T-DEPENDENCE OF THE AXION MASS WHEN THE $\mathrm{U}_A(1)$ AND CHIRAL SYMMETRY BREAKING ARE TIED*

Dubravko Klabučar, Davor Horvatić

Physics Department, Faculty of Science — PMF, University of Zagreb, Croatia

Dalibor Kekez

Ruđer Bošković Institute, Zagreb, Croatia

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Modulo the scale of spontaneous breaking of Peccei–Quinn symmetry, the axion mass $m_a(T)$ is given by the QCD topological susceptibility $\chi(T)$ at all temperatures T. From an approach tying the $\mathrm{U}_A(1)$ and chiral symmetry breaking and getting good T-dependence of η and η' mesons, we get $\chi(T)$ for an effective Dyson–Schwinger model of nonperturbative QCD. Comparison with lattice results for $\chi(T)$, and thus also for $m_a(T)$, shows good agreement for temperatures ranging from zero up to the double of the chiral restoration temperature T_{c}.

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1. Introduction

The fundamental theory of strong interactions, QCD, has the so-called Strong CP problem. Namely, there is no experimental evidence of any CP-symmetry violation in strong interactions, although there is in principle no reason why the QCD Lagrangian should not include the so-called Θ-term $\mathcal{L}^{\Theta}$, where gluon fields $F^b_{\mu\nu}(x)$ comprise the CP-violating combination $Q(x)$

$$\mathcal{L}^{\Theta}(x) \,=\, \Theta \, \frac{g^2}{64\,\pi^2} \, \epsilon^{\mu\nu\rho\sigma} \, F^b_{\mu\nu}(x) \, F^b_{\rho\sigma}(x) \,\equiv\, \Theta \, Q(x) \, . \tag{1}$$

Admittedly, $\mathcal{L}^{\Theta}$ can be rewritten as a total divergence, but, unlike in QED, this does not enable discarding it in spite of the gluon fields vanishing sufficiently fast as $|x| \to \infty$. This is because of nontrivial topological structures in QCD, such as instantons, which are important for, *e.g.* solving of the $\mathrm{U}_A(1)$ problem and yielding the anomalously large mass of the η' meson.

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Thus, there is no reason why the coefficient Θ of this term should be of a very different magnitude from the coefficients of the other, CP-symmetric terms comprising the usual CP-symmetric QCD Lagrangian. Nevertheless, the experimental bound on the coefficient of the term is extremely low, $|\Theta| < 10^{-10}$ [1], and consistent with zero. This is the mystery of the missing strong CP violation: why is Θ so small?

Various proposed theoretical solutions stood the test of time with varying success. A long-surviving solution, which is actually the preferred solution nowadays, is a new particle beyond the Standard Model — the axion. Important is also that axions turned out to be very interesting also for cosmology, as promising candidates for dark matter. (See, *e.g.*, [2, 3].)

2. Axion mass from the non-Abelian axial anomaly

Peccei and Quinn introduced [4, 5] a new axial global symmetry $\mathrm{U}(1)_{\mathrm{PQ}}$ which is broken spontaneously at some scale f_a. This presumably huge [6] but otherwise presently unknown scale is the key free parameter of axion theories, which determines the absolute value of the axion mass m_a. However, this constant cancels from combinations such as $m_a(T)/m_a(0)$. Hence, useful insights and applications are possible in spite of f_a being not known.

We have often, including applications at $T > 0$ [7–11], employed a chirally well-behaved relativistic bound-state approach to modeling nonperturbative QCD through Dyson–Schwinger equations (DSE) for Green's functions of the theory. (For reviews, see [12–14] for example.) Such calculations can yield model predictions on the QCD topological susceptibility, including its temperature dependence $\chi(T)$, which are correctly related to the QCD dynamical chiral symmetry breaking (DChSB) and restoration. It turns out that $\chi(T)$ is precisely that factor in the axion mass $m_a(T)$, which carries the nontrivial T-dependence.

2.1. Axions as quasi-Goldstone bosons

The pseudoscalar axion field $a(x)$ arises as the (would-be massless) Goldstone boson of the spontaneous breaking of the Peccei–Quinn symmetry [15, 16]. The axion contributes to the total Lagrangian its kinetic term and its interaction with the Standard-Model fermions. However, what is important for the resolution of the strong CP problem, is that the axion also couples to the topological charge density operator $Q(x)$ in Eq. (1). Then, the Θ-term in the QCD Lagrangian gets modified to

$$\mathcal{L}^{\Theta\,+}_{\mathrm{axion}} = \mathcal{L}^{\Theta} + \frac{a(x)}{f_a}\, Q(x) = \left(\Theta + \frac{a}{f_a}\right) \frac{g^2}{64\pi^2}\, \epsilon^{\mu\nu\rho\sigma} F^b_{\mu\nu}\, F^b_{\rho\sigma}\,. \tag{2}$$

Through this coupling of the axion to gluons, the $\mathrm{U}(1)_{\mathrm{PQ}}$ symmetry is also broken *explicitly* by the $\mathrm{U}_A(1)$ non-Abelian, gluon axial anomaly, so that the axion has a nonvanishing mass, $m_a \neq 0$ [15, 16].

Gluons generate an effective axion potential, and its minimization leads to the axion expectation value $\langle a \rangle$ such that the modified coefficient, multiplying the topological charge density $Q(x)$, should vanish

$$\Theta + \frac{\langle a \rangle}{f_a} = 0\,. \tag{3}$$

The strong CP problem is thereby solved, irrespective of the initial value of Θ. Relaxation from any Θ-value in the early Universe towards the minimum at Eq. (3) is known as misalignment production, and the resulting axion oscillation energy is a cold dark matter candidate (*e.g.*, see [2, 3]).

2.2. Axion mass from anomalous $U_A(1)$ breaking driven by DChSB

A direct measure of the $\mathrm{U}_A(1)$ symmetry breaking is the topological susceptibility χ, given by the convolution of the time-ordered product $\mathcal{T}$ of the topological charge densities $Q(x)$ defined by Eq. (1) [or Eq. (2)]

$$\chi = \int \mathrm{d}^4x \, \langle 0 | \, \mathcal{T} \, Q(x) \, Q(0) \, | 0 \rangle \,. \tag{4}$$

The expansion of the effective axion potential reveals in its quadratic term that the axion mass squared (times f_a^2) is equal[1] to the QCD topological susceptibility. This holds for all temperatures T

$$m_a^2(T) \, f_a^2 = \chi(T)\,. \tag{5}$$

On the other hand, in our study [11] of the T-dependence of the η and η' masses and the influence of the anomalous $\mathrm{U}_A(1)$ breaking and restoration, we used the light-quark-sector result [17–19]

$$\chi(T) = \frac{-1}{\frac{1}{m_u \langle \bar{u}u\,(T)\rangle} + \frac{1}{m_d \langle \bar{d}d\,(T)\rangle} + \frac{1}{m_s \langle \bar{s}s\,(T)\rangle}} + \mathcal{C}_m\,, \tag{6}$$

where $\mathcal{C}_m$ is a very small correction term of higher orders in the small current quark masses m_q ($q = u, d, s$), and in the present context, we do not consider it further. Thus, the overwhelming part, namely the leading term of χ, is given by the quark condensates $\langle \bar{q}q \rangle$ ($q = u, d, s$), which arise as order parameters of DChSB. Their temperature dependence determines that

[1] To a high level of accuracy, since corrections to Eq. (5) are of the order of M_π^2/f_a^2 [20], where the pion mass M_π is negligible.

of $\chi(T)$, which in turn determines the T-dependence of the anomalous part of the pseudoscalar meson masses in the η–η' complex. This is the mechanism of Ref. [11], how DChSB and chiral restoration drive, respectively, the anomalous breaking and restoration of the $\mathrm{U}_A(1)$ symmetry of QCD.

Now, Eqs. (5) and (6) show that this mechanism determines also the T-dependence of the axion mass.

To describe η' and η mesons, it is essential to include $\mathrm{U}_A(1)$ symmetry breaking at least at the level of the masses. This could be done simply [23–25], by adding the anomalous contribution to isoscalar meson masses as a perturbation, thanks to the fact that the $\mathrm{U}_A(1)$ anomaly is suppressed in the limit of large number of QCD colors N_c [26, 27]. Concretely, Ref. [25] adopted Shore's equations [28], where the $\mathrm{U}_A(1)$-anomalous contribution to the light pseudoscalar masses is expressed through the condensates of light quarks with nonvanishing current masses. They are thus used also in $\chi(T)$ (6), since this approach has recently been extended [11] to $T > 0$. This gave us our results for $\chi(T)$ depicted in Fig. 1.

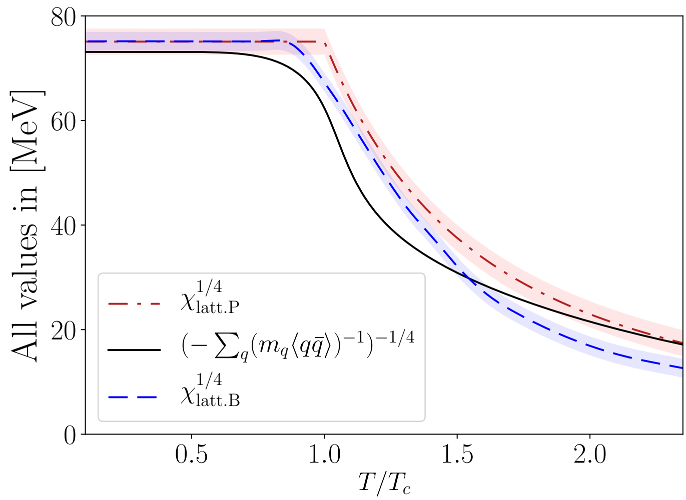

Fig. 1. The relative temperature T/T_c dependence of (the leading term of) $\chi(T)$ from our oft-adopted [7–11] chirally well-behaved DSE model (solid curve), and from lattice: dash-dotted curve extracted from Petreczky *et al.* [21] and dashed curve extracted from Borsany *et al.* [22].

Indeed, the now established smooth, crossover behavior around the pseudocritical temperature T_c for the chiral transition, is obtained for the DChSB condensates of realistically massive light quarks — *i.e.*, the quarks with realistic explicit chiral symmetry breaking [11]. In contrast, using in Eq. (6) the massless quark condensate $\langle\bar{q}q\rangle_0$ (which drops sharply to zero at T_c) instead of the "massive" ones, would dictate a sharp transition of the second order at T_c [10, 11] also for $\chi(T)$. Obviously, this would imply that axions are massless for $T > T_\mathrm{c}$.

In Fig. 1, we present (the leading term of) our model-calculated [11] $\chi(T)^{1/4}$, depicted as the solid curve. Due to Eq. (5), this is our model prediction for $\sqrt{m_a(T)\, f_a}$. For temperatures up to $T \approx 2.3\, T_\mathrm{c}$, we compare it to the lattice results of Petreczky *et al.* [21] and of Borsany *et al.* [22], rescaled to the relative temperature T/T_c.

3. Summary

The axion mass and its temperature dependence $m_a(T)$ can be calculated in an effective model of nonperturbative QCD (up to the constant scale parameter f_a) as the square root of the topological susceptibility $\chi(T)$. We obtained it from the condensates of u-, d- and s-quarks and antiquarks calculated in the SDE approach using a simplified nonperturbative model interaction [11]. Our prediction on $m_a(T)$ is thus supported by the fact that our topological susceptibility also yields the T-dependence of the $\mathrm{U}_A(1)$ anomaly-influenced masses of η' and η mesons which is consistent with experimental evidence [11].

Our result on $\chi(T)$ and the related axion mass is qualitatively similar to the one obtained in the framework of the NJL model [29]. Our topological susceptibility is also qualitatively similar to the pertinent lattice results [21, 22], except that our dynamical model could so far access only much smaller range of temperatures, $T < 2.4\, T_\mathrm{c}$. On the other hand, the lattice supports the smooth crossover transition of $\chi(T)$, which is, in our approach, the natural consequence of employing the massive-quark condensates exhibiting crossover around the chiral restoration temperature T_c. Hence, the (partial) $\mathrm{U}_A(1)$ restoration observed in Ref. [11] must also be a crossover, which in the present work, as well as in its longer counterpart [30] containing a detailed analysis of the model parameter dependence, translates into the corresponding smooth T-dependence of the axion mass.

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