma(\text{GeV})}}$ , in dependence on heavy 5-plet mass  $M_\Sigma$ .

The positive singly-charged  $\Sigma^+$  state has the following partial decay widths:

$$\begin{aligned} \Gamma(\Sigma^+ \rightarrow \ell^+ Z^0) &= \frac{g^2}{32\pi c_W^2} \left| \frac{1}{2} V_2^{\ell\Sigma} \right|^2 \frac{M_\Sigma^3}{M_Z^2} \left( 1 - \frac{M_Z^2}{M_\Sigma^2} \right)^2 \left( 1 + 2 \frac{M_Z^2}{M_\Sigma^2} \right), \\ \sum_{m=1}^3 \Gamma(\Sigma^+ \rightarrow \nu_m W^+) &= \frac{g^2}{32\pi} \sum_{\ell=\{e,\mu,\tau\}} (|\sqrt{3}V_1^{\ell\Sigma}|^2 + |\sqrt{2}V_2^{\ell\Sigma}|^2) \frac{M_\Sigma^3}{M_W^2} \left( 1 - \frac{M_W^2}{M_\Sigma^2} \right)^2 \\ &\quad \times \left( 1 + 2 \frac{M_W^2}{M_\Sigma^2} \right). \end{aligned} \quad (32)$$

Finally, the doubly charged  $\Sigma^{++}$  state decays exclusively via a charged current, with the partial decay width

$$\begin{aligned} \Gamma(\Sigma^{++} \rightarrow \ell^+ W^+) &= \frac{g^2}{32\pi} |\sqrt{3}V_2^{\ell\Sigma}|^2 \frac{M_\Sigma^3}{M_W^2} \left( 1 - \frac{M_W^2}{M_\Sigma^2} \right)^2 \left( 1 + 2 \frac{M_W^2}{M_\Sigma^2} \right). \end{aligned} \quad (33)$$

Let us stress that there is no such pointlike decay for the triply-charged  $\Sigma^{+++}$ . This state, instead, has some other interesting decays studied in the next subsections.

In Table III we list all possible events coming from the decays of the neutral and singly-charged 5-plet states to the SM charged leptons. This includes the same-sign dilepton events as a distinguished signature at the LHC.

### B. Decays which do not depend on any free parameter

Besides previous decays, there are also decays of  $\Sigma^i$  to a lighter  $\Sigma^j$  state. The decay rate for a single pion final state is given by

$$\begin{aligned} \Gamma(\Sigma^i \rightarrow \Sigma^j \pi^+) &= (g^{W\Sigma})_{ij}^2 \frac{2}{\pi} G_F^2 |V_{ud}|^2 f_\pi^2 (\Delta M_{ij})^3 \\ &\quad \times \sqrt{1 - \frac{m_\pi^2}{(\Delta M_{ij})^2}}. \end{aligned} \quad (34)$$

The corresponding leptonic decay is

$$\Gamma(\Sigma^i \rightarrow \Sigma^j l^+ \nu) = (g^{W\Sigma})_{ij}^2 \frac{2}{15\pi^3} G_F^2 (\Delta M_{ij})^5 \sqrt{1 - \frac{m_l^2}{(\Delta M_{ij})^2}} \quad (35)$$

where  $(g^{W\Sigma})_{ij}^2$  is given in Eq. (21) and  $\Delta M_{ij} = M_i - M_j$  is given in Eq. (24).

Such decays are suppressed by small mass differences, except for  $\Sigma^{+++}$  where the mass difference  $\Delta M_{32}$  between the  $\Sigma^{+++}$  and  $\Sigma^{++}$  state is large enough to enable also the two pions in the final state. This three-body decay is dominated by a  $\rho(770)$  resonance, and we obtain for it an analytical formula expressed in terms of the meson decay constants,  $f_\pi \simeq 130$  MeV and  $f_\rho \simeq 150$  MeV,

TABLE III. Decays of exotic leptons to SM particles including same sign dilepton events.

	$\Sigma^+ \rightarrow \ell^+ Z^0$	$\Sigma^0 \rightarrow \ell^+ W^-$	$\Sigma^0 \rightarrow \ell^- W^+$	$\Sigma^- \rightarrow \ell^+ Z^0$
$\Sigma^+ \rightarrow \ell^+ Z^0$	$\ell^+ \ell^- Z^0 Z^0$	$\ell^+ \ell^+ Z^0 W^-$	$\ell^+ \ell^- Z^0 W^+$	-
$\Sigma^0 \rightarrow \ell^- W^+$	$\ell^- \ell^- W^+ Z^0$	$\ell^- \ell^+ W^+ W^-$	$\ell^- \ell^- W^+ W^+$	$\ell^- \ell^+ W^+ Z^0$
$\Sigma^0 \rightarrow \ell^+ W^-$	$\ell^+ \ell^- W^- Z^0$	$\ell^+ \ell^+ W^- W^-$	$\ell^+ \ell^- W^- W^+$	$\ell^+ \ell^+ W^- Z^0$
$\Sigma^- \rightarrow \ell^- Z^0$	-	$\ell^- \ell^+ Z^0 W^-$	$\ell^- \ell^- Z^0 W^+$	$\ell^- \ell^+ Z^0 Z^0$

$$\begin{aligned}
 & \Gamma(\Sigma^{+++} \rightarrow \Sigma^{++} \pi^+ \pi^0) \\
 & \simeq \Gamma(\Sigma^{+++} \rightarrow \Sigma^{++} \rho^+) \\
 & = \frac{24}{\pi} G_F^2 V_{ud}^2 f_\rho^2 (\Delta M_{32})^3 \left(1 - \frac{4m_\pi^2}{m_\rho^2}\right)^{-1} \\
 & \quad \times \sqrt{1 - \frac{m_\rho^2}{(\Delta M_{32})^2}}. \tag{36}
 \end{aligned}$$

This decay shows an enhancement with respect to the corresponding two-body decay

$$\frac{\Gamma(\Sigma^{+++} \rightarrow \Sigma^{++} \pi^+ \pi^0)}{\Gamma(\Sigma^{+++} \rightarrow \Sigma^{++} \pi^+)} = 6 \frac{f_\rho^2}{f_\pi^2} \sqrt{1 - \frac{m_\rho^2}{(\Delta M_{32})^2}} \left(1 - \frac{4m_\pi^2}{m_\rho^2}\right)^{-1}. \tag{37}$$

These decays will serve as the referent decays for the golden decay mode of the triply-charged state studied in the following subsection.

### C. Golden $\Sigma^{+++} \rightarrow W^+ W^+ l^+$ decay

We now turn to evaluation of the decay rate for  $\Sigma^{+++} \rightarrow W^+ W^+ l^+$  with on-shell  $W$  bosons.

The Feynman diagram on Fig. 8 and its crossed, contribute with an off-shell  $\Sigma^{++}$  in the intermediate state.

The expression for the amplitude squared is lengthy so we give the analytical expression for decay width only in the limit  $M_\Sigma \gg M_W$ :

$$\Gamma(\Sigma^{+++} \rightarrow W^+ W^+ l^+) |_{M_\Sigma \gg M_W} = \frac{g^2}{384 \pi^2 s_W^2} |\sqrt{3} V_2^{\ell\Sigma}|^2 \frac{M_\Sigma^5}{M_W^4}. \tag{38}$$

Obviously, this decay is governed by the same mixing factor as in  $\Sigma^{++}$  decay in Eq. (33). Full numerical calculation, using FormCalc package [35], results in a partial width plotted on Figs. 9 and 10 for same choices of the mixing factor  $V_2$  as on previous figures. On the same figures we plot the partial widths of the decays of  $\Sigma^{+++}$  given in Eqs. (34)–(36).

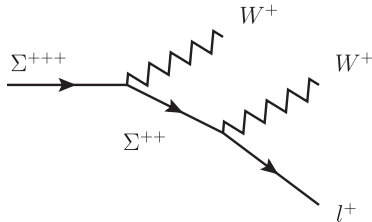


FIG. 8. Feynman diagram contributing to the triply-charged golden decay.

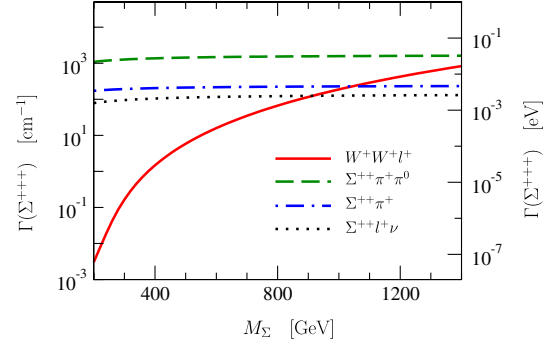


FIG. 9 (color online). Selected partial decay widths of  $\Sigma^{+++}$  5-plet lepton for  $|V_2^{\ell\Sigma}| = 10^{-6} \sqrt{\frac{20}{M_\Sigma(\text{GeV})}}$  in dependence of heavy 5-plet mass  $M_\Sigma$ .

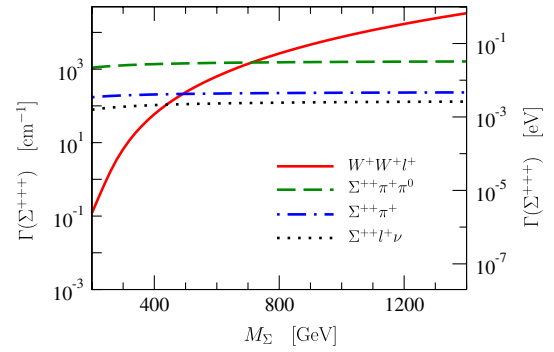


FIG. 10 (color online). Selected partial decay widths of  $\Sigma^{+++}$  5-plet lepton for  $|V_2^{\ell\Sigma}| = 10^{-6} \sqrt{\frac{800}{M_\Sigma(\text{GeV})}}$  in dependence of heavy 5-plet mass  $M_\Sigma$ .

For the sake of being definite, let us explicate the branching ratios for the choice of heavy-light mixing  $V_2$  taken on Fig. 10, for two values of heavy-lepton mass  $M_\Sigma$

$$\begin{aligned}
 \sum_l \text{BR}(\Sigma^{+++} \rightarrow W^+ W^+ l^+) |_{M_\Sigma=400 \text{ GeV}} &= 0.09 \\
 \sum_l \text{BR}(\Sigma^{+++} \rightarrow W^+ W^+ l^+) |_{M_\Sigma=800 \text{ GeV}} &= 0.80. \tag{39}
 \end{aligned}$$

As stated before, for  $1 \text{ fb}^{-1}$  of integrated luminosity expected to be collected from the present LHC run at  $\sqrt{s} = 7 \text{ TeV}$ , there will be  $\sim 100$  triply-charged  $\Sigma^{+++}$  or  $\overline{\Sigma}^{+++}$  fermions produced. This will result in  $\sim 10$  decays  $\Sigma^{+++}(\overline{\Sigma}^{+++}) \rightarrow W^\pm W^\pm l^\pm$  for  $M_\Sigma = 400 \text{ GeV}$  and  $|V_2^{\ell\Sigma}| = 10^{-6} \sqrt{\frac{800}{M_\Sigma(\text{GeV})}}$ .

## V. CONCLUSIONS

We have investigated the testability of exotic heavy leptons introduced to explain small neutrino masses. It is conceivable that such states additional to the SM leptons

may appear as higher representations, such as isospin 2 heavy fermions in conjunction with isospin 3/2 heavy scalar fields studied here. The LHC could provide enough energy and luminosity to produce and study such heavy particles if they mediate the proposed seesaw mechanism. In particular, the decays of new heavy leptons to SM particles depend on how light and heavy leptons mix with each other. Our seesaw model constrains the mixing to be small, given by the square root of the ratio of light and small masses,  $(m_\nu/M_\Sigma)^{1/2}$ . This suppresses their effects on the flavor physics, but the abundant production of new states invites us to investigate the phenomenology of these heavy leptons at the LHC.

For  $1 \text{ fb}^{-1}$  of integrated luminosity, the present LHC ( $\sqrt{s} = 7 \text{ TeV}$ ) run can produce in total 270 (7000)  $\Sigma^- - \bar{\Sigma}^+$  pairs of heavy leptons with mass  $M_\Sigma = 400 \text{ GeV}$  (200 GeV), among which there would be  $\sim 100$  (2850) triply-charged  $\Sigma^{+++}$  or  $\bar{\Sigma}^{+++}$  fermions. Thereby, the triply-charged  $\Sigma^{+++}$  has the biggest production cross-section,  $\sigma(\Sigma^{+++})|_{M_\Sigma=400 \text{ GeV}} = 63.1 \text{ fb}$ , and singly-charged  $\Sigma^-$  has the smallest but still comparable  $\sigma(\Sigma^-)|_{M_\Sigma=400 \text{ GeV}} = 43.7 \text{ fb}$ . The production cross sections for all other particles and antiparticles from a 5-plet are in between. Notably, the production of triply-charged leptons is followed with its golden decays  $\Sigma^{+++}(\bar{\Sigma}^{+++}) \rightarrow W^\pm W^\pm l^\pm$ . For a chosen mass  $M_\Sigma = 400 \text{ GeV}$  and the mixing  $|V_2^{\Sigma}| = 10^{-6} \sqrt{\frac{800}{M_\Sigma(\text{GeV})}}$  we expect  $\sim 10$  golden mode events.

The great advantage of the processes considered in this paper, common to most seesaw-inspired models, is the copious production of energetic leptons, which provides a clear experimental signature and relatively small SM background. Detailed studies of standard seesaw scenarios [36] demonstrate that final states with 2 (same sign) and 3 or 4 charged leptons can be increasingly easily distinguished from the SM background. For the model at hand, similar results are expected. For example, our prominent production channel  $p\bar{p} \rightarrow \Sigma^{+++}\bar{\Sigma}^{+++}$  results in 3 or more energetic charged leptons in roughly one third of cases. Additionally, when it is experimentally possible to separate final-state particles coming from each of the two produced heavy leptons, then the three same-sign leptons coming from a single heavy-lepton lead to distinctive signature of the presented model. The corresponding SM background is essentially negligible, similarly to what has been demonstrated in [37] in generic new physics scenario with lepton number violation.

To conclude, the exotic 5-plet leptons introduced to explain small neutrino masses can be observed at LHC. At the designed LHC luminosity and  $\sqrt{s} = 14 \text{ TeV}$  our 5-plet tree-level seesaw model is falsifiable for sub-TeV  $\Sigma^-$ -lepton masses, while for masses which are outside of the LHC reach the 5-plet leptons remain viable for generating radiative neutrino masses. In this sense, we hope that the

involved predictive and testable vectorlike fermion 5-plet may establish an exciting link between collider phenomenology and the origin of neutrino masses.

## ACKNOWLEDGMENTS

We thank Borut Bajc, Walter Grimus, and Goran Senjanović for discussions at the early stage of this research. K. K. and I. P. thank Jan O. Eeg for discussions and hospitality offered at the University in Oslo. This work is supported by the Croatian Ministry of Science, Education, and Sports under Contract No. 119-0982930-1016.

## APPENDIX A: THE INTERACTION VERTICES OF THE 5-PLET

The novel seesaw model employs, in addition to the SM particles,  $n_\Sigma$  vectorlike 5-plets of leptons with hypercharge two,  $\Sigma_{L,R} \sim (1, 5, 2)$  under  $SU(3)_C \times SU(2)_L \times U(1)_Y$ . We write the component fields as

$$\Sigma_L = \begin{pmatrix} \Sigma_L^{+++} \\ \Sigma_L^{++} \\ \Sigma_L^+ \\ \Sigma_L^0 \\ \Sigma_L^- \end{pmatrix}, \quad \Sigma_R = \begin{pmatrix} \Sigma_R^{+++} \\ \Sigma_R^{++} \\ \Sigma_R^+ \\ \Sigma_R^0 \\ \Sigma_R^- \end{pmatrix}. \quad (\text{A1})$$

The renormalizable Lagrangian involving  $\Sigma_L$  and  $\Sigma_R$  is given by

$$\mathcal{L} = \bar{\Sigma}_L i \not{D} \Sigma_L + \bar{\Sigma}_R i \not{D} \Sigma_R - \bar{\Sigma}_R M_\Sigma \Sigma_L - \bar{\Sigma}_L M_\Sigma^\dagger \Sigma_R + \bar{\Sigma}_R Y_1 L_L \Phi_1^* + (\bar{\Sigma}_L)^c Y_2 L_L \Phi_2 + \text{H.c.} \quad (\text{A2})$$

With a nonzero vacuum expectation value for the scalar fields, doublet leptons receive mass and mix with the 5-plet leptons. Lepton mass terms in the Lagrangian are

$$\begin{aligned} \mathcal{L}_m = & -\frac{1}{2} (\overline{(\nu_L)^c} (\Sigma_L^0)^c (\Sigma_R^0)^c) \begin{pmatrix} 0 & m_2^T & m_1^T \\ m_2 & 0 & M_\Sigma^T \\ m_1 & M_\Sigma & 0 \end{pmatrix} \begin{pmatrix} \nu_L \\ \Sigma_L^0 \\ (\Sigma_R^0)^c \end{pmatrix} + \text{H.c.} \\ & - (\overline{l_R} \Sigma_R^- (\Sigma_L^+)^c) \begin{pmatrix} m_1 & 0 & 0 \\ m_3 & M_\Sigma & 0 \\ m_4 & 0 & M_\Sigma^T \end{pmatrix} \begin{pmatrix} l_L \\ \Sigma_L^- \\ (\Sigma_R^+)^c \end{pmatrix} + \text{H.c.} \\ & - \overline{\Sigma_R^+} M_\Sigma \Sigma_L^{++} - \overline{\Sigma_R^{+++}} M_\Sigma \Sigma_L^{+++} + \text{H.c.}, \quad (\text{A3}) \end{aligned}$$

with

$$\begin{aligned}
m_1 &= \sqrt{\frac{1}{10}} Y_1 \mathbf{v}_{\Phi 1}^*, & m_2 &= -\sqrt{\frac{3}{20}} Y_2 \mathbf{v}_{\Phi 2}, \\
m_3 &= \sqrt{\frac{2}{5}} Y_1 \mathbf{v}_{\Phi 1}^*, & m_4 &= \sqrt{\frac{1}{10}} Y_2 \mathbf{v}_{\Phi 2}.
\end{aligned} \tag{A4}$$

For detailed studies, one needs to understand the mass matrices in Eq. (A3) and their diagonalization. The diagonalization of the mass matrices can be achieved by making the following unitary transformations on the leptons

$$\begin{aligned}
\begin{pmatrix} \nu_L \\ \Sigma_L^0 \\ (\Sigma_R^0)^c \end{pmatrix} &= U^0 \begin{pmatrix} \nu_{mL} \\ \Sigma_{mL}^0 \\ (\Sigma_{mR}^0)^c \end{pmatrix}, \\
\begin{pmatrix} l_L \\ \Sigma_L^- \\ (\Sigma_R^+)^c \end{pmatrix} &= U^L \begin{pmatrix} l_{mL} \\ \Sigma_{mL}^- \\ (\Sigma_{mR}^+)^c \end{pmatrix}, \\
\begin{pmatrix} l_R \\ \Sigma_R^- \\ (\Sigma_L^+)^c \end{pmatrix} &= U^R \begin{pmatrix} l_{mR} \\ \Sigma_{mR}^- \\ (\Sigma_{mL}^+)^c \end{pmatrix},
\end{aligned} \tag{A5}$$

different for a symmetric neutral and a nonsymmetric charged mass matrix

$$\begin{aligned}
U^{0T} \begin{pmatrix} 0 & m_2^T & m_1^T \\ m_2 & 0 & M_\Sigma^T \\ m_1 & M_\Sigma & 0 \end{pmatrix} U^0 &= \begin{pmatrix} m_\nu & 0 \\ 0 & M_0 \end{pmatrix}, \\
U^{R\dagger} \begin{pmatrix} m_l & 0 & 0 \\ m_3 & M_\Sigma & 0 \\ m_4 & 0 & M_\Sigma^T \end{pmatrix} U^L &= \begin{pmatrix} m_{\text{lepton}} & 0 \\ 0 & M \end{pmatrix}.
\end{aligned} \tag{A6}$$

For 3 light doublet fields and  $n_\Sigma$  heavy 5-plet vectorlike lepton fields, the matrices  $U_{L,R}$  and  $U_0$  are  $(3 + 2n_\Sigma) \times (3 + 2n_\Sigma)$  unitary matrices, which we decompose into the block matrices as follows:

$$U^0 \equiv \begin{pmatrix} U_{\nu\nu}^0 & U_{\nu\Sigma_L^0}^0 & U_{\nu(\Sigma_R^0)^c}^0 \\ U_{\Sigma_L^0\nu}^0 & U_{\Sigma_L^0\Sigma_L^0}^0 & U_{\Sigma_L^0(\Sigma_R^0)^c}^0 \\ U_{(\Sigma_R^0)^c\nu}^0 & U_{(\Sigma_R^0)^c\Sigma_L^0}^0 & U_{(\Sigma_R^0)^c(\Sigma_R^0)^c}^0 \end{pmatrix}, \tag{A7}$$

$$U^L \equiv \begin{pmatrix} U_{ll}^L & U_{l\Sigma_L^-}^L & U_{l(\Sigma_R^+)^c}^L \\ U_{\Sigma_L^-l}^L & U_{\Sigma_L^-\Sigma_L^-}^L & U_{\Sigma_L^-(\Sigma_R^+)^c}^L \\ U_{(\Sigma_R^+)^cl}^L & U_{(\Sigma_R^+)^c\Sigma_L^-}^L & U_{(\Sigma_R^+)^c(\Sigma_R^+)^c}^L \end{pmatrix}, \tag{A8}$$

$$U^R \equiv \begin{pmatrix} U_{ll}^R & U_{l\Sigma_R^-}^R & U_{l(\Sigma_L^+)^c}^R \\ U_{\Sigma_R^-l}^R & U_{\Sigma_R^-\Sigma_R^-}^R & U_{\Sigma_R^-(\Sigma_L^+)^c}^R \\ U_{(\Sigma_L^+)^cl}^R & U_{(\Sigma_L^+)^c\Sigma_R^-}^R & U_{(\Sigma_L^+)^c(\Sigma_L^+)^c}^R \end{pmatrix}. \tag{A9}$$

Here, we distinguish the entries  $U_{\nu\nu}^0$ ,  $U_{ll}^L$ , and  $U_{ll}^R$  which are  $3 \times 3$  matrices, the entries  $U_{\nu\Sigma_L^0}^0$ ,  $U_{\nu(\Sigma_R^0)^c}^0$ ,  $U_{l\Sigma_L^-}^L$ ,  $U_{l(\Sigma_R^+)^c}^L$ ,  $U_{\Sigma_R^-}^R$ , and  $U_{l(\Sigma_L^+)^c}^R$  which are  $3 \times n_\Sigma$  matrices, and the entries  $U_{\Sigma_L^0\nu}^0$ ,  $U_{(\Sigma_R^0)^c\nu}^0$ ,  $U_{\Sigma_L^-l}^L$ ,  $U_{(\Sigma_R^+)^cl}^L$ ,  $U_{\Sigma_R^-}^R$ , and  $U_{(\Sigma_L^+)^cl}^R$  which are  $n_\Sigma \times 3$  matrices. The remaining entries are  $n_\Sigma \times n_\Sigma$  matrices.

In principle, the matrices  $U_0$  and  $U_{L,R}$  can be expressed in terms of  $m_{1,2,3,4}$ ,  $m_l$  and  $M_\Sigma$  sub-blocks. Since, in order for the seesaw mechanism to work, the factors  $m_{1,2,3,4} M_\Sigma^{-1}$  should be small, one can expand  $U_0$  and  $U_{L,R}$  in powers of  $M_\Sigma^{-1}$  and keep a track of the leading order contributions. For this purpose, without loss of generality, it is convenient to write the leading order expressions up to  $M_\Sigma^{-2}$  in the basis where  $m_l$  and  $M_\Sigma$  are already diagonalized. Following the procedure of [18,24] we first obtain results for the entries  $U^0$ :

$$\begin{aligned}
U_{\nu\nu}^0 &= \left( 1 - \frac{1}{2} m_1^\dagger M_\Sigma^{-2} m_1 - \frac{1}{2} m_2^\dagger M_\Sigma^{-2} m_2 \right) V_{\text{PMNS}}, \\
U_{\nu\Sigma_L^0}^0 &= m_1^\dagger M_\Sigma^{-1}, \\
U_{\nu(\Sigma_R^0)^c}^0 &= m_2^\dagger M_\Sigma^{-1}, \\
U_{\Sigma_L^0\nu}^0 &= -M_\Sigma^{-1} m_1 V_{\text{PMNS}}, \\
U_{\Sigma_L^0\Sigma_L^0}^0 &= 1 - \frac{1}{2} M_\Sigma^{-1} m_1 m_1^\dagger M_\Sigma^{-1}, \\
U_{\Sigma_L^0(\Sigma_R^0)^c}^0 &= -\frac{1}{2} M_\Sigma^{-1} m_1 m_2^\dagger M_\Sigma^{-1}, \\
U_{(\Sigma_R^0)^c\nu}^0 &= -M_\Sigma^{-1} m_2 V_{\text{PMNS}}, \\
U_{(\Sigma_R^0)^c\Sigma_L^0}^0 &= -\frac{1}{2} M_\Sigma^{-1} m_2 m_1^\dagger M_\Sigma^{-1}, \\
U_{(\Sigma_R^0)^c(\Sigma_R^0)^c}^0 &= 1 - \frac{1}{2} M_\Sigma^{-1} m_2 m_2^\dagger M_\Sigma^{-1}.
\end{aligned} \tag{A10}$$

Here,  $V_{\text{PMNS}}$  is a  $3 \times 3$  unitary matrix which diagonalizes the effective light neutrino mass matrix:

$$\tilde{m}_\nu \simeq -m_2^T M_\Sigma^{-1} m_1 - m_1^T M_\Sigma^{-1} m_2, \quad V_{\text{PMNS}}^T \tilde{m}_\nu V_{\text{PMNS}} = m_\nu. \tag{A11}$$

The mass matrix for heavy neutral leptons acquires corrections of the order  $M_\Sigma^{-1}$ ,

$$\begin{aligned} \tilde{M}_0 \simeq & \begin{pmatrix} 0 & M_\Sigma \\ M_\Sigma & 0 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} m_2 m_1^\dagger M_\Sigma^{-1} & m_2 m_2^\dagger M_\Sigma^{-1} \\ m_1 m_1^\dagger M_\Sigma^{-1} & m_1 m_2^\dagger M_\Sigma^{-1} \end{pmatrix} \\ & + \frac{1}{2} \begin{pmatrix} M_\Sigma^{-1} m_1^* m_2^T & M_\Sigma^{-1} m_1^* m_1^T \\ M_\Sigma^{-1} m_2^* m_2^T & M_\Sigma^{-1} m_2^* m_1^T \end{pmatrix}, \end{aligned} \quad (\text{A12})$$

which, being of the order of light neutrino masses, are completely negligible for our phenomenological considerations. In principle, for every vectorlike 5-plet we would get two nearly-degenerate heavy Majorana fermions, with mass splitting of the order  $m_\nu$ . By neglecting these corrections, we treat heavy neutral leptons as Dirac fermions for all practical phenomenological considerations.

Next, we obtain the leading order expressions up to  $M_\Sigma^{-2}$  for the entries in  $U^L$ :

$$\begin{aligned} U_{ll}^L &= 1 - \frac{1}{2} m_3^\dagger M_\Sigma^{-2} m_3 - \frac{1}{2} m_4^\dagger M_\Sigma^{-2} m_4, \\ U_{l\Sigma_L^-}^L &= m_3^\dagger M_\Sigma^{-1}, \\ U_{l(\Sigma_R^+)^c}^L &= m_4^\dagger M_\Sigma^{-1}, \\ U_{\Sigma_L^- l}^L &= -M_\Sigma^{-1} m_3, \\ U_{\Sigma_L^- \Sigma_L^-}^L &= 1 - \frac{1}{2} M_\Sigma^{-1} m_3 m_3^\dagger M_\Sigma^{-1}, \\ U_{\Sigma_L^- (\Sigma_R^+)^c}^L &= -\frac{1}{2} M_\Sigma^{-1} m_3 m_4^\dagger M_\Sigma^{-1}, \\ U_{(\Sigma_R^+)^c l}^L &= -M_\Sigma^{-1} m_4, \\ U_{(\Sigma_R^+)^c \Sigma_L^-}^L &= -\frac{1}{2} M_\Sigma^{-1} m_4 m_3^\dagger M_\Sigma^{-1}, \\ U_{(\Sigma_R^+)^c (\Sigma_R^+)^c}^L &= 1 - \frac{1}{2} M_\Sigma^{-1} m_4 m_4^\dagger M_\Sigma^{-1}. \end{aligned} \quad (\text{A13})$$

Similarly, for  $U^R$ , the leading order expressions up to  $M_\Sigma^{-2}$  are:

$$\begin{aligned} U_{ll}^R &= 1, & U_{l\Sigma_R^-}^R &= m_l m_3^\dagger M_\Sigma^{-2}, \\ U_{l(\Sigma_L^+)^c}^R &= m_l m_4^\dagger M_\Sigma^{-2}, & U_{\Sigma_R^- l}^R &= -M_\Sigma^{-2} m_3 m_l, \\ U_{\Sigma_R^- \Sigma_R^-}^R &= 1, & U_{\Sigma_R^- (\Sigma_L^+)^c}^R &= 0, \\ U_{(\Sigma_L^+)^c l}^R &= -M_\Sigma^{-2} m_4 m_l, & U_{(\Sigma_L^+)^c \Sigma_R^-}^R &= 0, \\ & & U_{(\Sigma_L^+)^c (\Sigma_L^+)^c}^R &= 1. \end{aligned} \quad (\text{A14})$$

As a result, the mass matrix for the light singly-charged leptons gets corrections of the order  $M_\Sigma^{-2}$

$$\tilde{m}_l \simeq m_l \left( 1 - \frac{1}{2} m_3^\dagger M_\Sigma^{-2} m_3 - \frac{1}{2} m_4^\dagger M_\Sigma^{-2} m_4 \right), \quad (\text{A15})$$

while, the mass matrix for the heavy singly-charged leptons gets corrections of the order  $M_\Sigma^{-1}$

$$\tilde{M} \simeq \begin{pmatrix} M_\Sigma & 0 \\ 0 & M_\Sigma \end{pmatrix} + \frac{1}{2} \begin{pmatrix} m_3 m_3^\dagger M_\Sigma^{-1} & m_3 m_4^\dagger M_\Sigma^{-1} \\ m_4 m_3^\dagger M_\Sigma^{-1} & m_4 m_4^\dagger M_\Sigma^{-1} \end{pmatrix}. \quad (\text{A16})$$

Both corrections, in Eq. (A14) and (A15), are completely negligible for our phenomenological considerations.

Our studies of particle production require a knowledge of gauge couplings to leptonic fields. In the weak interaction basis, they can be written as

$$\begin{aligned} \mathcal{L}_{\text{gauge}} = & + e(3\overline{\Sigma}^{+++} \gamma^\mu \Sigma^{+++} + 2\overline{\Sigma}^{++} \gamma^\mu \Sigma^{++} + \overline{\Sigma}^+ \gamma^\mu \Sigma^+ - \overline{\Sigma}^- \gamma^\mu \Sigma^- - \bar{l} \gamma^\mu l) A_\mu + \frac{g}{c_W} ((2 - 3s_W^2) \overline{\Sigma}^{+++} \gamma^\mu \Sigma^{+++} \\ & + (1 - 2s_W^2) \overline{\Sigma}^{++} \gamma^\mu \Sigma^{++}) Z_\mu - \frac{g}{c_W} s_W^2 (\overline{\Sigma}^+ \gamma^\mu \Sigma^+ - \overline{\Sigma}^- \gamma^\mu \Sigma^- - \bar{l} \gamma^\mu l) Z_\mu + \frac{g}{c_W} (\bar{\nu}_L \gamma^\mu \nu_L + \overline{\Sigma}_L^0 \gamma^\mu \Sigma_L^0 \\ & - \overline{\Sigma}_R^0 \gamma^\mu \Sigma_R^0) Z_\mu + \frac{g}{c_W} \left( -\frac{1}{2} \bar{\nu}_L \gamma^\mu \nu_L - 2\overline{\Sigma}_L^0 \gamma^\mu \Sigma_L^0 - \frac{1}{2} \bar{l}_L \gamma^\mu l_L - 2\overline{\Sigma}_L^- \gamma^\mu \Sigma_L^- - 2\overline{\Sigma}_R^- \gamma^\mu \Sigma_R^- \right) Z_\mu \\ & + g[\sqrt{2}\overline{\Sigma}_L^{++} \gamma^\mu \Sigma_L^{++} + \sqrt{3}\overline{\Sigma}_L^+ \gamma^\mu \Sigma_L^+ + \sqrt{3}\overline{\Sigma}_L^0 \gamma^\mu \Sigma_L^0 + \sqrt{2}\overline{\Sigma}_L^- \gamma^\mu \Sigma_L^- + \sqrt{2}\overline{\Sigma}_R^+ \gamma^\mu \Sigma_R^+ + \sqrt{3}\overline{\Sigma}_R^+ \gamma^\mu \Sigma_R^+ \\ & + \sqrt{3}\overline{\Sigma}_R^0 \gamma^\mu \Sigma_R^0 + \sqrt{2}\overline{\Sigma}_R^- \gamma^\mu \Sigma_R^- + \frac{1}{\sqrt{2}} \bar{l}_L \gamma^\mu \nu_L] W_\mu^- + \text{H.c.}, \end{aligned} \quad (\text{A17})$$

where  $c_W = \cos\theta_W$  and  $s_W = \sin\theta_W$ .

In the mass-eigenstate basis we obtain the terms involving the heavy fermion, light fermion, and gauge boson fields that are relevant for the decays of heavy leptons. In the following, we focus only on the above terms and restrict to couplings of the order  $M_\Sigma^{-1}$ . Whereas the photon couplings to all leptons and  $Z$  couplings to triply and

doubly charged leptons are diagonal in the mass-eigenstate basis, the couplings of  $Z$ -boson to singly-charged and neutral leptons are more complicated. They are given by

$$\mathcal{L}_{\text{NCZ}} \equiv (\mathcal{L}_{\text{NCZ}}^A + \mathcal{L}_{\text{NCZ}}^B + \mathcal{L}_{\text{NCZ}}^C + \text{H.c.}), \quad (\text{A18})$$

where

$$\begin{aligned}
\mathcal{L}_{\text{NCZ}}^A &= \frac{g}{c_W} \bar{l}_m V_{Zl\Sigma^-}^L \gamma^\mu P_L \Sigma_{m'}^- Z_\mu^0, \\
\mathcal{L}_{\text{NCZ}}^B &= \frac{g}{c_W} (\bar{l}_m)^c V_{Zl\Sigma^+}^R \gamma^\mu P_R \Sigma_{m'}^+ Z_\mu^0, \\
\mathcal{L}_{\text{NCZ}}^C &= \frac{g}{c_W} [\bar{\nu}_m V_{Z\nu\Sigma^0}^L \gamma^\mu P_L \Sigma_{m'}^0 Z_\mu^0 \\
&\quad + \bar{\nu}_m V_{Z\nu\Sigma^0}^R \gamma^\mu P_R \Sigma_{m'}^0 Z_\mu^0].
\end{aligned} \tag{A19}$$

Here, the matrix couplings, up to order  $M_\Sigma^{-1}$ , are given by

$$\begin{aligned}
V_{Zl\Sigma^-}^L &= -2U_{\Sigma^- l}^{L\dagger} - \frac{1}{2}U_{l\Sigma^-}^L, \\
V_{Zl\Sigma^+}^R &= \frac{1}{2}U_{l(\Sigma^+)^c}^{L*}, \\
V_{Z\nu\Sigma^0}^L &= -2U_{\Sigma^0 \nu}^{0\dagger} - \frac{1}{2}U_{\nu\nu}^{0\dagger}U_{\nu\Sigma^0}^0, \\
V_{Z\nu\Sigma^0}^R &= \frac{1}{2}U_{\nu\nu}^{0T}U_{\nu(\Sigma^0)^c}^{0*}.
\end{aligned} \tag{A20}$$

Analogously, the charged current interactions are given by

$$\mathcal{L}_{\text{CC}} \equiv (\mathcal{L}_{\text{CC}}^A + \mathcal{L}_{\text{CC}}^B + \mathcal{L}_{\text{CC}}^C + \mathcal{L}_{\text{CC}}^D + \mathcal{L}_{\text{CC}}^E + \text{H.c.}), \tag{A21}$$

where

$$\begin{aligned}
\mathcal{L}_{\text{CC}}^A &= g[\bar{\nu}_m V_{\nu\Sigma^+}^L \gamma^\mu P_L \Sigma_{m'}^+ W_\mu^- + \bar{\nu}_m V_{\nu\Sigma^+}^R \gamma^\mu P_R \Sigma_{m'}^+ W_\mu^-], \\
\mathcal{L}_{\text{CC}}^B &= g[\bar{\Sigma}_m^- V_{\Sigma^- \nu}^L \gamma^\mu P_L \nu_{m'} W_\mu^- + \bar{\Sigma}_m^- V_{\Sigma^- \nu}^R \gamma^\mu P_R \nu_{m'} W_\mu^-], \\
\mathcal{L}_{\text{CC}}^C &= g\bar{l}_m V_{l\Sigma^0}^L \gamma^\mu P_L \Sigma_{m'}^0 W_\mu^-, \\
\mathcal{L}_{\text{CC}}^D &= g\bar{\Sigma}_m^0 V_{\Sigma^0 l}^R \gamma^\mu P_R (l_{m'})^c W_\mu^-, \\
\mathcal{L}_{\text{CC}}^E &= g(\bar{l}_m)^c V_{l\Sigma^{++}}^R \gamma^\mu P_R \Sigma_{m'}^{++} W_\mu^-,
\end{aligned} \tag{A22}$$

and up to order  $M_\Sigma^{-1}$

$$\begin{aligned}
V_{\nu\Sigma^+}^L &= \sqrt{3}U_{\Sigma^0 \nu}^{0\dagger}, \\
V_{\nu\Sigma^+}^R &= \sqrt{3}U_{(\Sigma^0)^c \nu}^{0T} - \frac{1}{\sqrt{2}}U_{\nu\nu}^{0T}U_{l(\Sigma^+)^c}^{L*}, \\
V_{\Sigma^- \nu}^L &= \frac{1}{\sqrt{2}}U_{l\Sigma^-}^{L\dagger}U_{\nu\nu}^0 + \sqrt{2}U_{\Sigma^0 \nu}^0, \\
V_{\Sigma^- \nu}^R &= \sqrt{2}U_{(\Sigma^0)^c \nu}^{0*}, \\
V_{l\Sigma^0}^L &= \sqrt{2}U_{\Sigma^- l}^{L\dagger} + \frac{1}{\sqrt{2}}U_{\nu\Sigma^0}^0, \\
V_{\Sigma^0 l}^R &= \sqrt{3}U_{(\Sigma^+)^c l}^{L*} - \frac{1}{\sqrt{2}}U_{\nu(\Sigma^0)^c}^{0T}, \\
V_{l\Sigma^{++}}^R &= \sqrt{3}U_{(\Sigma^+)^c l}^{LT}.
\end{aligned} \tag{A23}$$

Finally, by suppressing the indices  $m$  and  $m'$  and by defining

$$V_1 = \sqrt{\frac{1}{10}}Y_1^\dagger v_{\Phi_1} M_\Sigma^{-1}, \quad V_2 = \sqrt{\frac{1}{10}}Y_2^\dagger v_{\Phi_2}^* M_\Sigma^{-1}, \tag{A24}$$

we obtain more compact expressions

$$\begin{aligned}
\mathcal{L}_{\text{NCZ}} &= \frac{g}{c_W} \left[ \bar{\nu} \left( \frac{3}{2}V_{\text{PMNS}}^\dagger V_1 \gamma^\mu P_L + \frac{-\sqrt{3}}{2\sqrt{2}}V_{\text{PMNS}}^T V_2^* \gamma^\mu P_R \right) \Sigma^0 \right. \\
&\quad \left. + \bar{l} (3V_1 \gamma^\mu P_L) \Sigma^- + \bar{l}^c \left( \frac{1}{2}V_2^* \gamma^\mu P_R \right) \Sigma^+ \right] Z_\mu^0 + \text{H.c.}
\end{aligned} \tag{A25}$$

and

$$\begin{aligned}
\mathcal{L}_{\text{CC}} &= g \left[ \bar{\nu} (-\sqrt{3}V_{\text{PMNS}}^\dagger V_1 \gamma^\mu P_L + \sqrt{2}V_{\text{PMNS}}^T V_2^* \gamma^\mu P_R) \Sigma^+ \right. \\
&\quad \left. + \bar{\Sigma}^- (\sqrt{3}V_2^T V_{\text{PMNS}}^* \gamma^\mu P_R) \nu + \bar{l} \left( -\frac{3}{\sqrt{2}}V_1 \gamma^\mu P_L \right) \Sigma^0 \right. \\
&\quad \left. + \bar{\Sigma}^0 \left( -\frac{\sqrt{3}}{2}V_2^T \gamma^\mu P_R \right) l^c \right. \\
&\quad \left. + \bar{l}^c (-\sqrt{3}V_2^* \gamma^\mu P_R) \Sigma^{++} \right] W_\mu^- + \text{H.c.}
\end{aligned} \tag{A26}$$

Note that the interactions involving light neutrinos in the above have the additional  $V_{\text{PMNS}}$  factor compared with those involving light charged leptons. For easier comparison to the type-III seesaw case, we are keeping in this Appendix the notation and the style from [18] as much as possible.

- 
- [1] S. Weinberg, *Phys. Rev. Lett.* **43**, 1566 (1979).  
[2] E. Ma, *Phys. Rev. Lett.* **81**, 1171 (1998).  
[3] P. Minkowski, *Phys. Lett. B* **67**, 421 (1977); T. Yanagida, in *Proceedings Of The Workshop On Unified Theory And*

*The Baryon Number Of The Universe*, edited by O. Sawada and A. Sugamoto (KEK, Tsukuba, 1979), p. 95; M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity*, edited by P. van Nieuwenhuizen and

- D. Freedman (North Holland, Amsterdam, 1979), p. 315; S. L. Glashow, in *Quarks And Leptons*, edited by M. Lévy *et al.* (Plenum, New York, 1980), p. 707; R. N. Mohapatra and G. Senjanović, *Phys. Rev. Lett.* **44**, 912 (1980).
- [4] W. Konetschny and W. Kummer, *Phys. Lett. B* **70**, 433 (1977); M. Magg and C. Wetterich, *Phys. Lett. B* **94**, 61 (1980); J. Schechter and J. W. F. Valle, *Phys. Rev. D* **22**, 2227 (1980); T. P. Cheng and L. F. Li, *Phys. Rev. D* **22**, 2860 (1980); G. Lazarides, Q. Shafi, and C. Wetterich, *Nucl. Phys. B* **181**, 287 (1981); R. N. Mohapatra and G. Senjanović, *Phys. Rev. D* **23**, 165 (1981).
- [5] R. Foot, H. Lew, X. G. He, and G. C. Joshi, *Z. Phys. C* **44**, 441 (1989).
- [6] B. Bajc and G. Senjanovic, *J. High Energy Phys.* **08** (2007) 014; B. Bajc, M. Nemevsek, and G. Senjanovic, *Phys. Rev. D* **76**, 055011 (2007); I. Dorsner and P. Fileviez Perez, *J. High Energy Phys.* **06** (2007) 029.
- [7] E. Ma, *Phys. Rev. Lett.* **86**, 2502 (2001).
- [8] W. Grimus, L. Lavoura, and B. Radovčić, *Phys. Lett. B* **674**, 117 (2009).
- [9] Z. Z. Xing and S. Zhou, *Phys. Lett. B* **679**, 249 (2009).
- [10] I. Picek and B. Radovčić, *Phys. Lett. B* **687**, 338 (2010).
- [11] I. Picek and B. Radovčić, *Phys. Rev. D* **78**, 015014 (2008).
- [12] M. I. Vysotsky, *Phys. Lett. B* **644**, 352 (2007).
- [13] K. S. Babu, S. Nandi, and Z. Tavartkiladze, *Phys. Rev. D* **80**, 071702 (2009).
- [14] Y. Liao, G-Z Ning, and L. Ren, *Phys. Rev. D* **82**, 113003 (2010).
- [15] C-K Chua and S. S. C. Law, *Phys. Rev. D* **83**, 055010 (2011).
- [16] A. Delgado, C. G. Cely, T. Han, and Z. Wang, [arXiv:1105.5417](https://arxiv.org/abs/1105.5417).
- [17] R. Franceschini, T. Hambye, and A. Strumia, *Phys. Rev. D* **78**, 033002 (2008).
- [18] T. Li and X-G He, *Phys. Rev. D* **80**, 093003 (2009).
- [19] A. Arhrib *et al.*, *Phys. Rev. D* **82**, 053004 (2010).
- [20] M. Cirelli, N. Fornengo, and A. Strumia, *Nucl. Phys. B* **753**, 178 (2006); M. Cirelli and A. Strumia, *Phys. Rev. D* **80**, 071702 (2009).
- [21] A. de Gouvea and J. Jenkins, *Phys. Rev. D* **77**, 013008 (2008).
- [22] F. Bonnet, D. Hernandez, T. Ota, and W. Winter, *J. High Energy Phys.* **10** (2009) 076.
- [23] Y. Liao, *Phys. Lett. B* **694**, 346 (2011).
- [24] W. Grimus and L. Lavoura, *J. High Energy Phys.* **11** (2000) 042.
- [25] H. Hettmansperger, M. Lindner, and W. Rodejohann, *J. High Energy Phys.* **04** (2011) 123.
- [26] K. Nakamura *et al.* (Particle Data Group), *J. Phys. G* **37**, 075021 (2010).
- [27] A. Zee, *Phys. Lett. B* **93**, 389 (1980).
- [28] A. Zee, *Nucl. Phys. B* **264**, 99 (1986).
- [29] K. S. Babu, *Phys. Lett. B* **203**, 132 (1988).
- [30] L. M. Krauss, S. Nasri, and M. Trodden, *Phys. Rev. D* **67**, 085002 (2003); M. Aoki, S. Kanemura, and O. Seto, *Phys. Rev. Lett.* **102**, 051805 (2009).
- [31] P. M. Nadolsky *et al.*, *Phys. Rev. D* **78**, 013004 (2008).
- [32] M. R. Whalley, D. Bourilkov, and R. C. Group, [arXiv:hep-ph/0508110](https://arxiv.org/abs/hep-ph/0508110).
- [33] J. Alwall *et al.*, *J. High Energy Phys.* **09** (2007) 028.
- [34] N. D. Christensen and C. Duhr, *Comput. Phys. Commun.* **180**, 1614 (2009).
- [35] T. Hahn and M. Perez-Victoria, *Comput. Phys. Commun.* **118**, 153 (1999).
- [36] F. del Aguila and J. A. Aguilar-Saavedra, *Nucl. Phys. B* **813**, 22 (2009); J. A. Aguilar-Saavedra, *Nucl. Phys. B* **828**, 289 (2010).
- [37] B. Mukhopadhyaya and S. Mukhopadhyay, *Phys. Rev. D* **82**, 031501 (2010); S. Mukhopadhyay and B. Mukhopadhyaya, [arXiv:1108.4921](https://arxiv.org/abs/1108.4921).