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# T-DEPENDENCE OF THE AXION MASS WHEN THE $U_A(1)$ AND CHIRAL SYMMETRY BREAKING ARE TIED\*

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Modulo the scale of spontaneous breaking of Peccei–Quinn symmetry, the axion mass  $m_a(T)$  is given by the QCD topological susceptibility  $\chi(T)$  at all temperatures  $T$ . From an approach tying the  $U_A(1)$  and chiral symmetry breaking and getting good  $T$ -dependence of  $\eta$  and  $\eta'$  mesons, we get  $\chi(T)$  for an effective Dyson–Schwinger model of nonperturbative QCD. Comparison with lattice results for  $\chi(T)$ , and thus also for  $m_a(T)$ , shows good agreement for temperatures ranging from zero up to the double of the chiral restoration temperature  $T_c$ .

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## 1. Introduction

The fundamental theory of strong interactions, QCD, has the so-called Strong CP problem. Namely, there is no experimental evidence of any CP-symmetry violation in strong interactions, although there is in principle no reason why the QCD Lagrangian should not include the so-called  $\Theta$ -term  $\mathcal{L}^\Theta$ , where gluon fields  $F_{\mu\nu}^b(x)$  comprise the CP-violating combination  $Q(x)$

$$\mathcal{L}^\Theta(x) = \Theta \frac{g^2}{64\pi^2} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}^b(x) F_{\rho\sigma}^b(x) \equiv \Theta Q(x). \quad (1)$$

Admittedly,  $\mathcal{L}^\Theta$  can be rewritten as a total divergence, but, unlike in QED, this does not enable discarding it in spite of the gluon fields vanishing sufficiently fast as  $|x| \rightarrow \infty$ . This is because of nontrivial topological structures in QCD, such as instantons, which are important for, *e.g.* solving of the  $U_A(1)$  problem and yielding the anomalously large mass of the  $\eta'$  meson.

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Thus, there is no reason why the coefficient  $\Theta$  of this term should be of a very different magnitude from the coefficients of the other, CP-symmetric terms comprising the usual CP-symmetric QCD Lagrangian. Nevertheless, the experimental bound on the coefficient of the term is extremely low,  $|\Theta| < 10^{-10}$  [1], and consistent with zero. This is the mystery of the missing strong CP violation: why is  $\Theta$  so small?

Various proposed theoretical solutions stood the test of time with varying success. A long-surviving solution, which is actually the preferred solution nowadays, is a new particle beyond the Standard Model — the axion. Important is also that axions turned out to be very interesting also for cosmology, as promising candidates for dark matter. (See, *e.g.*, [2, 3].)

## 2. Axion mass from the non-Abelian axial anomaly

Peccei and Quinn introduced [4, 5] a new axial global symmetry  $U(1)_{\text{PQ}}$  which is broken spontaneously at some scale  $f_a$ . This presumably huge [6] but otherwise presently unknown scale is the key free parameter of axion theories, which determines the absolute value of the axion mass  $m_a$ . However, this constant cancels from combinations such as  $m_a(T)/m_a(0)$ . Hence, useful insights and applications are possible in spite of  $f_a$  being not known.

We have often, including applications at  $T > 0$  [7–11], employed a chirally well-behaved relativistic bound-state approach to modeling nonperturbative QCD through Dyson–Schwinger equations (DSE) for Green’s functions of the theory. (For reviews, see [12–14] for example.) Such calculations can yield model predictions on the QCD topological susceptibility, including its temperature dependence  $\chi(T)$ , which are correctly related to the QCD dynamical chiral symmetry breaking (DChSB) and restoration. It turns out that  $\chi(T)$  is precisely that factor in the axion mass  $m_a(T)$ , which carries the nontrivial  $T$ -dependence.

### 2.1. Axions as quasi-Goldstone bosons

The pseudoscalar axion field  $a(x)$  arises as the (would-be massless) Goldstone boson of the spontaneous breaking of the Peccei–Quinn symmetry [15, 16]. The axion contributes to the total Lagrangian its kinetic term and its interaction with the Standard-Model fermions. However, what is important for the resolution of the strong CP problem, is that the axion also couples to the topological charge density operator  $Q(x)$  in Eq. (1). Then, the  $\Theta$ -term in the QCD Lagrangian gets modified to

$$\mathcal{L}_{\text{axion}}^{\Theta+} = \mathcal{L}^{\Theta} + \frac{a(x)}{f_a} Q(x) = \left( \Theta + \frac{a}{f_a} \right) \frac{g^2}{64\pi^2} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}^b F_{\rho\sigma}^b. \quad (2)$$

Through this coupling of the axion to gluons, the  $U(1)_{\text{PQ}}$  symmetry is also broken *explicitly* by the  $U_A(1)$  non-Abelian, gluon axial anomaly, so that the axion has a nonvanishing mass,  $m_a \neq 0$  [15, 16].

Gluons generate an effective axion potential, and its minimization leads to the axion expectation value  $\langle a \rangle$  such that the modified coefficient, multiplying the topological charge density  $Q(x)$ , should vanish

$$\Theta + \frac{\langle a \rangle}{f_a} = 0. \tag{3}$$

The strong CP problem is thereby solved, irrespective of the initial value of  $\Theta$ . Relaxation from any  $\Theta$ -value in the early Universe towards the minimum at Eq. (3) is known as misalignment production, and the resulting axion oscillation energy is a cold dark matter candidate (*e.g.*, see [2, 3]).

*2.2. Axion mass from anomalous  $U_A(1)$  breaking driven by DChSB*

A direct measure of the  $U_A(1)$  symmetry breaking is the topological susceptibility  $\chi$ , given by the convolution of the time-ordered product  $\mathcal{T}$  of the topological charge densities  $Q(x)$  defined by Eq. (1) [or Eq. (2)]

$$\chi = \int d^4x \langle 0 | \mathcal{T} Q(x) Q(0) | 0 \rangle. \tag{4}$$

The expansion of the effective axion potential reveals in its quadratic term that the axion mass squared (times  $f_a^2$ ) is equal<sup>1</sup> to the QCD topological susceptibility. This holds for all temperatures  $T$

$$m_a^2(T) f_a^2 = \chi(T). \tag{5}$$

On the other hand, in our study [11] of the  $T$ -dependence of the  $\eta$  and  $\eta'$  masses and the influence of the anomalous  $U_A(1)$  breaking and restoration, we used the light-quark-sector result [17–19]

$$\chi(T) = \frac{-1}{\frac{1}{m_u \langle \bar{u}u \rangle(T)} + \frac{1}{m_d \langle \bar{d}d \rangle(T)} + \frac{1}{m_s \langle \bar{s}s \rangle(T)}} + \mathcal{C}_m, \tag{6}$$

where  $\mathcal{C}_m$  is a very small correction term of higher orders in the small current quark masses  $m_q$  ( $q = u, d, s$ ), and in the present context, we do not consider it further. Thus, the overwhelming part, namely the leading term of  $\chi$ , is given by the quark condensates  $\langle \bar{q}q \rangle$  ( $q = u, d, s$ ), which arise as order parameters of DChSB. Their temperature dependence determines that

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<sup>1</sup> To a high level of accuracy, since corrections to Eq. (5) are of the order of  $M_\pi^2/f_a^2$  [20], where the pion mass  $M_\pi$  is negligible.

of  $\chi(T)$ , which in turn determines the  $T$ -dependence of the anomalous part of the pseudoscalar meson masses in the  $\eta$ - $\eta'$  complex. This is the mechanism of Ref. [11], how DChSB and chiral restoration drive, respectively, the anomalous breaking and restoration of the  $U_A(1)$  symmetry of QCD.

Now, Eqs. (5) and (6) show that this mechanism determines also the  $T$ -dependence of the axion mass.

To describe  $\eta'$  and  $\eta$  mesons, it is essential to include  $U_A(1)$  symmetry breaking at least at the level of the masses. This could be done simply [23–25], by adding the anomalous contribution to isoscalar meson masses as a perturbation, thanks to the fact that the  $U_A(1)$  anomaly is suppressed in the limit of large number of QCD colors  $N_c$  [26, 27]. Concretely, Ref. [25] adopted Shore’s equations [28], where the  $U_A(1)$ -anomalous contribution to the light pseudoscalar masses is expressed through the condensates of light quarks with nonvanishing current masses. They are thus used also in  $\chi(T)$  (6), since this approach has recently been extended [11] to  $T > 0$ . This gave us our results for  $\chi(T)$  depicted in Fig. 1.

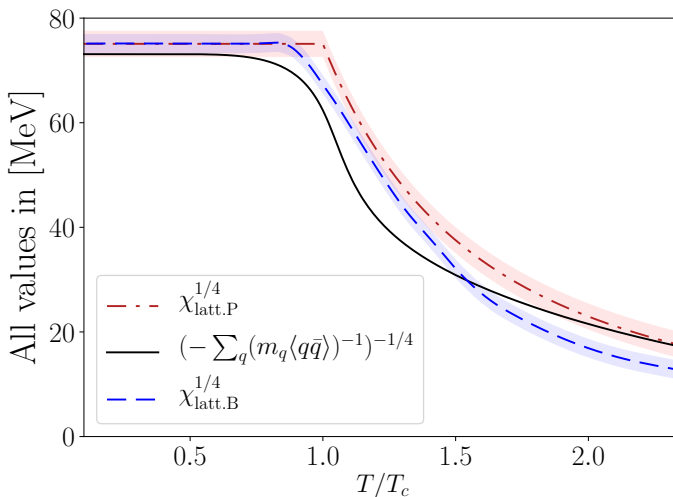


Fig. 1. The relative temperature  $T/T_c$  dependence of (the leading term of)  $\chi(T)$  from our oft-adopted [7–11] chirally well-behaved DSE model (solid curve), and from lattice: dash-dotted curve extracted from Petreczky *et al.* [21] and dashed curve extracted from Borsany *et al.* [22].

Indeed, the now established smooth, crossover behavior around the pseudocritical temperature  $T_c$  for the chiral transition, is obtained for the DChSB condensates of realistically massive light quarks — *i.e.*, the quarks with realistic explicit chiral symmetry breaking [11]. In contrast, using in Eq. (6) the massless quark condensate  $\langle \bar{q}q \rangle_0$  (which drops sharply to zero at  $T_c$ ) instead

of the “massive” ones, would dictate a sharp transition of the second order at  $T_c$  [10, 11] also for  $\chi(T)$ . Obviously, this would imply that axions are massless for  $T > T_c$ .

In Fig. 1, we present (the leading term of) our model-calculated [11]  $\chi(T)^{1/4}$ , depicted as the solid curve. Due to Eq. (5), this is our model prediction for  $\sqrt{m_a(T) f_a}$ . For temperatures up to  $T \approx 2.3 T_c$ , we compare it to the lattice results of Petreczky *et al.* [21] and of Borsany *et al.* [22], rescaled to the relative temperature  $T/T_c$ .

### 3. Summary

The axion mass and its temperature dependence  $m_a(T)$  can be calculated in an effective model of nonperturbative QCD (up to the constant scale parameter  $f_a$ ) as the square root of the topological susceptibility  $\chi(T)$ . We obtained it from the condensates of  $u$ -,  $d$ - and  $s$ -quarks and antiquarks calculated in the SDE approach using a simplified nonperturbative model interaction [11]. Our prediction on  $m_a(T)$  is thus supported by the fact that our topological susceptibility also yields the  $T$ -dependence of the  $U_A(1)$  anomaly-influenced masses of  $\eta'$  and  $\eta$  mesons which is consistent with experimental evidence [11].

Our result on  $\chi(T)$  and the related axion mass is qualitatively similar to the one obtained in the framework of the NJL model [29]. Our topological susceptibility is also qualitatively similar to the pertinent lattice results [21, 22], except that our dynamical model could so far access only much smaller range of temperatures,  $T < 2.4 T_c$ . On the other hand, the lattice supports the smooth crossover transition of  $\chi(T)$ , which is, in our approach, the natural consequence of employing the massive-quark condensates exhibiting crossover around the chiral restoration temperature  $T_c$ . Hence, the (partial)  $U_A(1)$  restoration observed in Ref. [11] must also be a crossover, which in the present work, as well as in its longer counterpart [30] containing a detailed analysis of the model parameter dependence, translates into the corresponding smooth  $T$ -dependence of the axion mass.

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