Measurements of Nonsinglet Moments of the Nucleon Structure Functions and Comparison to Predictions from Lattice QCD for $Q^2 = 4 \text{ GeV}^2$


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We present extractions of the nucleon nonsinglet moments utilizing new precision data on the deuteron $F_2$ structure function at large Bjorken-$x$ determined via the Rosenbluth separation technique at Jefferson Lab Experimental Hall C. These new data are combined with a complementary set of data on the proton previously measured in Hall C at similar kinematics and world datasets on the proton and deuteron at lower $Q^2$. We then compare these extractions to predictions from lattice QCD for $Q^2 = 4 \text{ GeV}^2$. The extracted moments are described as sums of a quark-model term and a gluonic term, with the latter being expanded in powers of $Q^2$. The single-twist term is dominated by the quark contribution, while the double-twist term is largely gluonic. The predicted contributions of the quark models are found to be in good agreement with the data, while the lattice predictions are compared with the quark-model fits and found to be in good agreement as well.

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In the framework of quantum chromodynamics (QCD), the partonic structure of hadrons may be studied through moments (or Bjorken x weighted integrals) of the hadron structure functions. The difference of the u and d quark distributions is a flavor nonsinglet quantity with the N even (considered in this work) nonsinglet moments of these parton distribution functions (PDF) defined as

$$\langle x^{N-1} \rangle_{u-d} = \int dx x^{N-1} [u(x) - d(x) + \bar{u}(x) - \bar{d}(x)].$$  \hspace{1cm} (1)

A successful lattice computation of the nucleon nonsinglet moment is a fundamental test of QCD [1]. Precise lattice QCD (LQCD) predictions of these moments [2–10] are now available. These recent calculations include those that approach the physical pion mass and employ smaller lattice spacing to calculate the moments [2] and those that use an innovative approach to directly calculate the PDFs and from these moments [9]. For a discussion of the connection between PDFs and lattice calculations see Ref. [11]. Experimentally, the nonsinglet moments can be determined from the difference of proton and neutron moments [12,13]. For a discussion of the connection

$$\frac{d^2 \sigma}{d \Omega d E'} = \Gamma(\sigma_T + \epsilon \sigma_L) = \Gamma \sigma_r.$$ \hspace{1cm} (2)

Here, $\Gamma = K(\alpha/2\pi^2Q^2)(E'/E)/(1 - \epsilon)$ is the flux of transverse virtual photons with the total flux $K = \nu(1 - x)$ in the Hand convention [14], $\epsilon$ the relative longitudinal flux, $d \Omega$ the differential solid angle and $E$ ($E'$) the energy of the incoming (scattered) electron with four-momentum transfer $Q^2$ and energy transfer $\nu = E - E'$. On the right-hand side, $\sigma_r$ is called the reduced cross section. Fitting $\sigma_r$ linearly in $\epsilon$ yields $\sigma_L$ as the slope, and $\sigma_T$ as the intercept. The $F_2$ structure function can then be obtained from

$$F_2(x, Q^2) = \frac{K\nu}{4\pi^2\alpha(1 + \nu^2/Q^2)}[\sigma_T(x, Q^2) + \sigma_L(x, Q^2)].$$ \hspace{1cm} (3)

At leading order, the structure function $F_2$ can be written in terms of the light-cone momentum distribution of partons in the Bjorken limit, ($Q^2 \rightarrow \infty$ and at fixed $x$) as

$$F_2 = x \sum_i e_i^2 [q_i(x, Q^2) + \bar{q}_i(x, Q^2)].$$ \hspace{1cm} (4)

The moments of $F_2$, defined as $\int x^{N-2} d^2 \sigma / d \Omega d E'$, only receive contributions from operators with spin $N$. This is not true at finite $Q^2$, where operators with other spins can contribute. However, Nachtmann [15] showed that the contribution to the moments from operators with spin $N$ can be projected out by defining moments in terms of the Nachtmann scaling variable $\xi$ as

$$M_2^{(N)}(Q^2) = \int_0^1 dx \frac{x^{N+1}}{3} \left[ (3 + 3(N + 1)r + N(N + 2)r^2) x^3 \right] F_2(x, Q^2),$$ \hspace{1cm} (5)

where $N$ is the order of the moment, $\xi = (2x/r + 1 + r)$ is the fraction of the light cone momentum of the struck quark, and $r = \sqrt{1 + 4M^2x^2/Q^2}$. It is the Nachtmann moments of the data that must be employed for a meaningful
comparison to quark distribution moments calculated from LQCD or those determined from perturbative QCD (pQCD) fits.

In the Bjorken limit, structure function moments are independent of $Q^2$ (a phenomenon called scaling). At finite $Q^2$, gluon radiative effects, which give rise to scaling violations, and higher twist effects (i.e., interactions between the struck quark and remaining quarks) which give rise to the $Q^2$ dependence of the structure functions, become important. The $Q^2$ dependence of the moments can be studied within the framework of pQCD, but at lower $Q^2$, pQCD loses its applicability and one must consider finite $Q^2$ effects as well to study the hadronic structure and revert to effective theories or LQCD.

Current LQCD calculations have focused on nonsinglet $u−d$ quantities using moments of the PDFs, which are calculationally simpler because the complicated disconnected diagrams cancel. Experimentally, the integrated nonsinglet distribution can be determined from $2p−d$, which is approximately $p−n$, where $p$, $d$, and $n$ denote the proton, deuteron, and neutron moments, respectively. From Eq. (4), the nonsinglet structure function is

$$F_2^p − F_2^n = \frac{1}{3} (u−d + \bar{u}−\bar{d}) \approx 2F_2^u − F_2^d,$$

where $u$ and $d$ are the up and down quark distributions, respectively. Similarly, the nonsinglet Nachtmann moments can be determined as $M_2^{NS} = M_2^p − M_2^n \approx 2M_2^u − M_2^d$, where $M_2^{p+n}$ is obtained from deuteron data as described below. In the $\overline{\text{MS}}$ renormalization scheme, the nonsinglet moments of the PDFs, $\langle x \rangle_{u−d}$, as calculated in LQCD, which describes the soft, nonperturbative physics, in terms of the nonsinglet $N=2$ moment of the $F_2$ structure function can be written as

$$\langle x \rangle_{u−d} = \frac{3}{C_{\text{U}}^N} M_2^{NS},$$

where $C_{\text{U}}^N$ are Wilson coefficients which represent the hard, perturbatively calculable coefficient functions. Since PDFs describe nonperturbative behavior, they cannot be directly calculated in perturbative QCD, but they can be calculated using LQCD, or extracted from global fits to a variety of data, for example, Refs. [16–18].

Although there exist previous deuteron $F_2$ measurements in the nucleon resonance region, those presented in this work are the most precise and accurate determinations to date for several reasons. First, the moments presented here are the first to utilize deuteron and proton $F_2$ values extracted from precision Rosenbluth separations of the structure functions, while previous moment determinations [12] relied on models of the longitudinal contribution.

Second, the quasielastic (QE) contribution was precisely determined and then subtracted utilizing the same dataset. This is important, because inelastic and quasielastic are treated separately in theory. Third, the deuteron data were corrected for nuclear effects such as Fermi motion, enabling a clean extraction of $p+n$. In all, comparison of these new measurements to the previous $F_2$ moments from Refs. [12] and [13] shows an order of magnitude reduction in the uncertainties.

As noted above, inelastic and QE contributions were separated first by removing the latter utilizing the shape of the QE given in Ref. [19] with the magnitude determined from the experimental data by scaling up the shape to match the data while the inelastic shape given by a global fit [20] to the available deuteron data. The elastic contribution then was added back at $x=1$. Figure 1 shows the deuteron structure function $F_2$ in the QE region before and after the QE subtraction. Systematic uncertainties for this subtraction were determined by the following procedure: First, the QE contribution was scaled up and down until the chi-squared value between the data and the fit (QE and inelastic) becomes +1 and −1, and then the difference of the fit from the data was used as the systematic uncertainty.

Since the deuteron is a bound nucleus and not a pure $p+n$ state, $F_2^d$ was corrected for nuclear effects such as Fermi motion momentum smearing, which washes out the resonant structure, as shown in the top curve of Fig. 2. $F_2^{p+n}$ is obtained from $F_2^d$ as $F_2^{p+n} = f(x)F_2^d$, where the correction factor is given by

$$f(x) = \frac{(F_2^p + F_2^n)_{\text{fit}}}{(F_2^d)_{\text{fit}}}. \quad (8)$$

The global fit to all deuteron data (which determines the neutron) in the resonance region is from Ref. [20]. This utilizes the weak binding approximation convolution

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FIG. 1. Jefferson Lab experiment E06-009 data on deuteron $F_2$ before and after subtraction of the QE contribution. The band at the bottom represents the estimated systematic uncertainty from this procedure. The dot dashed curve is the QE model, the short dashed curve is the total (QE + inelastic) model and the solid curve is the inelastic model.
and structure is now quite visible. For moment E06-009 at large precision resonance region data from Jefferson Lab experiment E06-009 [26,27] which different wave functions based on different nucleon-nucleon corrections are estimated as the difference of the nominal about 4\% contribution from nuclear corrections is quite small being less than 0.1\%, while the systematic uncertainties due to the absolute angle and Fermi corrections provide the largest contributions to the total uncertainty and are given in the tables below. The E06-009 data have been determined from the global fit [20] to be normalized to better than 1\% relative to that of SLAC [23] and thus the relative normalization uncertainty contribute negligibly to the uncertainties on the moments.

Results for the experimental Nachtmann moments of the unpolarized structure function $F_2^{n+p}$ are given in Table I for $Q^2 = 4$ GeV$^2$. The systematic uncertainties labeled as Sys1, Sys2, and Sys3 are due to the absolute angle uncertainty, wave function dependence of the Fermi corrections, and off-shell corrections. Utilizing the previous proton structure function moment determinations given in Table II from Refs. [29,30], the nonsinglet moments of the structure functions were extracted from $M_2^{SS} = 2M_2^p - M_2^{p+n}$.

Since the previous proton moments [29,30] were evaluated at $Q^2 = 3.75$ GeV$^2$, they needed to be brought to the common $Q^2$ value of 4 GeV$^2$. This was accomplished utilizing moments calculated from fits to the global data set with the results given in Table II. The uncertainty from this procedure was estimated to be negligible. The nonsinglet Nachtmann moments obtained from $2M_2^p - M_2^{p+n}$ are given in Table III, together with the recent LQCD calculations from Refs. [9] and [2]. The experimental $u - d$ moments given in the fourth column are obtained from Eq. (7) as explained below after adding the elastic

![FIG. 2. Top curve: Data on $F_2^D$ at $Q^2 = 4$ GeV$^2$ from SLAC, CERN, and Jefferson Lab experiment E06-009. Top curve is shifted up for comparison to bottom. Bottom curve: Same as top data after Fermi correction.](image)

| TABLE I. Experimental Nachtmann moments of $F_2^{n+p}$ (i.e., the deuteron after nuclear corrections) at $Q^2 = 4$ GeV$^2$. Sys1, Sys2, and Sys3 are the estimated systematic uncertainties due to the absolute angle, wave function, and off-shell parameter in the Fermi correction, respectively. The column labeled P2P indicates the quadrature sum of statistical and uncorrelated systematic uncertainties. |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| $N$ | $M_2^{p+n}$ $(10^{-3})$ | P2P Unc. $(10^{-3})$ | Sys1 $(10^{-3})$ | Sys2 $(10^{-3})$ | Sys3 $(10^{-3})$ |
| 2 | 301.1 | 0.6 | 0.3 | 0.6 | 3 |
| 4 | 31.4 | 0.1 | 0.2 | 0 | 0.8 |
| 6 | 7.8 | 0.02 | 0.1 | 0 | 0.3 |
The calculation proceeds directly from the virtual Compton amplitude, as outlined in Ref. [35], in very much the same way as the moments are extracted from the experimental data, rather than from the leading twist operator matrix element [36]. No renormalization is needed. This skirts the issue of renormalization and mixing with operators of higher twist [37], which impair previous lattice calculations. The moments (\chi^{N-1}_{LQCD}) refer to \(Q^2 = 4\) GeV\(^2\) and are obtained by factoring out the appropriate Wilson coefficient in the \(\overline{\text{MS}}\) scheme, just like in Eq. (7). In contrast, the last column, labeled as LQCD2, is one of the most recent calculations from Ref. [2] performed at the physical pion mass. LQCD1 provides a great improvement in the agreement with the experimental data while the latter, although performed at the physical pion mass, is systematically higher than the experimental value.

Figure 3 shows a collection of recent lattice QCD calculations of \(N = 2\) nonsinglet moments performed at various pion masses at \(Q^2 = 4\) GeV\(^2\) together with the experimental results obtained in this analysis. The experimental results are shown with (red diamond) and without (black diamond) the elastic contribution. The one without the elastic contribution (black diamond) is shifted left for clarity purposes. LQCD calculations for twisted mass fermion results (filled triangles, filled inverse triangles, and open triangle) are taken from Ref. [2]. Also shown are the results from RBC and UKQCD Collaborations (filled circle) [3], LHPC Collaborations (filled rectangles) [4], QCDSF and UKQCD Collaborations (open rectangles) [5], LHPC (plus marker) [6] and (star) [7], and RQCD (open circle) [38,39]. The open diamond is the average of the moments obtained from three different PDF sets determined from pQCD fits [8,40,41], with the band indicating the range. Finally, the inverted black triangle is the newest calculations from QCDSF [9], which uses a new approach, as explained earlier.

At the time the E06-009 experiment was proposed, all LQCD calculations of moments were at large pion mass and disagreed with the values extracted from the available data. Increasing computing power in recent years has made it possible to perform LQCD calculations approaching the real pion mass, thus eliminating the need for calculation at higher values and subsequent extrapolations. The availability of the new precision nonsinglet quark moments

<table>
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<th>(N)</th>
<th>(M_{2S}^N (Q^2 = 3.75)) (10(^{-3}))</th>
<th>Unc. (10(^{-3}))</th>
<th>(M_{2S}^N (Q^2 = 4)) (10(^{-3}))</th>
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<td>6</td>
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<td>5.1</td>
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</table>

TABLE III. Nonsinglet (NS) Nachtmann moments of the unpolarized structure function \(F_2\) at \(Q^2 = 4.0\) GeV\(^2\). The moments including the elastic contribution are given in the third column. The fourth column is the experimental \(u - \bar{d}\) nonsinglet moments obtained as shown in Eq. (7). Columns five and six are recent LQCD calculations from Refs. [9] and [2], respectively.
presented in this work allow several important points that can be gleaned from Fig. 3. First, it is clear that new calculations pushing down to the physical pion mass have not fully resolved the systematic differences with the lower values given by data. Second, new alternative LQCD methods \[9\], allowing calculation of the quark distributions directly, give improved agreement with the data and are found to be in agreement within the 1σ uncertainties for not only the \(N = 2\) moments, but for the \(N = 4\) moments as well. The agreement between the data and PDF extractions may indicate a suppression of higher twists (HT), as recently predicted in Ref. \[42\], due to the fact that the cuts applied to remove low \(W\) and \(Q^2\) from the global data set used to extract these PDFs, the HT is relatively suppressed and likely smaller than the actual resonance region data used here. A systematic study of HT in the moment requires a range in \(Q^2\) which can now be performed with our precision dataset covering the range \(0 < Q^2 < 5 \text{ GeV}^2\). This will be the subject of a forthcoming publication.

In this Letter, we have presented new experimental extractions of the nonsinglet \(F_2\) structure function moments, as well as nonsinglet quark distribution moments with precision many times better than previous extractions. This improved precision is afforded by new precision data on the deuteron \(F_2\) structure function from Jefferson Lab experiment E06-009, as well as previous proton and deuteron data from Jefferson Lab, CERN, and SLAC. The results have been compared to recent LQCD calculations including those carried out at the real pion mass and those new calculations from QCDSF at a higher pion mass which use a new approach. Although experimental measurements were found to be smaller than the LQCD calculations performed at the real pion mass, they were found to be in far better agreement with those from QCDSF. While there are still problems to overcome in comparing lattice calculations to data, such as residual finite volume effects, renormalization, and mixing, these new QCDSF results hint at a breakthrough in LQCD calculations after several decades. Improvements are necessary, as high precision data now exist for comparison. The nonsinglet quark moments presented in this Letter provide a benchmark for LQCD and the study of nucleon structure within QCD and highlight the need to study the differences in LQCD calculations. Additionally, results for higher moments for \(N = 4\) and 6 are also presented in this work which can be utilized for confronting future LQCD calculations.

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\[9\] G. Schierholz (private communication).
\[16\] www.cteq.org.
[29] P. Monaghan (private communication).