T-dependence of the Axion Mass when the U_A(1) and Chiral Symmetry Breaking Are Tied

Klabučar, D.; Horvatić, D.; Kekez, D.

T-DEPENDENCE OF THE AXION MASS WHEN THE U$_A$(1) AND CHIRAL SYMMETRY BREAKING ARE TIED*

Dubravko Klabučar, Davor Horvatić

Physics Department, Faculty of Science — PMF, University of Zagreb, Croatia

Dalibor Kekez

Ruđer Bošković Institute, Zagreb, Croatia

Modulo the scale of spontaneous breaking of Peccei–Quinn symmetry, the axion mass $m_a(T)$ is given by the QCD topological susceptibility $\chi(T)$ at all temperatures T. From an approach tying the U$_A$(1) and chiral symmetry breaking and getting good T-dependence of η and η' mesons, we get $\chi(T)$ for an effective Dyson–Schwinger model of nonperturbative QCD. Comparison with lattice results for $\chi(T)$, and thus also for $m_a(T)$, shows good agreement for temperatures ranging from zero up to the double of the chiral restoration temperature T_c.

1. Introduction

The fundamental theory of strong interactions, QCD, has the so-called Strong CP problem. Namely, there is no experimental evidence of any CP-symmetry violation in strong interactions, although there is in principle no reason why the QCD Lagrangian should not include the so-called Θ-term $\mathcal{L}^\Theta$, where gluon fields $F_{\mu\nu}^b(x)$ comprise the CP-violating combination $Q(x)$

$$\mathcal{L}^\Theta(x) = \Theta \, \frac{g^2}{64\,\pi^2} \, \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}^b(x) \, F_{\rho\sigma}^b(x) \equiv \Theta \, Q(x) \,. \tag{1}$$

Admittedly, $\mathcal{L}^\Theta$ can be rewritten as a total divergence, but, unlike in QED, this does not enable discarding it in spite of the gluon fields vanishing sufficiently fast as $|x| \to \infty$. This is because of nontrivial topological structures in QCD, such as instantons, which are important for, *e.g.* solving of the U$_A$(1) problem and yielding the anomalously large mass of the η' meson.

---

* Presented at "Excited QCD 2019", Schladming, Austria, January 30–February 3, 2019.

Thus, there is no reason why the coefficient Θ of this term should be of a very different magnitude from the coefficients of the other, CP-symmetric terms comprising the usual CP-symmetric QCD Lagrangian. Nevertheless, the experimental bound on the coefficient of the term is extremely low, $|\Theta| < 10^{-10}$ [1], and consistent with zero. This is the mystery of the missing strong CP violation: why is Θ so small?

Various proposed theoretical solutions stood the test of time with varying success. A long-surviving solution, which is actually the preferred solution nowadays, is a new particle beyond the Standard Model — the axion. Important is also that axions turned out to be very interesting also for cosmology, as promising candidates for dark matter. (See, *e.g.*, [2, 3].)

2. Axion mass from the non-Abelian axial anomaly

Peccei and Quinn introduced [4, 5] a new axial global symmetry $U(1)_{PQ}$ which is broken spontaneously at some scale f_a. This presumably huge [6] but otherwise presently unknown scale is the key free parameter of axion theories, which determines the absolute value of the axion mass m_a. However, this constant cancels from combinations such as $m_a(T)/m_a(0)$. Hence, useful insights and applications are possible in spite of f_a being not known.

We have often, including applications at $T > 0$ [7–11], employed a chirally well-behaved relativistic bound-state approach to modeling nonperturbative QCD through Dyson–Schwinger equations (DSE) for Green's functions of the theory. (For reviews, see [12–14] for example.) Such calculations can yield model predictions on the QCD topological susceptibility, including its temperature dependence $\chi(T)$, which are correctly related to the QCD dynamical chiral symmetry breaking (DChSB) and restoration. It turns out that $\chi(T)$ is precisely that factor in the axion mass $m_a(T)$, which carries the nontrivial T-dependence.

2.1. Axions as quasi-Goldstone bosons

The pseudoscalar axion field $a(x)$ arises as the (would-be massless) Goldstone boson of the spontaneous breaking of the Peccei–Quinn symmetry [15, 16]. The axion contributes to the total Lagrangian its kinetic term and its interaction with the Standard-Model fermions. However, what is important for the resolution of the strong CP problem, is that the axion also couples to the topological charge density operator $Q(x)$ in Eq. (1). Then, the Θ-term in the QCD Lagrangian gets modified to

$$\mathcal{L}_{\text{axion}}^{\Theta+} = \mathcal{L}^{\Theta} + \frac{a(x)}{f_a}\,Q(x) = \left(\Theta + \frac{a}{f_a}\right)\frac{g^2}{64\pi^2}\,\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}^{b}\,F_{\rho\sigma}^{b}\,. \qquad (2)$$

Through this coupling of the axion to gluons, the $U(1)_{PQ}$ symmetry is also broken *explicitly* by the $U_A(1)$ non-Abelian, gluon axial anomaly, so that the axion has a nonvanishing mass, $m_a \neq 0$ [15, 16].

Gluons generate an effective axion potential, and its minimization leads to the axion expectation value $\langle a \rangle$ such that the modified coefficient, multiplying the topological charge density $Q(x)$, should vanish

$$\Theta + \frac{\langle a \rangle}{f_a} = 0 \,. \tag{3}$$

The strong CP problem is thereby solved, irrespective of the initial value of Θ. Relaxation from any Θ-value in the early Universe towards the minimum at Eq. (3) is known as misalignment production, and the resulting axion oscillation energy is a cold dark matter candidate (*e.g.*, see [2, 3]).

2.2. Axion mass from anomalous $U_A(1)$ breaking driven by DChSB

A direct measure of the $U_A(1)$ symmetry breaking is the topological susceptibility χ, given by the convolution of the time-ordered product $\mathcal{T}$ of the topological charge densities $Q(x)$ defined by Eq. (1) [or Eq. (2)]

$$\chi = \int \mathrm{d}^4 x \, \langle 0| \, \mathcal{T} \, Q(x) \, Q(0) \, |0\rangle \,. \tag{4}$$

The expansion of the effective axion potential reveals in its quadratic term that the axion mass squared (times f_a^2) is equal[1] to the QCD topological susceptibility. This holds for all temperatures T

$$m_a^2(T) \, f_a^2 = \chi(T) \,. \tag{5}$$

On the other hand, in our study [11] of the T-dependence of the η and η' masses and the influence of the anomalous $U_A(1)$ breaking and restoration, we used the light-quark-sector result [17–19]

$$\chi(T) = \frac{-1}{\frac{1}{m_u \langle \bar{u}u(T) \rangle} + \frac{1}{m_d \langle \bar{d}d(T) \rangle} + \frac{1}{m_s \langle \bar{s}s(T) \rangle}} + \mathcal{C}_m \,, \tag{6}$$

where $\mathcal{C}_m$ is a very small correction term of higher orders in the small current quark masses m_q ($q = u, d, s$), and in the present context, we do not consider it further. Thus, the overwhelming part, namely the leading term of χ, is given by the quark condensates $\langle \bar{q}q \rangle$ ($q = u, d, s$), which arise as order parameters of DChSB. Their temperature dependence determines that

---

[1] To a high level of accuracy, since corrections to Eq. (5) are of the order of M_π^2/f_a^2 [20], where the pion mass M_π is negligible.

of $\chi(T)$, which in turn determines the T-dependence of the anomalous part of the pseudoscalar meson masses in the η–η' complex. This is the mechanism of Ref. [11], how DChSB and chiral restoration drive, respectively, the anomalous breaking and restoration of the $U_A(1)$ symmetry of QCD.

Now, Eqs. (5) and (6) show that this mechanism determines also the T-dependence of the axion mass.

To describe η' and η mesons, it is essential to include $U_A(1)$ symmetry breaking at least at the level of the masses. This could be done simply [23–25], by adding the anomalous contribution to isoscalar meson masses as a perturbation, thanks to the fact that the $U_A(1)$ anomaly is suppressed in the limit of large number of QCD colors N_c [26, 27]. Concretely, Ref. [25] adopted Shore's equations [28], where the $U_A(1)$-anomalous contribution to the light pseudoscalar masses is expressed through the condensates of light quarks with nonvanishing current masses. They are thus used also in $\chi(T)$ (6), since this approach has recently been extended [11] to $T > 0$. This gave us our results for $\chi(T)$ depicted in Fig. 1.
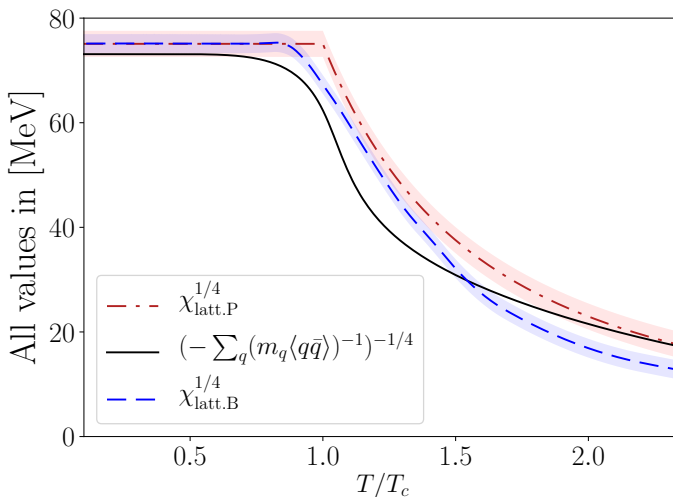

Fig. 1. The relative temperature T/T_c dependence of (the leading term of) $\chi(T)$ from our oft-adopted [7–11] chirally well-behaved DSE model (solid curve), and from lattice: dash-dotted curve extracted from Petreczky *et al.* [21] and dashed curve extracted from Borsany *et al.* [22].

Indeed, the now established smooth, crossover behavior around the pseudocritical temperature T_c for the chiral transition, is obtained for the DChSB condensates of realistically massive light quarks — *i.e.*, the quarks with realistic explicit chiral symmetry breaking [11]. In contrast, using in Eq. (6) the massless quark condensate $\langle \bar{q}q \rangle_0$ (which drops sharply to zero at T_c) instead

of the "massive" ones, would dictate a sharp transition of the second order at T_c [10, 11] also for $\chi(T)$. Obviously, this would imply that axions are massless for $T > T_c$.

In Fig. 1, we present (the leading term of) our model-calculated [11] $\chi(T)^{1/4}$, depicted as the solid curve. Due to Eq. (5), this is our model prediction for $\sqrt{m_a(T)\,f_a}$. For temperatures up to $T \approx 2.3\,T_c$, we compare it to the lattice results of Petreczky *et al.* [21] and of Borsany *et al.* [22], rescaled to the relative temperature T/T_c.

3. Summary

The axion mass and its temperature dependence $m_a(T)$ can be calculated in an effective model of nonperturbative QCD (up to the constant scale parameter f_a) as the square root of the topological susceptibility $\chi(T)$. We obtained it from the condensates of u-, d- and s-quarks and antiquarks calculated in the SDE approach using a simplified nonperturbative model interaction [11]. Our prediction on $m_a(T)$ is thus supported by the fact that our topological susceptibility also yields the T-dependence of the $U_A(1)$ anomaly-influenced masses of η' and η mesons which is consistent with experimental evidence [11].

Our result on $\chi(T)$ and the related axion mass is qualitatively similar to the one obtained in the framework of the NJL model [29]. Our topological susceptibility is also qualitatively similar to the pertinent lattice results [21, 22], except that our dynamical model could so far access only much smaller range of temperatures, $T < 2.4\,T_c$. On the other hand, the lattice supports the smooth crossover transition of $\chi(T)$, which is, in our approach, the natural consequence of employing the massive-quark condensates exhibiting crossover around the chiral restoration temperature T_c. Hence, the (partial) $U_A(1)$ restoration observed in Ref. [11] must also be a crossover, which in the present work, as well as in its longer counterpart [30] containing a detailed analysis of the model parameter dependence, translates into the corresponding smooth T-dependence of the axion mass.

REFERENCES

[1] C.A. Baker *et al.*, *Phys. Rev. Lett.* **97**, 131801 (2006).

[2] O. Wantz, E.P.S. Shellard, *Phys. Rev. D* **82**, 123508 (2010).

[3] J.E. Kim, S. Nam, Y.K. Semetzidis, *Int. J. Mod. Phys. A* **33**, 1830002 (2018).

[4] R.D. Peccei, H.R. Quinn, *Phys. Rev. Lett.* **38**, 1440 (1977).

[5] R.D. Peccei, H.R. Quinn, *Phys. Rev. D* **16**, 1791 (1977).

[6] M. Tanabashi *et al.* [Particle Data Group], *Phys. Rev. D* **98**, 030001 (2018).

[7] D. Horvatić, D. Klabučar, A. Radzhabov, *Phys. Rev. D* **76**, 096009 (2007).

[8] D. Horvatić, D. Blaschke, D. Klabučar, A.E. Radzhabov, *Phys. Part. Nucl.* **39**, 1033 (2008).

[9] D. Horvatić *et al.*, *Eur. Phys. J. A* **38**, 257 (2008).

[10] S. Benić, D. Horvatić, D. Kekez, D. Klabučar, *Phys. Rev. D* **84**, 016006 (2011).

[11] D. Horvatić, D. Kekez, D. Klabučar, *Phys. Rev. D* **99**, 014007 (2019).

[12] R. Alkofer, L. von Smekal, *Phys. Rep.* **353**, 281 (2001).

[13] C.D. Roberts, S.M. Schmidt, *Prog. Part. Nucl. Phys.* **45**, S1 (2000).

[14] C.S. Fischer, *J. Phys. G* **32**, R253 (2006).

[15] S. Weinberg, *Phys. Rev. Lett.* **40**, 223 (1978).

[16] F. Wilczek, *Phys. Rev. Lett.* **40**, 279 (1978).

[17] P. Di Vecchia, G. Veneziano, *Nucl. Phys. B* **171**, 253 (1980).

[18] H. Leutwyler, A.V. Smilga, *Phys. Rev. D* **46**, 5607 (1992).

[19] S. Dürr, *Nucl. Phys. B* **611**, 281 (2001).

[20] M. Gorghetto, G. Villadoro, *J. High Energy Phys.* **1903**, 033 (2019).

[21] P. Petreczky, H.P. Schadler, S. Sharma, *Phys. Lett. B* **762**, 498 (2016).

[22] S. Borsanyi *et al.*, *Nature* **539**, 69 (2016).

[23] D. Klabučar, D. Kekez, *Phys. Rev. D* **58**, 096003 (1998).

[24] D. Kekez, D. Klabučar, *Phys. Rev. D* **73**, 036002 (2006).

[25] S. Benić, D. Horvatić, D. Kekez, D. Klabučar, *Phys. Lett. B* **738**, 113 (2014).

[26] E. Witten, *Nucl. Phys. B* **156**, 269 (1979).

[27] G. Veneziano, *Nucl. Phys. B* **159**, 213 (1979).

[28] G.M. Shore, *Nucl. Phys. B* **744**, 34 (2006).

[29] Z.Y. Lu, M. Ruggieri, *Phys. Rev. D* **100**, 014013 (2019) [arXiv:1811.05102 [hep-ph]].

[30] D. Horvatić, D. Kekez, D. Klabučar, *Universe* **5**, 208 (2019) [arXiv:1909.09879 [hep-ph]].