

Prostori funkcija

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`\begin{sazetak}`

U ovom diplomskom radu proučavali smo prostore funkcija. U prvom uvodnom poglavlju podsjetili smo se osnovnih pojmova iz topologije. U drugom poglavlju bavili smo se prostorima omeđenih funkcija sa skupa A u metrički prostor (X, d) . Definirali smo metriku d_∞ na $B(A, (X, d))$ i vidjeli da je metrički prostor $(B(A, (X, d)), d_\infty)$ potpun. Također smo promatrali skup neprekidnih funkcija kao podskup prostora omeđenih funkcija i pokazali da je on zatvoren skup u $(B(A, (X, d)), d_\infty)$ te da je skup neprekidnih funkcija zajedno sa metrikom d_∞ restringiranom na njega također potpun. U trećem poglavlju uveli smo produktnu topologiju na produktu indeksirane familije topoloških prostora. Zatim smo pokazali da su koordinatne projekcije p_β neprekidne funkcije te da su otvorena preslikavanja. Zatim smo promatrali podbazu produktne topologije. Dobili smo također karakterizaciju neprekidnosti funkcije sa topološkog prostora Y u produkt topoloških prostora preko koordinatnih projekcija. Zatim smo, za skup X i topološki prostor Y , definirali topologiju otvorenu po točkama na Y^X . Zatim smo promatrali podbazu te topologije na Y^X . Zatim smo promatrali konvergenciju u topologiji otvorenoj po točkama i vidjeli da je konvergencija niza funkcija (f_n) funkciji g u topologiji otvorenoj po točkama ekvivalentna konvergenciji po točkama. Nadalje, u četvrtom poglavlju promatrali smo kompaktno otvorenu topologiju na Y^X , za dva topološka prostora X i Y . Vidjeli smo kojeg je oblika podbaza te topologije te smo pokazali da je Y^X redom T_0, T_1 ili T_2 prostor ako i samo ako je Y redom T_0, T_1 ili T_2 prostor. Također smo pokazali da, ukoliko je Y regularan prostor, tada je $C(X, Y)$ s kompaktno otvorenom topologijom, regularan prostor. Zatim smo vidjeli da se, za topološki prostor X i metrički prostor Y kompaktno otvorena topologija na $C((X, \tau), (Y, d))$ i topologija inducirana restrikcijom metrike d_∞ podudaraju. Nakon toga smo promatrali izravni produkt dva topološka prostora, i pokazali da je homeomorfan produktu indeksirane familije topoloških prostora, za dva skupa, kakav je bio općenito definiran u trećem poglavlju. Zatim smo promatrali lokalnu kompaktnost i dobili neke zanimljive rezultate koji povezuju lokalnu kompaktnost sa aksiomima separacije i sa neprekidnošću. Zatim smo promatrali produkt dva topološka prostora i, pored još nekih pomoćnih rezultate, dobili zanimljive rezultate o neprekidnosti funkcije f sa produkta dva topološka prostora u topološki prostor. U zadnjem poglavlju proučavali smo Hilbertove prostore. Nakon definiranja Hilbertovog prostora, pokazali smo da je on separabilan metrički prostor. Također smo pokazali da je potpun. Vidjeli smo također da Hilbertov prostor nema svojstvo lokalne kompaktnosti, te da je strogo konveksan bez grananja. Zatim smo promatrali potprostor Hilbertovog prostora, Hilbertovu kocku, i dobili zanimljiv rezultat o njezinoj homeomorfности sa jednim produktom topoloških prostora.

`\end{sazetak}`

`\begin{summary}`

In this diploma thesis we studied function spaces. In the first chapter we recalled some of the basic facts in topology. In the second chapter we were dealing with spaces of bounded functions from a set A to a metric space (X, d) . We defined metric d_∞ on $B(A, (X, d))$ and we saw that the metric space $(B(A, (X, d)), d_\infty)$ is a complete metric space. We also observed the set of

continuous functions as a subset of the space of bounded functions and we showed that it is a closed set in $(B(A, (X, d)), d_\infty)$ and that the set of continuous functions, together with the metrics d_∞ restricted to it, is also a complete metric space. In third chapter we introduced product topology on the product of indexed family of topological spaces. Then we showed that projection maps, p_β , are continuous functions and that they are open mappings. Then we observed the subbasis of product topology. We also saw a characterisation of continuity of a function from a topological space Y to a product of topological spaces, that uses coordinate projection maps. After that we defined product topology, or the point open topology, on Y^X , where X is a set and Y is a topological space. We showed what the subbasis of product topology on Y^X looks like. We then observed the convergence in point open topology and we saw that the convergence of a sequence of functions (f_n) to a function g in point open topology is equivalent to the point-wise convergence. Further, in the fourth chapter we watched the compact-open topology on Y^X , for two topological spaces X and Y . We saw what is the form of the subbasis of such topology, and we showed that Y^X is T_0, T_1 or T_2 space respectively if and only if Y is T_0, T_1 or T_2 space respectively. We also showed that, if Y is a regular space, then $C(X, Y)$ with compact-open topology is also a regular space. Then we saw that, for a topological space X and a metric space Y , compact-open topology on $C((X, \tau), (Y, d))$ coincides with the topology induced by the restriction of d_∞ . After that we observed the direct product of two topological spaces, and we showed that it is homeomorphic to the product of indexed family of topological spaces, for two sets, like it was defined in third chapter in general case. Then we observed the local compactness and we got some interesting results which connect local compactness to separation axioms and continuity. Then we observed the product of two topological spaces, and beside some auxiliary results, we got some interesting results about continuity of a function f from a product of two topological spaces to a third topological space, in context of a compact-open topology. In the last chapter we studied Hilbert spaces. After defining Hilbert space, we showed that it is a separable metric space. We also showed that it is a complete metric space. We also showed that Hilbert space doesn't have the property of local compactness, and that it is strictly convex without ramifications. Then we considered a subspace of Hilbert space, the Hilbert cube, and we showed an interesting result about homeomorphism from Hilbert cube to a product of topological spaces.

\end{summary}