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## Energy Dependence of Moments of Net-Proton Multiplicity Distributions at RHIC

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We report the beam energy ( $\sqrt{s_{NN}} = 7.7\text{--}200$  GeV) and collision centrality dependence of the mean ( $M$ ), standard deviation ( $\sigma$ ), skewness ( $S$ ), and kurtosis ( $\kappa$ ) of the net-proton multiplicity distributions in Au + Au collisions. The measurements are carried out by the STAR experiment at midrapidity ( $|y| < 0.5$ )

and within the transverse momentum range  $0.4 < p_T < 0.8$  GeV/ $c$  in the first phase of the Beam Energy Scan program at the Relativistic Heavy Ion Collider. These measurements are important for understanding the quantum chromodynamic phase diagram. The products of the moments,  $S\sigma$  and  $\kappa\sigma^2$ , are sensitive to the correlation length of the hot and dense medium created in the collisions and are related to the ratios of baryon number susceptibilities of corresponding orders. The products of moments are found to have values significantly below the Skellam expectation and close to expectations based on independent proton and antiproton production. The measurements are compared to a transport model calculation to understand the effect of acceptance and baryon number conservation and also to a hadron resonance gas model.

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The Beam Energy Scan (BES) program at the Relativistic Heavy-Ion Collider (RHIC) facility aims at studying in detail the QCD phase structure. This enables us to map the phase diagram, temperature ( $T$ ) versus baryonic chemical potential ( $\mu_B$ ), of strong interacting matter. Important advancements have been made towards the understanding of the QCD phase structure at small  $\mu_B$ . Theoretically, it has been found that at high temperatures there occurs a crossover transition from hadronic matter to a deconfined state of quarks and gluons at  $\mu_B = 0$  MeV [1]. Experimental data from RHIC and the Large Hadron Collider have provided evidence of the formation of QCD matter with quark and gluon degrees of freedom [2]. Several studies have been done to estimate the quark-hadron transition temperature at  $\mu_B = 0$  [3]. Interesting features of the QCD phase structure are expected to appear at larger  $\mu_B$  [4]. These include the QCD critical point ( $CP$ ) [5,6] and a first-order phase boundary between quark-gluon and hadronic phases [7].

Previous studies of net-proton multiplicity distributions suggest that the possible  $CP$  region is unlikely to be below  $\mu_B = 200$  MeV [8]. The versatility of the RHIC machine has permitted the center of mass energy ( $\sqrt{s_{NN}}$ ) to be varied below the injection energy ( $\sqrt{s_{NN}} = 19.6$  GeV), thereby providing the possibility to scan the QCD phase diagram above  $\mu_B \sim 250$  MeV. The  $\mu_B$  value is observed to increase with decreasing  $\sqrt{s_{NN}}$  [9]. The goal of the BES program at RHIC is to look for the experimental signatures of a first-order phase transition and the critical point by colliding Au ions at various  $\sqrt{s_{NN}}$  [10].

Nonmonotonic variations of observables related to the moments of the distributions of conserved quantities such as net-baryon, net-charge, and net-strangeness [11] number with  $\sqrt{s_{NN}}$  are believed to be good signatures of a phase transition and a  $CP$ . The moments are related to the correlation length ( $\xi$ ) of the system [12]. The signatures of phase transition or  $CP$  are detectable if they survive the evolution of the system [13]. Finite size and time effects in heavy-ion collisions put constraints on the significance of the desired signals. A theoretical calculation suggests a non-equilibrium  $\xi \approx 2-3$  fm for heavy-ion collisions [14]. Hence, it is proposed to study the higher moments [like skewness,  $S = \langle(\delta N)^3\rangle/\sigma^3$ , and kurtosis,  $\kappa = [\langle(\delta N)^4\rangle/\sigma^4] - 3$ , with  $\delta N = N - \langle N \rangle$ ] of distributions of conserved quantities due to a stronger dependence on  $\xi$  [12]. Both the magnitude and

the sign of the moments [15], which quantify the shape of the multiplicity distributions, are important for understanding phase transition and  $CP$  effects. Furthermore, products of the moments can be related to susceptibilities associated with the conserved numbers. The product  $\kappa\sigma^2$  of the net-baryon number distribution is related to the ratio of fourth-order ( $\chi_B^{(4)}$ ) to second-order ( $\chi_B^{(2)}$ ) baryon number susceptibilities [16,17]. The ratio  $\chi_B^{(4)}/\chi_B^{(2)}$  is expected to deviate from unity near the  $CP$ . It has different values for the hadronic and partonic phases [17].

This Letter reports measurements of the energy dependence of higher moments of the net-proton multiplicity ( $N_p - N_{\bar{p}} = \Delta N_p$ ) distributions from Au + Au collisions. The aim is to search for signatures of the  $CP$  over a broad range of  $\mu_B$  in the QCD phase diagram. Theoretical calculations have shown that  $\Delta N_p$  fluctuations reflect the singularity of the charge and baryon number susceptibility, as expected at the  $CP$  [18]. The measurements presented here are within a finite acceptance range and use only the protons among the produced baryons. References [19,20] discuss the advantages of using net-baryon measurements and effects of acceptance.

The data presented in this Letter were obtained using the Time Projection Chamber (TPC) of the Solenoidal Tracker at RHIC (STAR) [21]. The event-by-event proton ( $N_p$ ) and antiproton ( $N_{\bar{p}}$ ) multiplicities are measured for Au + Au minimum-bias events at  $\sqrt{s_{NN}} = 11.5, 19.6, 27, 39, 62.4,$  and  $200$  GeV for collisions occurring within  $|\Delta Z| = 30$  cm from the TPC center along the beam line. For  $7.7$  GeV,  $|\Delta Z|$  is  $50$  cm. The  $19.6$  and  $27$  GeV data were collected in the year 2011 and the other energies were taken in 2010. Interactions of the beam with the beam pipe are rejected by choosing events with a radial vertex position in the transverse plane of less than  $2$  cm. The numbers of events analyzed are  $3 \times 10^6, 6.6 \times 10^6, 15 \times 10^6, 30 \times 10^6, 86 \times 10^6, 47 \times 10^6,$  and  $238 \times 10^6$  for  $\sqrt{s_{NN}} = 7.7, 11.5, 19.6, 27, 39, 62.4,$  and  $200$  GeV, respectively. Similar studies have also been carried out in  $p + p$  collisions with  $0.6 \times 10^6$  and  $7 \times 10^6$  events at  $\sqrt{s_{NN}} = 62.4$  and  $200$  GeV, respectively. The centrality selection utilizes the uncorrected charged particle multiplicity other than identified protons and antiprotons within pseudorapidity  $|\eta| < 1.0$  measured by the TPC. It is found that the measured

net-proton moment values depend on the choice of the pseudorapidity range for the centrality selection. However, the values of the moments do not change if the centrality selection range is further increased to the full acceptance of the TPC (which leads to a 15% increase in charged particle multiplicity). In the UrQMD [22] studies, after increasing the  $\eta$  range used for centrality selection to two units, it is observed that the maximum decrease of moments is  $\sim 2.5\%$  and  $35\%$  for  $\sqrt{s_{NN}} = 200$  and  $7.7$  GeV, respectively [23]. There is minimal change for central collisions compared to other centralities. For each centrality, the average number of participants ( $\langle N_{\text{part}} \rangle$ ) is obtained by Glauber model calculations. The  $\Delta N_p$  measurements are carried out at midrapidity ( $|y| < 0.5$ ) in the range  $0.4 < p_T < 0.8$  GeV/c. Ionization energy loss ( $dE/dx$ ) of charged particles in the TPC is used to identify the inclusive ( $p(\bar{p})$ ) [24]. The minimum  $p_T$  cut and a maximum distance of closest approach to the collision vertex of 1 cm for each  $p(\bar{p})$  candidate track suppress contamination from secondaries [24]. To have a good purity of the proton sample (better than 98%) for all beam energies, the maximum  $p_T$  is taken to be 0.8 GeV/c. This  $p_T$  interval accounts for approximately 50% of the total uncorrected  $p + \bar{p}$  multiplicity at midrapidity. The average proton reconstruction efficiency for the  $p_T$  range studied is between 70%–78% and 83%–86% for central and peripheral collisions, respectively, at different  $\sqrt{s_{NN}}$ .

$\Delta N_p$  distributions from 70%–80%, 30%–40%, and 0%–5% Au + Au collision centralities are shown in Fig. 1. The  $\Delta N_p$  is not corrected for reconstruction efficiency. The distributions are also not corrected for the finite centrality width effect [23]. The subsequent analysis in this Letter is corrected for the centrality width effect. The difference of two Poisson distributions is a Skellam distribution. The corresponding Skellam distributions are also shown,

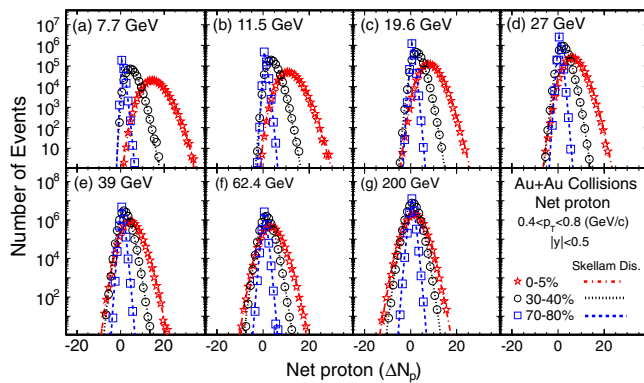


FIG. 1 (color online).  $\Delta N_p$  multiplicity distributions in Au + Au collisions at various  $\sqrt{s_{NN}}$  for 0%–5%, 30%–40%, and 70%–80% collision centralities at midrapidity. The statistical errors are small and within the symbol size. The lines are the corresponding Skellam distributions. The distributions are not corrected for the finite centrality width effect and  $N_p(N_{\bar{p}})$  reconstruction efficiency.

$P(\Delta N_p) = (M_p/M_{\bar{p}})^{N/2} I_N(2\sqrt{M_p M_{\bar{p}}}) \exp[-(M_p + M_{\bar{p}})]$ , where  $I_N(x)$  is a modified Bessel function of the first kind, and  $M_p$  and  $M_{\bar{p}}$  are the measured mean multiplicities of proton and antiprotons [25]. The data seem to closely follow the Skellam distributions. To study the detailed shape of the distribution, we discuss the various order cumulants ( $C_n$ ), where  $C_1 = M$ ,  $C_2 = \sigma^2$ ,  $C_3 = S\sigma^3$ , and  $C_4 = \kappa\sigma^4$ . For a Skellam  $\Delta N_p$  distribution,  $C_3/C_2 = (M_p - M_{\bar{p}})/(M_p + M_{\bar{p}})$  and  $C_4/C_2 = 1$ .

The four cumulants that describe the shape of  $\Delta N_p$  distributions at various collision energies are plotted as a function of  $\langle N_{\text{part}} \rangle$  in Fig. 2. We use the Delta theorem approach to obtain statistical errors [26]. The typical statistical error values for  $C_2$ ,  $C_3$ , and  $C_4$  for central Au + Au collisions at 7.7 GeV are 0.3%, 2.5%, and 25%, respectively, and those for high statistics 200 GeV results are 0.04%, 1.2%, and 2.0%, respectively. Most of the cumulant values show a linear variation with  $\langle N_{\text{part}} \rangle$ . The  $C_1$  values increase as  $\sqrt{s_{NN}}$  decreases, in accordance with the energy and centrality dependence of baryon transport.  $C_2$  and  $C_4$  have similar values as a function of  $\langle N_{\text{part}} \rangle$  for a given  $\sqrt{s_{NN}}$ .  $C_1$  and  $C_3$  follow each other closely as a function of  $\langle N_{\text{part}} \rangle$  at any given  $\sqrt{s_{NN}}$ . The differences between these groupings decrease as  $\sqrt{s_{NN}}$  decreases. The decrease in the  $C_3$  values with increasing beam energy indicates that the distributions become symmetric for the higher beam energies. The particle production at any given centrality can be considered a superposition of several identically distributed independent sources the number of which is proportional to  $N_{\text{part}}$  [8]. For the cumulants, this means a linear increase with  $\langle N_{\text{part}} \rangle$  as the system volume increases. This reflects that the cumulants are extensive quantities that are proportional to system volume. The lines in Fig. 2 are linear fits to the cumulants, which provide a reasonable description of the centrality dependence. This

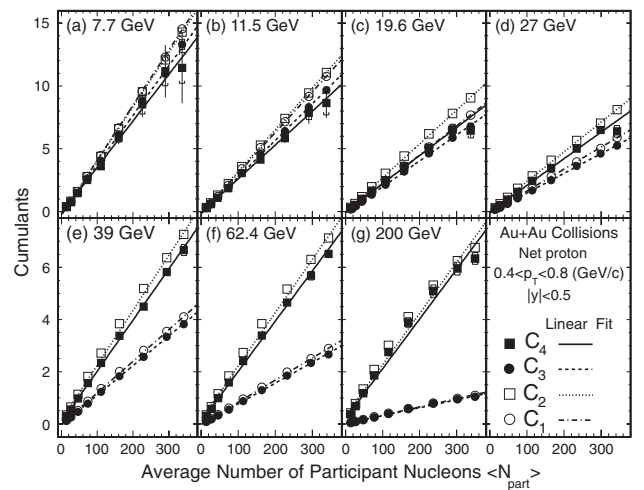


FIG. 2. Centrality dependence of the cumulants of  $\Delta N_p$  distributions for Au + Au collisions. Error bars are statistical and caps are systematic errors.

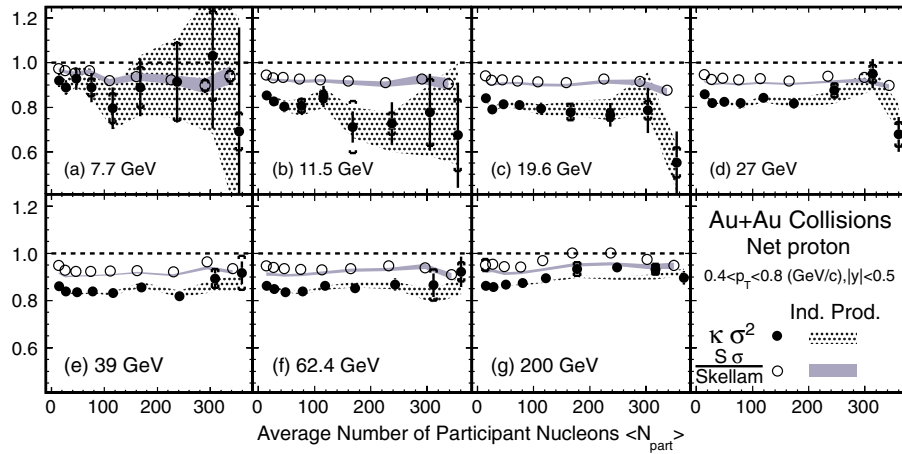


FIG. 3 (color online). Centrality dependence of  $S\sigma/\text{Skellam}$  and  $\kappa\sigma^2$  for  $\Delta N_p$  in Au + Au collisions at  $\sqrt{s_{NN}} = 7.7, 11.5, 19.6, 27, 39, 62.4,$  and  $200$  GeV. The results are corrected for the  $p(\bar{p})$  reconstruction efficiency. The error bars are statistical and caps are systematic errors. The shaded bands are expectations assuming the approach of independent proton and antiproton production, as described in the text. The width of the bands represents statistical uncertainty.

indicates that the volume effect dominates the measured cumulant values. The  $\chi^2/\text{ndf}$  (number of degrees of freedom) between the linear fit and data are smaller than 3.2 for all cumulants presented. The slight deviation of some cumulants in most central collisions from the fit line are due to the corresponding proton distributions.

In order to cancel the volume effect to first order and to understand the collision dynamics, we present the ratios of the cumulants  $C_3/C_2$  ( $= S\sigma$ ) and  $C_4/C_2$  ( $= \kappa\sigma^2$ ) as a function of  $\langle N_{\text{part}} \rangle$  for all collision energies, in Fig. 3. The  $S\sigma$  are normalized to the corresponding Skellam expectations. Results with correction for the  $p(\bar{p})$  reconstruction efficiency are presented. The correction for a finite track reconstruction efficiency is done by assuming a binomial distribution for the probability to reconstruct  $n$  particles out of  $N$  produced [20,27]. These observables are related to the ratio of baryon number susceptibilities ( $\chi_B$ ) at a given temperature ( $T$ ) computed in QCD motivated models as  $S\sigma = (\chi_B^{(3)}/T)/(\chi_B^{(2)}/T^2)$  and  $\kappa\sigma^2 = (\chi_B^{(4)})/(\chi_B^{(2)}/T^2)$  [16,17]. Close to the  $CP$ , QCD-based calculations predict the net-baryon number distributions to be non-Gaussian and susceptibilities to diverge, causing  $S\sigma$  and  $\kappa\sigma^2$  to have nonmonotonic variations with  $\langle N_{\text{part}} \rangle$  and/or  $\sqrt{s_{NN}}$  [6,12].

We observe in Fig. 3 that the  $\kappa\sigma^2$  and the  $S\sigma$  normalized to Skellam expectations are below unity for all of the Au + Au collision data sets presented. The deviations below unity of the order of 1%–3% [28] as seen for the central collisions for energies above 27 GeV are expected from quantum statistical effects. The measured  $S\sigma$  and  $\kappa\sigma^2$  are compared to expectations in which the cumulants of  $\Delta N_p$  distributions are constructed by considering independent production of protons and antiprotons. For independent production, the various order ( $n = 1, 2, 3,$  and  $4$ ) net-proton cumulants are given as  $C_n(\Delta N_p) = C_n(N_p) + (-1)^n C_n(N_{\bar{p}})$ , where

$C_n(N_p)$  and  $C_n(N_{\bar{p}})$  are cumulants of the measured distributions of  $N_p$  and  $N_{\bar{p}}$ , respectively. This approach breaks intraevent correlations between  $N_p$  and  $N_{\bar{p}}$ . The results from independent production are found to be in good agreement with the data. However, for  $\sqrt{s_{NN}} < 39$  GeV, the  $C_n$  of net protons are dominated by the corresponding values from the proton distributions. The assumption that  $N_p$  and  $N_{\bar{p}}$  have independent binomial distributions [29] also leads to a good description of the measurements (similar to independent production, but not plotted in Fig. 3).

Systematic errors are estimated by varying the following requirements for  $p(\bar{p})$  tracks: distance of closest approach, track quality reflected by the number of fit points used in track reconstruction, and the  $dE/dx$  selection criteria for  $p(\bar{p})$  identification. The typical systematic errors are of the order of 4% for  $M$  and  $\sigma$ , 5% for  $S$ , and 12% for  $\kappa$ . A 5% uncertainty in reconstruction efficiency estimation is also considered. The statistical and systematic (caps) errors are presented separately in the figures.

Figure 4 shows the energy dependence of  $S\sigma$  and  $\kappa\sigma^2$  for  $\Delta N_p$  for Au + Au collisions for two collision centralities (0%–5% and 70%–80%), corrected for ( $p(\bar{p})$ ) reconstruction efficiency. The  $S\sigma$  values normalized to the corresponding Skellam expectations are shown in the bottom panel of Fig. 4. The Skellam expectations reflect a system of totally uncorrelated, statistically random particle production. The corresponding results from  $p + p$  collisions at  $\sqrt{s_{NN}} = 62.4$  and  $200$  GeV are also shown and found to be similar to peripheral Au + Au collisions within the statistical errors. For  $\sqrt{s_{NN}}$  below 39 GeV, differences are observed between the 0%–5% central Au + Au collisions and the peripheral collisions. The results are closer to unity for  $\sqrt{s_{NN}} = 7.7$  GeV. The significance of deviations of 0%–5% Au + Au data from Skellam expectations,  $(|data - \text{Skellam}|)/\sqrt{\text{err}_{\text{stat}}^2 + \text{err}_{\text{sys}}^2}$ , are found to be greatest

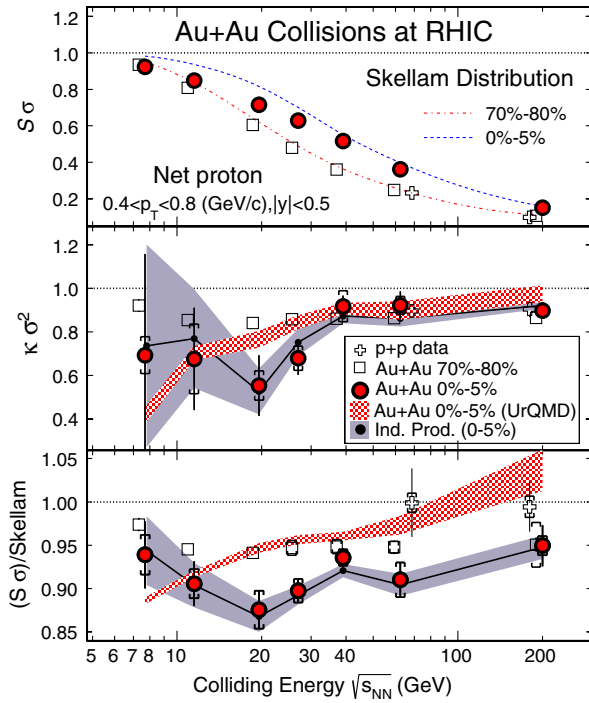


FIG. 4 (color online). Collision energy and centrality dependence of the net proton  $S\sigma$  and  $\kappa\sigma^2$  from Au + Au and  $p + p$  collisions at RHIC. Crosses, open squares, and filled circles are for the efficiency corrected results of  $p + p$ , 70%–80%, and 0%–5% Au + Au collisions, respectively. Skellam distributions for corresponding collision centralities are shown in the top panel. Shaded hatched bands are the results from UrQMD [22]. In the middle and lower panels, the shaded solid bands are the expectations assuming independent proton and antiproton production. The width of the bands represents statistical uncertainties. The HRG values for  $\kappa\sigma^2$  and  $S\sigma/\text{Skellam}$  are unity [30,31]. The error bars are statistical and caps are systematic errors. For clarity,  $p + p$  and 70%–80% Au + Au results are slightly displaced horizontally.

for 19.6 and 27 GeV, with values of 3.2 and 3.4 for  $\kappa\sigma^2$  and 4.5 and 5.6 for  $S\sigma$ , respectively. The significance of deviations for 5%–10% Au + Au data are smaller for  $\kappa\sigma^2$  with values of 2.0 and 0.6 and are 5.0 and 5.4 for  $S\sigma$ , for 19.6 and 27 GeV, respectively. Higher statistics data for  $\sqrt{s_{NN}} < 19.6$  GeV will help in quantitatively understanding the energy dependence of  $\kappa\sigma^2$  and  $S\sigma$ . A reasonable description of the measurements is obtained from the independent production approach. The data also show deviations from the hadron resonance gas model [30,31], which predict  $\kappa\sigma^2$  and  $S\sigma/\text{Skellam}$  to be unity. The effect of decay is less than 2% as per the hadron resonance gas model (HRG) calculations in Ref. [31]. To understand the effects of baryon number conservation [32] and experimental acceptance, UrQMD model calculations (a transport model which does not include a  $CP$ ) [22] for 0%–5% Au + Au collisions are shown in the middle and bottom panels of Fig. 4. The UrQMD model shows a monotonic decrease with decreasing beam energy [23]. The centrality dependence of

the  $\kappa\sigma^2$  and  $S\sigma$  from UrQMD [23] (not shown in the figures) closely follows the data at the lower beam energies of 7.7 and 11.5 GeV. Their values are, in general, larger compared to data for the higher beam energies.

The current data provide the most relevant measurements over the widest range in  $\mu_B$  (20–450 MeV) to date for the  $CP$  search, and for comparison with the baryon number susceptibilities computed from QCD to understand the various features of the QCD phase structure [6,16,17]. The deviations of  $S\sigma$  and  $\kappa\sigma^2$  below the Skellam expectation are qualitatively consistent with a QCD-based model which includes a  $CP$  [33]. However, the UrQMD model which does not include a  $CP$  also shows deviations from the Skellam expectation. Hence, conclusions on the existence of  $CP$  can be made only after comparison to QCD calculations with  $CP$  behavior which include the dynamics associated with heavy-ion collisions, such as finite correlation length and freeze-out effects.

In summary, measurements of the higher moments and their products ( $S\sigma$  and  $\kappa\sigma^2$ ) of the net-proton distributions at midrapidity ( $|y| < 0.5$ ) within  $0.4 < p_T < 0.8$  GeV/c in Au + Au collisions over a wide range of  $\sqrt{s_{NN}}$  and  $\mu_B$  have been presented to search for a possible  $CP$  and signals of a phase transition in the collisions. These observables show a centrality and energy dependence, which are not reproduced by either non- $CP$  transport model calculations or by a hadron resonance gas model. For  $\sqrt{s_{NN}} > 39$  GeV,  $S\sigma$  and  $\kappa\sigma^2$  values are similar for central, peripheral Au + Au collisions, and  $p + p$  collisions. Deviations for both  $\kappa\sigma^2$  and  $S\sigma$  from HRG and Skellam expectations are observed for  $\sqrt{s_{NN}} \leq 27$  GeV. The measurements are reasonably described by assuming independent production of  $N_p$  and  $N_{\bar{p}}$ , indicating that there are no apparent correlations between the protons and antiprotons for the observable presented. However, at the lower beam energies, the net-proton distributions are dominated by the shape of the proton distributions only. The data presented here also provide information to extract freeze-out conditions in heavy-ion collisions using QCD-based approaches [34,35].

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